

# Chapter 1

## Tables

### 1.1 Binomial Probabilities

Tabulated is the value

$$P(Y \leq y) = \sum_{x=0}^y \binom{n}{x} p^x (1-p)^{n-x},$$

where  $Y \sim b(n, p)$ .

- To obtain the value of  $P(Y \leq y)$  when  $p > 0.50$ , use the formula

$$P(Y \leq y) = 1 - P(W \leq n - y - 1),$$

where  $W \sim b(n, 1 - p)$ .

- To obtain the value of the probability mass function,  $p_Y(y)$ , use the formula

$$\begin{aligned} p_Y(y) &= P(Y \leq 0), y = 0, \\ &= P(Y \leq y) - P(Y \leq y - 1), y > 0. \end{aligned}$$





n	y	p																		
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	
20	0	0.8179	0.6676	0.5438	0.4420	0.3585	0.2901	0.2342	0.1887	0.1516	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000	
	1	0.9831	0.9401	0.8802	0.8103	0.7358	0.6605	0.5869	0.5169	0.4516	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000	
	2	0.9990	0.9929	0.9790	0.9561	0.9245	0.8850	0.8390	0.7879	0.7334	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002	
	3	1.0000	0.9994	0.9973	0.9926	0.9841	0.9710	0.9529	0.9294	0.9007	0.8677	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013	
	4	1.0000	1.0000	0.9997	0.9990	0.9974	0.9944	0.9893	0.9817	0.9710	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059	
	5	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207	
	6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577	
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316	
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987	0.9900	0.9591	0.8667	0.7624	0.5956	0.4143	0.2517	
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119	
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881	
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483	
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684	
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9835	0.9786	0.9423	
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9884	0.9936	0.9793	
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9884	0.9936	0.9793	
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9884	0.9936	0.9793	
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9884	0.9936	0.9793	
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9884	0.9936	0.9793	
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9884	0.9936	0.9793	
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9884	0.9936	0.9793	
25	0	0.7778	0.6135	0.4670	0.3604	0.2774	0.2129	0.1630	0.1244	0.0946	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	
	1	0.9742	0.9114	0.8280	0.7358	0.6424	0.5527	0.4696	0.3947	0.3286	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000	
	2	0.9980	0.9868	0.9620	0.9235	0.8729	0.8129	0.7466	0.6768	0.6063	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000	
	3	0.9999	0.9986	0.9938	0.9835	0.9659	0.9402	0.9064	0.8649	0.8169	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001	
	4	1.0000	0.9999	0.9992	0.9972	0.9928	0.9850	0.9726	0.9549	0.9314	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005	
	5	1.0000	1.0000	0.9999	0.9996	0.9988	0.9969	0.9935	0.9877	0.9790	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020	
	6	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9987	0.9972	0.9946	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073	
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9989	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216	
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539	
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148	
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122	
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450	
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000	
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550	
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878	
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9985	0.9971	0.9868	0.9560	0.8852	
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9957	0.9826	0.9461	0.9784	
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9957	0.9826	0.9461	

## 1.2 Poisson Probabilities

Tabulated is the value

$$P(Y \leq y) = \sum_{x=0}^y \frac{\lambda^x e^{-\lambda}}{x!},$$

where  $Y$  has a Poisson distribution with parameter  $\lambda$ .

To obtain the value of the probability mass function,  $p_Y(y)$ , use the formula

$$\begin{aligned} p_Y(y) &= P(Y \leq 0), y = 0, \\ &= P(Y \leq y) - P(Y \leq y - 1), y > 0. \end{aligned}$$



### 1.3 Areas Under the Standard Normal Density

Tabled are the areas under the  $N(0, 1)$  density curve below  $z$ . This represents the proportion of a  $N(0, 1)$  population which takes on values less than or equal to  $z$ . The tabled value is illustrated by the shaded region in Figure 1.1.

As examples, the proportion of a  $N(0, 1)$  population which takes on values less than or equal to  $-1.83$  is .0336, and the proportion of a  $N(0, 1)$  population which takes on values less than or equal to  $2.38$  is .9913.

By reading the table in reverse, the value  $z_p$ , below which lies a proportion  $p$  of a  $N(0, 1)$  population may be found. For example, to find the value  $z_{.25}$ , the point below which lies proportion .25 (or  $1/4$ ) of a  $N(0, 1)$  population, look in the body of the table to find values as close to .25 as possible. The two values are .2514, corresponding to  $z = -.67$ , and .2483, corresponding to  $z = -.68$ . Therefore,  $z_{.25}$  lies between  $-.67$  and  $-.68$ . As a second example,  $z_{.84}$  lies between .99 and 1.00.

