MA3832 Name: Solutions

Quiz 2 B Term, 2011

Show all work needed to reach your answers.

1. (10 points) Suppose that $f, g : [a, b] \to \mathbb{R}$ are both Riemann integrable functions, and that $\exists\, m, M \in \mathbb{R}$ s.t. $m \le f(x) \le g(x) \le M \;\forall\, x \in [a, b]$. What can be said about the integrals of f and g?

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$$m(b-a) \le \int_a^b f(x)\,dx \le \int_a^b g(x)\,dx \le M(b-a)$$

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2. (15 points) Suppose that $f : [0, 1] \to \mathbb{R}$.
   (a) Please give an example of a partition P such that $|P| < 1/2$.

   5 $P = \{0, 1/3, 2/3, 1\}$

   Other choices are possible.

   (b) Please give the Riemann sum corresponding to your partition and using the right endpoints of each subinterval as the sampling points.

   5 $S = f(1/3)\frac{1}{3} + f(\frac{2}{3})\frac{1}{3} + f(1)\,1/3$

   (c) If f is integrable, then a certain limit must converge. What is this limit, and what types of partitions and sampling points must be considered?

5 If f is integrable, then

$$\lim_{|P|\to 0} \sum_{i=1}^{N} f(\xi_i)\,\Delta x_i$$

(+1)

exists, independent of the choice of sampling points $\xi_i \in [x_{i-1}, x_i]$ and for all sufficiently fine partitions.

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