MA3832 Name: *Solutions*

Quiz 1 B Term, 2011

Show all work needed to reach your answers.

1. (15 points) Consider the function

$$f(x) = \begin{cases} \cos x - 1 & x \neq 0 \\ 1 & x = 0 \end{cases}$$

1. Set up the limit which, assuming it exists, is $f'(0)$, the derivative of f at $x_0 = 0$.

<u>If</u> f is differentiable, then

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{(\cos x - 1) - (1)}{x}$$

$$= \lim_{x\to 0} \frac{\cos x - 2}{x} \quad (+10)$$

2. Is f differentiable at $x = 0$? Please explain why or why not.

But this limit DNE since the denominator goes to zero, while the numerator goes to -1. Also one can observe that f is not continuous at $x=0$, so it can not be differentiable either. (+5)

2. (10 points) Please prove that a differentiable real-valued function on $\mathbb{R}$ with bounded derivative is uniformly continuous.

Let f be the real-valued function in question. Since f' is bounded, $\exists M>0$ s.t. $|f'(x)|<M$ $\forall x \in \mathbb{R}$. (+2) Since f is differentiable on $\mathbb{R}$, it is also continuous on $\mathbb{R}$, (+2) then by the Mean Value theorem, $\forall x,y \in \mathbb{R}$, $x<y$, $\exists c \in (x,y)$ (+2) s.t. $f(y)-f(x) = f'(c)(y-x)$.
So $|f(y)-f(x)| = |f'(c)||y-x| \leq M|y-x|$. (+2) Thus given any $\epsilon>0$, let $\delta = \epsilon/M$. (+1) Then $|y-x|<\delta = \epsilon/M \Rightarrow |f(y)-f(x)|<\epsilon$.
Therefore f is (by definition) uniformly (+1) continuous.