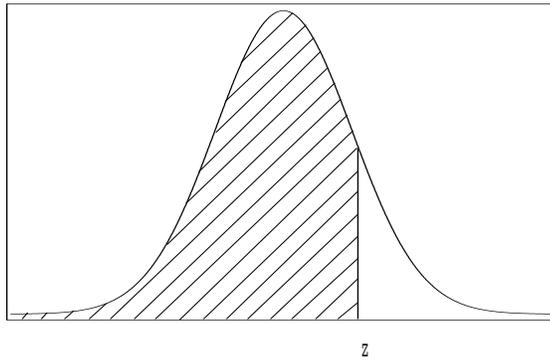


Figure 1.1:  $N(0, 1)$  Curve: Shaded Area is Tabled

$N(0, 1)$ Probabilities										
$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0014	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0227	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1094	.1075	.1057	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1921	.1894	.1867
-0.7	.2420	.2388	.2358	.2327	.2297	.2266	.2236	.2207	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2482	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4841	.4801	.4761	.4721	.4681	.4641



## 1.4 Critical Values of the $t$ Distribution

The critical value  $t_{k,q}$  is the value below which lies an area  $q$  under the density curve of the  $t$  distribution with  $k$  degrees of freedom. That is, quantile  $q$  of the  $t_k$  distribution. Tabled are these critical values  $t_{k,q}$  for selected degrees of freedom  $k$  and quantiles  $q$ . This is shown graphically in Figure 1.2, in which the curve represents a  $t_k$  density curve. The critical value  $t_{k,q}$  is the value  $t$  in the figure for which the shaded area equals  $q$ .

Note that a  $t$  distribution with degrees of freedom  $\infty$  is a  $N(0,1)$  distribution.

