Confidence Intervals for the Mean of A Single Population

Case 1: Known Variance

Assumptions

1. The data are $Y_1, Y_2, \ldots, Y_n$ where $Y_j = \mu + \epsilon_j$.
2. Either
   (a) n is large, or
   (b) The ϵ_j are from a $N(0, \sigma^2)$ population.
3. σ^2 is known.

Formulas

A level L confidence interval for μ is $\left(\overline{Y} - \sigma(\overline{Y})z_{\frac{1+L}{2}},\ \overline{Y} + \sigma(\overline{Y})z_{\frac{1+L}{2}}\right)$, where $\sigma(\overline{Y}) = \sqrt{\frac{\sigma^2}{n}}$, and $z_{\frac{1+L}{2}}$ may be obtained from a table of normal quantiles (click here).

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Case 2: Unknown Variance, n Large

This case is treated exactly as Case 1 except that $\sigma(\overline{Y})$ is replaced by $\hat{\sigma}(\overline{Y}) = \sqrt{\frac{S^2}{n}}$, where S is the sample standard deviation.

Case 3: Unknown Variance, n Small

Assumptions

1. The data are $Y_1, Y_2, \ldots, Y_n$ where $Y_j = \mu + \epsilon_j$.
2. The ϵ_j are from a $N(0, \sigma^2)$ population.
3. σ^2 is unknown.

Formulas

A level L confidence interval for μ is $\left(\overline{Y} - \hat{\sigma}(\overline{Y})t_{n-1,\frac{1+L}{2}},\ \overline{Y} + \hat{\sigma}(\overline{Y})t_{n-1,\frac{1+L}{2}}\right)$, where $\hat{\sigma}(\overline{Y})$ is defined as in Case 2, and $t_{n-1,\frac{1+L}{2}}$ may be obtained from a table of quantiles of the t distribution (click here).

Click Here for An Example