MA3832 **Name:** ______________________________

Final **B Term, 2011**

Show all work needed to reach your answers. You may use any theorem proven in our text, but cite any theorem that you use by name or description.

1. (15 points) Suppose that $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b). If $f' \equiv 0$, please explain why f is constant.

2. (15 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous. Please compute $F'(x)$ for
$$F(x) := \int_{x^2}^{x} f(t)\, dt$$
and please cite which theorem(s) allows you to compute this derivative.

3. (20 points) Suppose that $\forall\, i \in \mathbb{Z}^+$, $a_i \in \mathbb{R}$. Please give ϵ–N definition for *convergence* for the series $\displaystyle\sum_{i=1}^{\infty} a_i$. **Hint**: What sequence should one consider?

4. (20 points) Please explain why the function $f : \mathbb{R} \to \mathbb{R}$ (defined below) is integrable on the interval $[0, 1]$. You may assume that the sine function is continuous, and you may cite the appropriate homework problem.

$$f(x) := \begin{cases} \sin \dfrac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

5. (20 points) Suppose that $\forall\ n \in \mathbb{Z}^+ \cup \{0\}$, $c_n \in \mathbb{R}$. For the power series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$, please show that if r is the radius of convergence, then

$$r = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right|$$

provided this limit exists.

6. (10 points) Please prove or give a counterexample: If a function $f : [a, b] \to \mathbb{R}$ is differentiable on (a, b) and strictly increasing on $[a, b]$, then $f'(x) > 0\ \forall\ x \in (a, b)$.