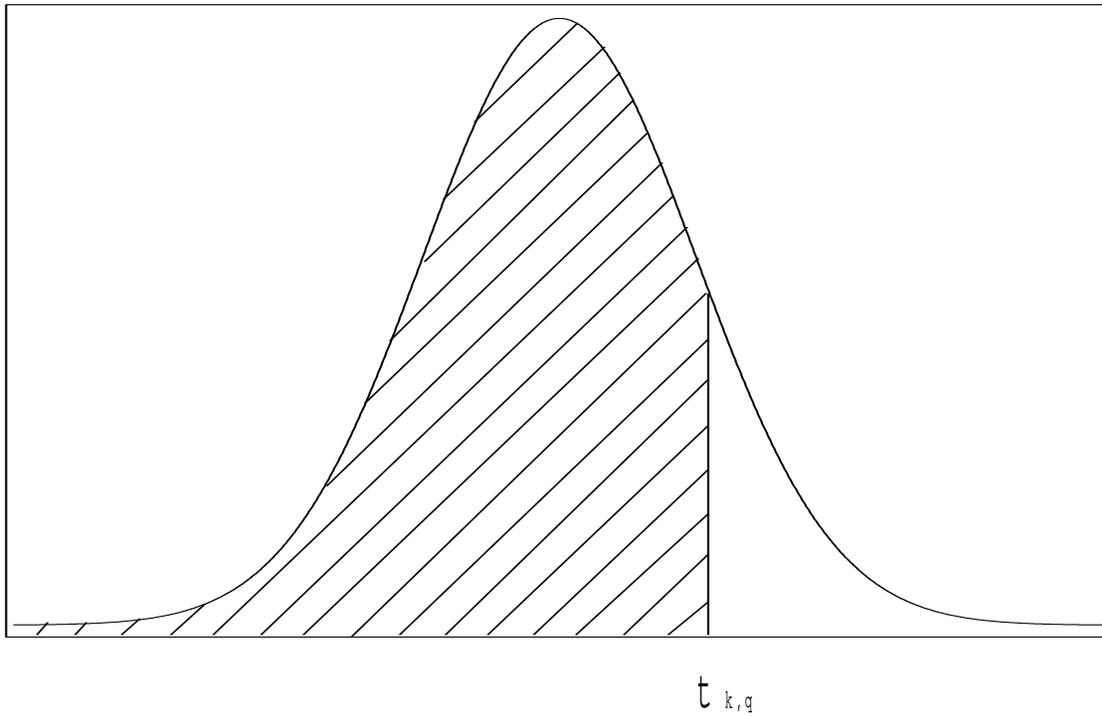


Figure 1.2:  $t_k$  Curve: Shaded Area is  $q$ ;  $t_{k,q}$  is Tabled

		Critical Values of the t Distribution						
Degrees of Freedom, $k$	$t_{k,.90}$	$t_{k,.95}$	$t_{k,.975}$	$t_{k,.99}$	$t_{k,.995}$	$t_{k,.999}$	$t_{k,.9995}$	
1	3.0777	6.3137	12.7062	31.8205	63.6567	318.3090	636.6190	
2	1.8856	2.9200	4.3027	6.9646	9.9248	22.3270	31.5990	
3	1.6377	2.3534	3.1824	4.5407	5.8409	10.2150	12.9240	
4	1.5332	2.1319	2.7764	3.7469	4.6041	7.1730	8.6100	
5	1.4759	2.0150	2.5706	3.3649	4.0321	5.8930	6.8690	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.2080	5.9590	
7	1.4149	1.8946	2.3646	2.9980	3.4995	4.7850	5.4080	
8	1.3968	1.8595	2.3060	2.8965	3.3554	4.5010	5.0410	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.2970	4.7810	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.1440	4.5870	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.0250	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	3.9300	4.3180	
13	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2210	
14	1.3450	1.7613	2.1448	2.6245	2.9768	3.7870	4.1400	
15	1.3406	1.7530	2.1314	2.6025	2.9467	3.7330	4.0730	
16	1.3368	1.7459	2.1199	2.5835	2.9208	3.6860	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.6460	3.9650	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.6100	3.9220	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.5790	3.8830	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.5520	3.8500	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.5270	3.8190	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7920	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7680	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.4670	3.7450	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.4500	3.7250	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460	
35	1.3062	1.6896	2.0301	2.4377	2.7238	3.3400	3.5912	
40	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510	
50	1.2987	1.6759	2.0086	2.4033	2.6778	3.2614	3.4960	
60	1.2958	1.6707	2.0003	2.3901	2.6603	3.2317	3.4602	
70	1.2938	1.6669	1.9944	2.3808	2.6479	3.2108	3.4350	
80	1.2922	1.6641	1.9901	2.3739	2.6387	3.1953	3.4163	
90	1.2910	1.6620	1.9867	2.3685	2.6316	3.1833	3.4019	
100	1.2901	1.6602	1.9840	2.3642	2.6259	3.1737	3.3905	
$\infty$	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	

## 1.5 Critical Values of the $\chi^2$ Distribution

The critical value  $\chi_{k,q}^2$  is the value below which lies an area  $q$  under the density curve of the  $\chi^2$  distribution with  $k$  degrees of freedom. That is, quantile  $q$  of the  $\chi_k^2$  distribution. Tabled are these critical values  $\chi_{k,q}^2$  for selected degrees of freedom  $k$  and quantiles  $q$ . This is shown graphically in Figure 1.3, in which the curve represents a  $\chi_k^2$  density curve. The critical value  $\chi_{k,q}^2$  is the value  $\chi^2$  in the figure for which the shaded area equals  $q$ .

