MA3832 Name: Solutions

Quiz 5 B Term, 2011

Show all work needed to reach your answers.

1. (10 points) Suppose that $[a.b] \subset \mathbb{R}$ is a finite interval, and that $f_n : [a, b] \to \mathbb{R}$ is a sequence of functions. What conditions will guarantee that

$$\lim_{n\to\infty}\left(\int_a^b f_n(x)dx\right) = \int_a^b \left(\lim_{n\to\infty} f_n(x)\right) dx$$

Two conditions are sufficient:

6. The $\lim_{n\to\infty} f_n(x)$ converges uniformly.

4. The functions f_n are integrable.

This is the uniform convergence theorem.

2. (5 points) Please give an example of a series that is convergent, but not absolutely convergent.

5/ $\sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{i}$ This is the alternating harmonic series;

$\sum_{i=1}^{\infty} \left|\frac{(-1)^{i-1}}{i}\right| = \sum_{i=1}^{\infty} \frac{1}{i}$ is the harmonic series which diverges.

3. (10 points) Suppose that for the series $\sum_{n=1}^{\infty} a_n$, there is a real number $\rho < 1$ such that $|a_n|^{\frac{1}{n}} < \rho$. Please show that the series converges absolutely. **Hint**: Comparison test to a geometric series.

Since $|a_n|^{\frac{1}{n}} < \rho$, we know that $|a_n| < \rho^n$. (+2) Since $\rho < 1$, $\sum_{n=1}^{\infty} \rho^n$ (+1) $= \frac{\rho}{1-\rho}$ is a convergent geometric series. (+3) So by the comparison test, (+1) $\sum_{n=1}^{\infty} |a_n|$ (+2) converges, and thus by definition $\sum_{n=1}^{\infty} a_n$ converges absolutely. (+1)