MA3832 Name: Solutions

Quiz 4 B Term, 2011

Show all work needed to reach your answers.

(1) (~~10~~ 12 points) Please prove that $\log(1+x) < x$ for all $x > 0$. **Hint**: Apply the mean value theorem to $h(x) = x - \log(1+x)$.

Let $h(x) = x - \log(1+x)$; then $h'(x) = 1 - \frac{1}{1+x} = \frac{x}{1+x}$ (+2)

By the mean value theorem, (+2)

$$h(x) - h(0) = h'(c)(x-0) \quad \text{for some } c \in (0,x)$$ (+2) (+1)

$\Rightarrow h(x) = x\,h'(c)$. Since $h'(c) > 0\ \forall c > 0$ and $x > 0$ (+2) (+1)

$h(x) > 0 \Rightarrow \log(1+x) < x$ (+1) (+1)

QED

(2) (~~10~~ 13 points) Suppose $f_n : \mathbb{R} \to \mathbb{R}$ is uniformly continuous $\forall\, n$. If $f_n \to f$ uniformly, please show that f is also uniformly continuous.

Proof: Since $f_n \to f$ uniformly, given $\epsilon > 0, \exists\, N \in \mathbb{Z}^+$ such that $\forall\, x \in \mathbb{R}$, if $n > N$ (+1) then $|f_n(x) - f(x)| < \epsilon/3$ (+2). Now since for each n the function f_n is continuous, $\exists$ $\delta > 0$ (+1) such that $\forall\, x, y \in \mathbb{R}$, if $|x - y|$ $< \delta$ (+1), then $|f_n(x) - f_n(y)| < \epsilon/3$ (+2). Next using the triangle (+1) inequality, one finds that whenever $|x - y|$ $< \delta$ (+1) and n (+1) $> N$,

n.b.

$$|f(x) - f(y)| \leq |f(x) - f_n(x)| + |f_n(x) - f_n(y)| + |f_n(y) - f(y)|$$ (+2)

$$< \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} = \epsilon.$$ (+1)

So f is continuous. $\square$