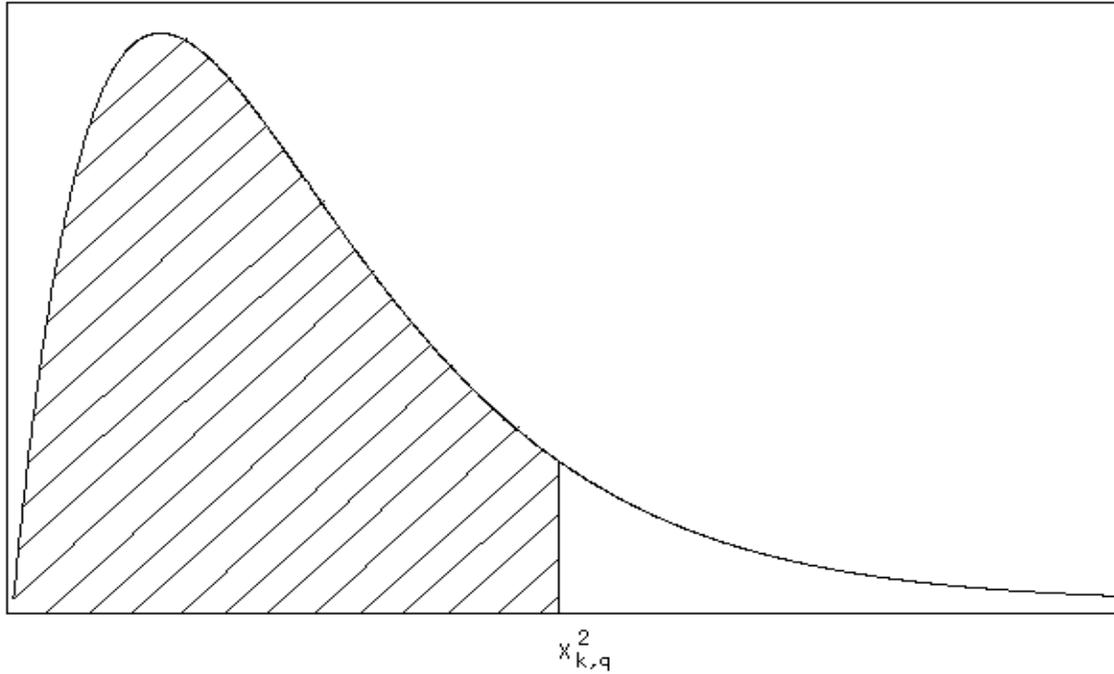


Figure 1.3:  $\chi_k^2$  Curve: Shaded Area is  $q$ ;  $\chi_{k,q}^2$  is Tabled

Critical Values of the $\chi^2$ Distribution										
$k$	$\chi^2_{k,0.005}$	$\chi^2_{k,0.010}$	$\chi^2_{k,0.025}$	$\chi^2_{k,0.050}$	$\chi^2_{k,0.100}$	$\chi^2_{k,0.900}$	$\chi^2_{k,0.950}$	$\chi^2_{k,0.975}$	$\chi^2_{k,0.990}$	$\chi^2_{k,0.995}$
1	0.000 <sup>1</sup>	0.000 <sup>2</sup>	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.60
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09	16.75
6	0.676	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81	18.55
7	0.989	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	20.28
8	1.344	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	21.95
9	1.735	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.034	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.643	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.260	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.886	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	17.19	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
45	24.31	25.90	28.37	30.61	33.35	57.51	61.66	65.41	69.96	73.17
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

<sup>1</sup>This value is actually  $3.9 \times 10^{-5}$ .<sup>2</sup>This value is actually  $1.6 \times 10^{-4}$ .

## 1.6 Critical Values of the $F$ Distribution

The critical value  $F_{k,l;q}$  is the value below which lies an area  $q$  under the density curve of the  $F$  distribution with  $k$  and  $l$  degrees of freedom. That is, quantile  $q$  of the  $F_{k,l}$  distribution. Tabled are these critical values  $F_{k,l;q}$  for selected degrees of freedom  $k$  and  $l$  and quantiles  $q$ . This is shown graphically in Figure 1.4, in which the curve represents a  $F_{k,l}$  density curve. The critical value  $F_{k,l;q}$  is the value  $F$  in the figure for which the shaded area equals  $q$ .

Two tables are given, the first for  $q = 0.99$  and the second for  $q = 0.95$ .

