MA3832 **Name:** _____

Final **D Term, 2004**

Show all work needed to reach your answers. You may use any theorem proven in our text, but cite any theorem by name and/or page number that you use.

1. (20 points) Suppose that $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b). If $f' \equiv 0$, please explain why f is constant.

2. (25 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is uniformly continous, and then define $f_n(x) := f(x + \frac{1}{n})$. Please show that f_n is uniformly convergent (i.e., give an ϵ–M proof).

Proof: Since f is uniformly continous, given $\epsilon > 0$, _____ such that $\forall\, x, y \in \mathbb{R}$, if $|x - y|$ ____, then _____ . By LUB 2. (p. 26), _____ such that $\frac{1}{N} <$ ____. So if $n > N$, then _____. Thus $\forall\ x \in \mathbb{R}$ and $\forall\ n > N$, $|x - ($_____$)| <$ ____ implies _____. So _____ which implies $f_n \to f$ uniformly.

3. (25 points) Please discuss the continuity and differentiability at $x = 0$ of the function $f : \mathbb{R} \to \mathbb{R}$ (defined below). Is each function itself continuous at $x = 0$? Does its derivative exist and is it continuous? Please explain your answer.

$$f(x) := \begin{cases} \dfrac{\sin x}{\sqrt[3]{x}} & x \neq 0 \\ \\ 0 & x = 0 \end{cases}$$

4. (20 points) Consider the two series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} (a_n)^n$. What can be said about the convergence/divergence relationship between these two series? If either one converges or diverges, must the other also converge or diverge? You may give a counterexample to show that one series can diverge while the other converges.

5. (10 points) Consider the following indefinite integral (antiderivative):

$$2\int \cos x \sin x \, dx$$

On seeing this integral, one student makes the substitution $u = \cos x \Rightarrow du = -\sin x \, dx$, and, using the fundamental theorem of calculus, finds that the value of this integral is $-\cos^2 x + C$. But another student makes the substitution $u = \sin x \Rightarrow du = \cos x \, dx$, and finds that the value of this integral is $\sin^2 x + C$. Has either student made a mistake? Please explain why or why not.