

Final

C Term, 2004

Show all work needed to reach your answers. You may use any theorem proven in our text or on the compactness sheet, but cite by page number or name any theorem that you use.

1. (30 points) Suppose that (E, d) and (E', d') are metric spaces. Let a function $f : E \rightarrow E'$ be *continuous* on E . Please make three distinct statements about f that are equivalent to “continuous”.

(a) _____

(b) _____

(c) _____

2. (20 points)

(a) Please explain precisely why $\mathbb{Q}$ is incomplete.

(b) Please explain precisely why $\mathbb{C}$ is not ordered.

3. (25 points) Please show that the set $S = \{ (x, y) \in \mathbb{R}^2 \mid y \leq -x \}$ is closed.

Hint: Construct an appropriate open ball.

4. (25 points) Let (E, d) be a metric space, and let F_1 and F_2 be nonempty, disjoint compact sets. Please show that there is a positive distance between these two sets, i.e., there exists $\epsilon > 0$ such that $d(p, q) > \epsilon$ for all $p \in F_1$ and $q \in F_2$.

Hint: Give a proof-by-contradiction; explain why there exists sequences of points from each set, $\{p_n\} \subset F_1$ and $\{q_n\} \subset F_2$, such that $d(p_n, q_n) < 1/n \forall n \in \mathbb{Z}^+$, then explain why these sequences are helpful.