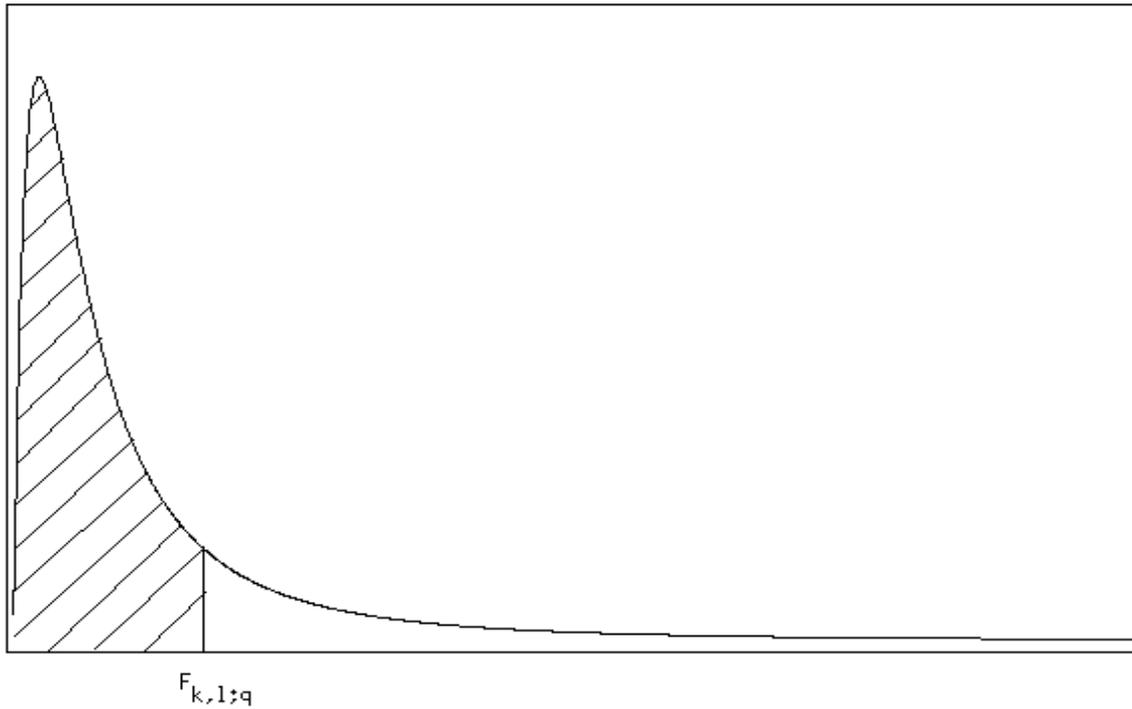


Figure 1.4:  $F$  Curve: Shaded Area is  $q$ ;  $F_{k,l;q}$  is Tabled

Critical Values of the F Distribution:  $F_{k, l; 0.99}$

Denominator Degrees of Freedom <i>l</i>	Numerator Degrees of Freedom, <i>k</i>																	
	1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	50	100	$\infty$
1	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6157	6209	6240	6261	6287	6303	6334	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	26.87	26.69	26.58	26.50	26.41	26.35	26.24	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02	13.91	13.84	13.75	13.69	13.58	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.722	9.553	9.449	9.379	9.291	9.238	9.130	9.020
6	13.75	10.92	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874	7.559	7.396	7.229	7.143	7.091	6.987	6.880	6.775
7	12.25	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620	6.314	6.155	6.058	5.992	5.908	5.858	5.755	5.650
8	11.26	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.515	5.359	5.263	5.198	5.116	5.065	4.963	4.859
9	10.56	8.022	6.992	6.422	6.057	5.805	5.613	5.467	5.351	5.257	4.962	4.808	4.713	4.649	4.567	4.517	4.415	4.311
10	10.04	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849	4.558	4.405	4.311	4.247	4.165	4.115	4.014	3.909
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.251	4.099	4.005	3.941	3.860	3.810	3.708	3.602
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296	4.010	3.858	3.765	3.701	3.619	3.569	3.467	3.361
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100	3.815	3.665	3.571	3.507	3.425	3.375	3.272	3.165
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939	3.656	3.505	3.412	3.348	3.266	3.215	3.112	3.004
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.522	3.372	3.278	3.214	3.132	3.081	2.977	2.868
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.889	3.780	3.691	3.409	3.259	3.165	3.101	3.018	2.967	2.863	2.753
17	8.400	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593	3.312	3.162	3.068	3.003	2.920	2.869	2.764	2.653
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.227	3.077	2.983	2.919	2.835	2.784	2.678	2.566
19	8.185	5.926	5.010	4.500	4.171	3.939	3.765	3.631	3.523	3.434	3.153	3.003	2.909	2.844	2.761	2.709	2.602	2.489
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.088	2.938	2.843	2.778	2.695	2.643	2.535	2.421
21	8.017	5.780	4.874	4.369	4.042	3.812	3.640	3.506	3.398	3.310	3.030	2.880	2.785	2.720	2.636	2.584	2.475	2.360
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	2.978	2.827	2.733	2.667	2.583	2.531	2.422	2.305
23	7.881	5.664	4.765	4.264	3.939	3.710	3.539	3.406	3.299	3.211	2.931	2.781	2.686	2.620	2.535	2.483	2.373	2.256
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	2.889	2.738	2.643	2.577	2.492	2.440	2.329	2.211
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	2.850	2.699	2.604	2.538	2.453	2.400	2.289	2.169
26	7.721	5.526	4.637	4.140	3.818	3.591	3.421	3.288	3.182	3.094	2.815	2.664	2.569	2.503	2.417	2.364	2.252	2.131
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062	2.783	2.632	2.536	2.470	2.384	2.330	2.218	2.097
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.120	3.032	2.753	2.602	2.506	2.440	2.354	2.300	2.187	2.064
29	7.598	5.420	4.538	4.045	3.725	3.499	3.330	3.198	3.092	3.005	2.726	2.574	2.478	2.412	2.325	2.271	2.158	2.034
30	7.562	5.390	4.510	4.018	3.699	3.473	3.304	3.173	3.067	2.979	2.700	2.549	2.453	2.386	2.299	2.245	2.131	2.006
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.522	2.369	2.271	2.203	2.114	2.058	1.938	1.805
50	7.171	5.057	4.199	3.720	3.408	3.186	3.020	2.890	2.785	2.698	2.419	2.265	2.167	2.098	2.007	1.949	1.825	1.683
100	6.895	4.824	3.984	3.513	3.206	2.988	2.823	2.694	2.590	2.503	2.223	2.067	1.965	1.893	1.797	1.735	1.598	1.427
$\infty$	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321	2.039	1.878	1.773	1.696	1.592	1.523	1.358	1.001

