MA3831 **Name:** ____________________

Final **C Term, 2003**

Show all work needed to reach your answers. You may use any theorem proven in our text or on the compactness sheet, but cite by page number or name any theorem that you use.

1. (30 points) Let a set $S \subset \mathbb{R}$ be *compact*. Please make three distinct statements about S that are equivalent to "compact". **Hint**: The first two are things we talked about a lot in class; the third you need to think about yourself. For the third, you may want to consider a sequence in S.

   (a) ____________________

   (b) ____________________

   (c) ____________________

2. (20 points) For each of the following sets $S \subset \mathbb{R}$, please give $\bar{S}$ (the closure of S), S^o (the interior of S), and ∂S (the boundary of S).

   (a) $S = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1, y > 0\}$

   $\bar{S} =$ ____________ $S^o =$ ____________ $\partial S =$ ____________

   (b) $S = \{x \in \mathbb{R} \mid x = 1/n \text{ for } n \in \mathbb{Z}^+\}$

   $\bar{S} =$ ____________ $S^o =$ ____________ $\partial S =$ ____________

   (c) $S = \mathbb{Q}$

   $\bar{S} =$ ____________ $S^o =$ ____________ $\partial S =$ ____________

   (d) If $\bar{S} = S^o$, what can be said about S?

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3. (25 points) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2 - x + y}{x - y} & x \neq y \\ a & x = y \end{cases}$$

where a is a constant. Considering the definition of continuity, please discuss as completely as possible the continuity of this function at all points $(x, y) \in \mathbb{R}^2$. For what (if any) value of a is f continuous at $(0, 0)$?

4. (25 points) Consider the sequence $\{p_n\} = \{p_1, p_2, p_3, ..., p_n, ...\}$. Suppose that this sequence is Cauchy and has a convergent subsequence $\{p_{n_k}\}$ which converges to a point p. Please prove or disprove: the sequence $\{p_n\}$ converges to p.