MA3832 **Name:** ______________________

Final **D Term, 2002**

Show all work needed to reach your answers. You may use any theorem proven in our text, but cite any theorem by page number that you use.

1. (30 points) Recall the definition of $\ln(x)$:
$$\ln(x) := \int_1^x \frac{dt}{t}$$
   (a) Please explain why $\ln(x)$ is positive for $x > 1$.

   (b) Please explain why $\ln(x)$ is increasing for $x > 0$.

   (c) Consider the function $f_n(x) := x^{1/n}$ for $n \in \mathbb{Z}^+$. Which grows faster, $\ln(x)$ or $f_n(x)$? Please justify your answer.

2. (25 points) Please use the definition of the derivative (for $x = 0$) and the rules for differentiation (for $x \neq 0$) to discuss the derivative of $f : \mathbb{R} \to \mathbb{R}$ defined as
$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$
Does its derivative exist and is it continuous? Please explain your answer. Is this function itself continuous? Again, please explain.

3. (15 points) Suppose that g is continuous on $[a, b]$ and differentiable on (a, b). If $g'(x) \neq 0 \; \forall\, x \in (a, b)$, please prove that g is one-to-one on (a, b). **Hint**: The definition of one-to-one is on p. 10 of our text.

4. (15 points) For $a_n \in \mathbb{R}$, suppose that $\sum_{n=0}^{\infty} a_n$ converges. Please show that $\sum_{n=0}^{\infty} a_n^2$ also converges.

5. (15 points) Suppose that $T : \mathbb{R} \to \mathbb{R}$ is continuous. Please explain why the following equality holds $\forall\, x \in \mathbb{R}$:

$$\int_{-h}^{x} \int_{-h}^{t} T(s)\, ds\, dt \;=\; x \int_{-h}^{x} T(s)\, ds \;-\; \int_{-h}^{x} sT(s)\, ds$$