Critical Values of the  $F$  Distribution:  $F_{k, l; 0.95}$

Denominator Degrees of Freedom $l$	Numerator Degrees of Freedom, $k$																		
	1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	50	100	$\infty$	
1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9	245.9	248.0	249.3	250.1	251.1	251.8	253.0	254.3	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.50	
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.703	8.660	8.634	8.617	8.594	8.581	8.554	8.526	
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.858	5.803	5.769	5.746	5.717	5.699	5.664	5.628	
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.619	4.558	4.521	4.496	4.464	4.444	4.405	4.365	
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	3.938	3.874	3.835	3.808	3.774	3.754	3.712	3.669	
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.511	3.445	3.404	3.376	3.340	3.319	3.275	3.230	
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347	3.218	3.150	3.108	3.079	3.043	3.020	2.975	2.928	
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.006	2.936	2.893	2.864	2.826	2.803	2.756	2.707	
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.845	2.774	2.730	2.700	2.661	2.637	2.588	2.538	
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.719	2.646	2.601	2.570	2.531	2.507	2.457	2.404	
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.617	2.544	2.498	2.466	2.426	2.401	2.350	2.296	
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.533	2.459	2.412	2.380	2.339	2.314	2.261	2.206	
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.463	2.388	2.341	2.308	2.266	2.241	2.187	2.131	
15	4.543	3.682	3.287	3.055	2.901	2.790	2.707	2.641	2.588	2.544	2.403	2.328	2.280	2.247	2.204	2.178	2.123	2.066	
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.352	2.276	2.227	2.194	2.151	2.124	2.068	2.010	
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.308	2.230	2.181	2.148	2.104	2.077	2.020	1.960	
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.269	2.191	2.141	2.107	2.063	2.035	1.978	1.917	
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.234	2.155	2.105	2.071	2.026	1.999	1.940	1.878	
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.203	2.124	2.074	2.039	1.994	1.966	1.907	1.843	
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321	2.176	2.096	2.045	2.010	1.965	1.936	1.876	1.812	
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.151	2.071	2.020	1.984	1.938	1.909	1.849	1.783	
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.128	2.048	1.996	1.960	1.914	1.885	1.823	1.757	
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.108	2.027	1.975	1.939	1.892	1.863	1.800	1.733	
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.089	2.007	1.955	1.919	1.872	1.842	1.779	1.711	
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220	2.072	1.990	1.938	1.901	1.853	1.823	1.760	1.691	
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.056	1.974	1.921	1.884	1.836	1.806	1.742	1.672	
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.041	1.959	1.906	1.869	1.820	1.790	1.725	1.654	
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.027	1.945	1.891	1.854	1.806	1.775	1.710	1.638	
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.015	1.932	1.878	1.841	1.792	1.761	1.695	1.622	
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077	1.924	1.839	1.783	1.744	1.693	1.660	1.599	1.509	
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.073	2.026	1.871	1.784	1.727	1.687	1.634	1.599	1.525	1.438	
100	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032	1.975	1.927	1.768	1.676	1.616	1.573	1.515	1.477	1.392	1.283	
$\infty$	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.666	1.571	1.506	1.459	1.394	1.350	1.243	1.001	

## 1.7 Table of Random Digits

Row	Table of Random Digits							
1	489603	272922	751103	838823	401597	317816	790344	068359
2	041863	967007	569984	953158	757081	448552	017030	351661
3	513514	228641	557839	698999	469379	271538	616180	253473
4	904689	406237	701811	371597	516605	980334	958803	386477
5	528769	757338	937577	396178	253543	971588	786185	202369
6	788073	386933	609113	120803	916516	361977	796978	426263
7	639884	576786	965372	471465	757173	308181	251927	427225
8	673943	180225	957878	379661	525909	370665	761640	235731
9	854865	608652	918005	143028	728641	299685	123340	047942
10	915293	464235	335633	907463	957808	660544	342004	919893
11	437657	025151	092608	802778	873908	776887	268507	554492
12	698639	806163	348833	561221	348944	085930	937793	661819
13	077082	487109	812230	233418	329963	467593	935007	369100
14	724791	790427	045150	299427	483723	151689	825020	550721
15	915804	020017	035659	014875	136581	914757	251523	324302
16	801090	776488	758060	009593	953851	051906	191535	835793
17	541139	180959	688369	587411	670140	060728	063536	981934
18	761488	953842	244255	441300	202967	416538	027735	980551
19	715543	367628	312767	823131	003390	494916	169074	591611
20	836362	701590	717950	011142	927065	873018	025973	688799
21	647717	348660	156030	994120	130391	472637	779721	993061
22	170765	651215	544782	920823	577834	397489	174953	623506
23	584722	606916	737474	237188	444032	724970	805305	583948
24	571153	600700	323852	739006	869074	001168	583831	487444
25	641015	162408	228825	962199	126730	078131	867864	378588
26	158144	143895	140256	741463	953107	633758	167651	387283
27	329018	685704	774802	545398	907609	507065	399132	891188
28	629521	024388	771689	252067	629788	190523	526414	879135
29	702713	905189	986457	123364	112119	548757	358302	062234
30	673654	489334	901445	556856	301996	488234	045365	083238
31	929213	326126	030722	710942	562356	379837	899403	646112
32	472826	011733	962183	700323	878016	611298	636870	963250
33	154724	137837	974746	892535	109686	840304	927262	387193
34	415016	095417	745763	464257	976284	946303	640398	396757
35	472006	147412	658634	608750	600425	959376	739741	300837
36	970761	990010	908518	934971	839006	661593	254040	602082
37	059485	462195	165393	770430	840679	665101	101645	228384
38	299570	508640	211724	080373	987658	472198	932342	169682
39	338696	350187	234563	573347	969443	123359	585432	823584
40	464579	243007	059461	676200	654081	342355	193796	324126

## Using the Table of Random Digits

The numbers in the table of random digits are completely random. That is, the individual digits are equally likely, and strings of digits of the same length are equally likely. The digits are arranged in sets of six just for convenience in reading.

