MA3832 **Name:** ______________________

Final **D Term, 2003**

Show all work needed to reach your answers. You may use any theorem proven in our text, but cite any theorem by page number that you use.

1. (20 points) Suppose that $f, g : [a, b] \to \mathbb{R}$ are both Riemann integrable functions, and that $\exists\ m, M \in \mathbb{R}$ s.t. $m \leq f(x) \leq g(x) \leq M\ \forall\ x \in [a, b]$. What can be said about the integrals of f and g?

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2. (30 points) Please discuss the continuity and differentiability at $x = 0$ of the functions $f, g : \mathbb{R} \to \mathbb{R}$ (defined below). Is each function itself continuous at $x = 0$? Does its derivative exist and is it continuous? Please explain your answer.

(a)

$$g(x) := \begin{cases} x \exp(\frac{1}{x}) & x > 0 \\ 0 & x \leq 0 \end{cases}$$

(b)

$$f(x) := \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

3. (40 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable, $f < 0$, f is strictly increasing, and $\lim_{x\to\infty} f(x) = 0$.

   (a) Suppose that $\lim_{x\to\infty} f'(x)$ exist and is finite. Please show that $\lim_{x\to\infty} f'(x) = 0$ (i.e., give an ϵ–M proof).

   (b) Please give an example to show that the limit may not exist. **Hint**: A sketch of the graph of f and a brief explanation of the key properties is sufficient.

4. (10 points) Suppose that $a_1,\ a_2,\ a_3,\ .\ .\ .\ \ a_n,\ .\ .\ .$ is a *decreasing* sequence of positive reals. If

$$\sum_{n=1}^{\infty} a_n$$

converges, please show that $\lim_{n\to\infty} n\, a_n = 0$.