MA4473 Name: *Solutions*

Final, Part 1 B Term, 2016

1. (15 points) Please define/describe/state each of the following:

   (a) Dirichlet Conditions

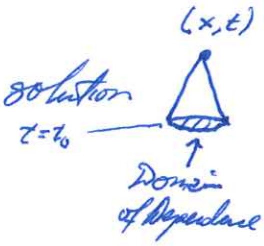
   Boundary conditions for a PDE-system whereby the solution u is specified on the boundary.

   (b) Wave Equation

   $u_{tt} = c^2 \Delta u$

   (c) Domain of Dependence

   The region in the $t=t_0$ hyperplane which can effect the solution at (x,t).

   (d) Test Function

   Any $\varphi \in C_0^\infty(\Omega)$ for some Ω.

   (e) Soliton

   A solitary wave solution where $u(x,t) = f(x-ct)$ for some function f and some constant c.

2. (10 points) Suppose that u satisfies the heat equation ($u_t = u_{xx}$) for $0 < x < 1$ and $t > 0$. What does the strong maximum principle guarantee?

   The strong maximum principle states that unless u is constant, then the maximum (and the minimum) of u occurs on the lower boundary: $\{(x,t) \mid x=0 \text{ or } x=1 \text{ or } t=0\}$.

3. (10 points) Please explain the Dirichlet principle.

   Given a bounded domain Ω, among all functions satisfying the Dirichlet BC $u(\vec{x}) = h(\vec{x})$ for $\vec{x} \in \partial\Omega$, the lowest energy is achieved when u is harmonic on Ω: $\forall w$, $E[u] \le E[w]$ when $\Delta u = 0$.

4. (15 points) Please describe three major differences that one would expect between solutions to hyperbolic equations (e.g., the wave equations) and parabolic equations (e.g., the heat equations).

	Hyperbolic	Parabolic
①	Finite Propagation Speed.	Infinite Propagation Speed.
②	Jumps/Shocks Preserved	Jumps/Shocks immediately smoothed out.
③	Energy conserved	Energy dissipated.
④	No maximum principle	Strong Maximum principle