To illustrate the use of the random number table, consider the assignment of Example 3.4 in Chapter 3.

- Since there are 15 fabric samples, we will consider the table to consist of two digit numbers (if there were fewer than 11 fabric samples, we could consider it to consist of single digit numbers, if between 100 and 1001, of three digit numbers, etc.).
- Pick a 6-digit number at random in the table. (This is done by not looking at the table.)
- Suppose we pick 994120 (row 21, column 4); start here and relabel the digits as 2-digit numbers (i.e. as 99, 41 and 20). This gives the first 3 random numbers. To get the rest, continue on to the next columns, obtaining numbers 13, 03, 91, 47, 26, 37, 77, 97, 21, 99, 30 and 61. Fifteen numbers have been selected in all, but there are only 14 distinct numbers, since the number 99 appears twice. Since we need 15 distinct numbers, we must select at least one more. As the last number selected, 61, was at the end of row 21, we move on to the first column of row 22 for the next selection. The result is 17. As 17 is not among the numbers previously selected, we stop here. Discarding the second 99, we have 15 distinct random numbers.
- Assign the first number selected to fabric sample 1, the second to fabric sample 2, etc. Assign the fabric samples with the 5 lowest random numbers to treatment 1, the fabric samples with the next 5 lowest random numbers to treatment 2, and the fabric samples with the 5 highest random numbers to treatment 3. Table 1.1 shows the resulting assignment.

	Treatment														
	1					2					3				
sorted order	3	13	17	20	21	26	30	37	41	47	61	77	91	97	99
fabric sample	5	4	15	3	12	8	13	9	2	7	14	10	6	11	1

Table 1.1: *Random Assignment of Fabric Samples to Treatment*

## 1.8 Table of Constants for Normal-Theory Tolerance Intervals

n	L=.90			L=.95			L=.99		
	$\gamma = .90$	$\gamma = .95$	$\gamma = .99$	$\gamma = .90$	$\gamma = .95$	$\gamma = .99$	$\gamma = .90$	$\gamma = .95$	$\gamma = .99$
2	15.980	18.800	24.170	32.020	37.670	48.430	160.200	188.500	242.300
3	5.847	6.919	8.974	8.380	9.916	12.860	18.930	22.400	29.060
4	4.166	4.943	6.440	5.369	6.370	8.299	9.398	11.10	14.530
5	3.494	4.152	5.423	4.275	5.079	6.634	6.612	7.855	10.260
6	3.131	3.723	4.870	3.712	4.414	5.775	5.337	6.345	8.301
7	2.902	3.452	4.521	3.369	4.007	5.248	4.613	5.448	7.187
8	2.743	3.264	4.278	3.136	3.732	4.891	4.147	4.936	6.468
9	2.626	3.125	4.098	2.967	3.532	4.631	3.822	4.550	5.966
10	2.535	3.018	3.959	2.829	3.379	4.433	3.582	4.265	5.594
11	2.463	2.933	3.849	2.737	3.259	4.277	3.397	4.045	5.308
12	2.404	2.863	3.758	2.655	3.162	4.150	3.250	3.870	5.079
13	2.355	2.805	3.682	2.587	3.081	4.044	3.130	3.727	4.893
14	2.314	2.756	3.618	2.529	3.012	3.955	3.029	3.608	4.737
15	2.278	2.713	3.562	2.480	2.954	3.878	2.945	3.507	4.605
16	2.246	2.676	3.514	2.437	2.903	3.812	2.872	3.421	4.492
17	2.219	2.643	3.471	2.400	2.858	3.754	2.808	3.345	4.393
18	2.194	2.614	3.433	2.366	2.819	3.702	2.753	3.279	4.307
19	2.172	2.588	3.399	2.337	2.784	3.656	2.703	3.221	4.230
20	2.152	2.564	3.368	2.310	2.752	3.615	2.659	3.168	4.161
21	2.135	2.543	3.340	2.286	2.723	3.577	2.620	3.121	4.100
22	2.118	2.524	3.315	2.264	2.697	3.543	2.584	1.078	4.044
23	2.103	2.506	3.292	2.244	2.673	3.512	2.551	3.040	3.993
24	2.089	2.489	3.270	2.225	2.651	3.483	2.522	3.004	3.947
25	2.077	2.474	3.251	2.208	2.631	3.457	2.494	2.972	3.904
26	2.065	2.460	3.232	2.193	2.612	3.432	2.469	2.941	3.865
27	2.054	2.447	3.215	2.178	2.595	3.409	2.446	2.914	3.828
28	2.044	2.435	3.199	2.164	2.579	3.388	2.424	2.888	3.794
29	2.034	2.424	3.184	2.152	2.554	3.368	2.404	2.864	3.763
30	2.025	2.413	3.170	2.140	2.549	3.350	2.385	2.841	3.733
35	1.988	2.368	3.112	2.090	2.490	3.272	2.306	2.748	3.611
40	1.959	2.334	3.066	2.052	2.445	3.213	2.247	2.677	3.518
50	1.916	2.284	3.001	1.996	2.379	3.126	2.162	2.576	3.385
60	1.887	2.248	2.955	1.958	2.333	3.066	2.103	2.506	3.293
80	1.848	2.202	2.894	1.907	2.272	2.986	2.026	2.414	3.173
100	1.822	2.172	2.854	1.874	2.233	2.934	1.977	2.355	3.096
200	1.764	2.102	2.762	1.798	2.143	2.816	1.865	2.222	2.921
500	1.717	2.046	2.689	1.737	2.070	2.721	1.777	2.117	2.783
1000	1.695	2.019	2.654	1.709	2.036	2.676	1.736	2.068	2.718
$\infty$	1.645	1.960	2.576	1.645	1.960	2.576	1.645	1.960	2.576

