Comparing Two Population Means: Independent Populations

A company buys cutting blades used in its manufacturing process from two suppliers. In order to decide if there is a difference in blade life, the lifetimes of 10 blades from manufacturer 1 and 13 blades from manufacturer 2 used in the same application are compared. A summary of the data shows the following (units are hours):

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>n</th>
<th>( \bar{y} )</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>118.4</td>
<td>26.9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>134.9</td>
<td>18.4</td>
</tr>
</tbody>
</table>

The investigators generated histograms and normal quantile plots of the two data sets and found no evidence of nonnormality or outliers. The point estimate of \( \mu_1 - \mu_2 \) is \( \bar{y}_1 - \bar{y}_2 = 118.4 - 134.9 = -16.5 \). They decided to obtain a level 0.90 confidence interval to compare the mean lifetimes of blades from the two manufacturers.

- **Pooled variance interval** The pooled variance estimate is

  \[
  s_p^2 = \frac{(10 - 1)(26.9)^2 + (13 - 1)(18.4)^2}{10 + 13 - 2} = 503.6.
  \]

  This gives the estimate of the standard error of \( \bar{Y}_1 - \bar{Y}_2 \) as

  \[
  \hat{\sigma}_p(\bar{Y}_1 - \bar{Y}_2) = \sqrt{503.6 \left( \frac{1}{10} + \frac{1}{13} \right)} = 9.44.
  \]

  Finally, \( t_{21,0.95} = 1.7207 \). So a level 0.90 confidence interval for \( \mu_1 - \mu_2 \) is

  \[
  (-16.5 - (9.44)(1.7207), -16.5 + (9.44)(1.7207))
  \]

  \[
  = (-32.7, -0.3).
  \]

- **Separate variance interval** The estimate of the standard error of \( \bar{Y}_1 - \bar{Y}_2 \) is

  \[
  \hat{\sigma}(\bar{Y}_1 - \bar{Y}_2) = \sqrt{\frac{(26.9)^2}{10} + \frac{(18.4)^2}{13}} = 9.92.
  \]

  The degrees of freedom \( \nu \) is computed as the greatest integer less than or equal to

  \[
  \frac{(26.9)^2}{10 - 1} + \frac{(18.4)^2}{13 - 1} = 15.17,
  \]

  so \( \nu = 15 \). Finally, \( t_{15,0.95} = 1.7530 \). So a level 0.90 confidence interval for \( \mu_1 - \mu_2 \) is

  \[
  (-16.5 - (9.92)(1.753), -16.5 + (9.92)(1.753))
  \]

  \[
  = (-33.9, 0.89).
  \]

There seems to be a problem here. The pooled variance interval, \((-32.7, -0.3)\), does not contain 0, and so suggests that \( \mu_1 \neq \mu_2 \). On the other hand, the separate variance interval, \((-33.9, 0.89)\), contains 0, and so suggests we cannot conclude that \( \mu_1 \neq \mu_2 \). What to do?

Since both intervals are similar and have upper limits very close to 0, I would suggest taking more data to resolve the ambiguity.