MA500 **Name:** _____

Midterm (closed book, closed notes, no internet, no discussion) **Fall 2014**

I affirm that I have not consulted my text, notes or any reference, paper or electronic, or any person once I opened the envelope and began this midterm exam.

Signature: _____

Show all work needed to reach your answers. You may use any result we discussed in class, but cite by name any named result you use.

1. (15 points) Suppose that $f : X \to Y$ and that A and B are subsets of X. Please show that $f(A \cap B) \subset f(A) \cap f(B)$.

2. (15 points) Suppose $S \subset \mathbb{R}$ is a nonempty set that is bounded below. Can S have more than one greatest lower bound? Please explain in detail why or why not.

3. (20 points) Consider the sequence of real numbers

$$\frac{1}{4}, \quad \frac{1}{4+\frac{1}{4}}, \quad \frac{1}{4+\frac{1}{4+\frac{1}{4}}}, \quad \ldots$$

Please show that this sequence is convergent and find its limit.

4. (20 points) Let (E, d) be any metric space. If $F \subset E$, then F is closed iff F contains its cluster points (limit points).

5. (15 points) Let (E, d_0) be a metric space, and for any $x, y \in E$, define

$$d(x, y) := 2d_0(x, y)\,.$$

Please prove or disprove in detail that (E, d) is also a metric space.

6. (15 points) Consider the set $S = \{0, 1\}$ along with the operations $+$ and $\times$ defined by the tables below. With these operations, S is a field. Can this field be ordered? Please explain in detail why or why not.

$+$	0	1
0	0	1
1	1	0

$\times$	0	1
0	0	0
1	0	1