MA4473 **Name:** ______________________________

Final, Part 2 (open book and notes, but no quizzes/homework) D Term, 2005

Show all work needed to reach your answers. You may use any theorem we discussed in the course, but please cite any result you use.

1. (25 points) Consider the following heat-flow system:

$$\begin{array}{rcl} \text{PDE:} & u_t = \kappa \triangle u & \boldsymbol{x} \in \Omega, \quad t > 0 \\ \text{BC:} & u(x,t) = 0 & \boldsymbol{x} \in \partial\Omega, \ t > 0 \\ \text{IC:} & u(x,0) = u_0(x) & \boldsymbol{x} \in \Omega \end{array}$$

where $\kappa > 0$ is constant and u_0 is a given function. Suppose that *energy* for this system is defined as

$$E(u) := \frac{1}{2}\int_\Omega u^2 dV$$

where the single integral sign represents a multiple integral. Please show that E is decreasing. What is the maximum value of this energy?

2. (25 points) For $\boldsymbol{x} \in \mathbb{R}^3$, please give the solution (in symbols) of the following wave problem:

$$\text{PDE:} \qquad \Box u = 0 \qquad\qquad \boldsymbol{x} \in \mathbb{R}^3,\ t > 0$$

$$\text{IC:} \quad \begin{array}{rl} u(x,0) \equiv 0 & \qquad \boldsymbol{x} \in \mathbb{R}^3 \\ u_t(x,0) = 1 & \qquad |\boldsymbol{x}| = 1 \\ u_t(x,0) = 0 & \qquad |\boldsymbol{x}| \neq 1 \end{array}$$

So if u represents light, the initial conditions are zero everywhere except on the spherical shell with radius 1 centered at the origin where at time $t = 0$ there is a flash (nonzero time derivative of u). Please describe in words what the solution looks like.