## 1.9 Studentized Range Distribution Quantiles

Tabled are the values  $q(L, k, \nu)$ , below which lie a proportion  $L$  of the studentized range distribution based on  $k$  populations and  $\nu$  degrees of freedom. There are three tables, one for each of  $L = 0.90, 0.95$  and  $0.99$ .

	$q(0.90, k, \nu)$								
	$k$								
$\nu$	2	3	4	5	6	7	8	9	10
1	8.93	13.44	16.36	18.49	20.15	21.50	22.64	23.62	24.48
2	4.13	5.73	6.77	7.54	8.14	8.63	9.05	9.41	9.72
3	3.33	4.47	5.20	5.74	6.16	6.51	6.81	7.06	7.29
4	3.01	3.98	4.59	5.03	5.39	5.68	5.93	6.14	6.33
5	2.85	3.72	4.26	4.66	4.98	5.24	5.46	5.65	5.82
6	2.75	3.56	4.07	4.44	4.73	4.97	5.17	5.34	5.50
7	2.68	3.45	3.93	4.28	4.55	4.78	4.97	5.14	5.28
8	2.63	3.37	3.83	4.17	4.43	4.65	4.83	4.99	5.13
9	2.59	3.32	3.76	4.08	4.34	4.54	4.72	4.87	5.01
10	2.56	3.27	3.70	4.02	4.26	4.47	4.64	4.78	4.91
11	2.54	3.23	3.66	3.96	4.20	4.40	4.57	4.71	4.84
12	2.52	3.20	3.62	3.92	4.16	4.35	4.51	4.65	4.78
13	2.50	3.18	3.59	3.88	4.12	4.30	4.46	4.60	4.72
14	2.49	3.16	3.56	3.85	4.08	4.27	4.42	4.56	4.68
15	2.48	3.14	3.54	3.83	4.05	4.23	4.39	4.52	4.64
16	2.47	3.12	3.52	3.80	4.03	4.21	4.36	4.49	4.61
17	2.46	3.11	3.50	3.78	4.00	4.18	4.33	4.46	4.58
18	2.45	3.10	3.49	3.77	3.98	4.16	4.31	4.44	4.55
19	2.45	3.09	3.47	3.75	3.97	4.14	4.29	4.42	4.53
20	2.44	3.08	3.46	3.74	3.95	4.12	4.27	4.40	4.51
25	2.42	3.04	3.42	3.68	3.89	4.06	4.20	4.32	4.43
30	2.40	3.02	3.39	3.65	3.85	4.02	4.16	4.28	4.38
35	2.39	3.00	3.36	3.62	3.82	3.99	4.12	4.24	4.34
40	2.38	2.99	3.35	3.60	3.80	3.96	4.10	4.21	4.32
45	2.38	2.98	3.34	3.59	3.79	3.95	4.08	4.19	4.30
50	2.37	2.97	3.33	3.58	3.77	3.93	4.06	4.18	4.28
60	2.36	2.96	3.31	3.56	3.75	3.91	4.04	4.16	4.25
70	2.36	2.95	3.30	3.55	3.74	3.90	4.03	4.14	4.24
80	2.35	2.94	3.29	3.54	3.73	3.89	4.01	4.13	4.22
90	2.35	2.94	3.29	3.53	3.72	3.88	4.01	4.12	4.21
100	2.35	2.94	3.28	3.53	3.72	3.87	4.00	4.11	4.20
$\infty$	2.33	2.90	3.24	3.48	3.66	3.81	3.93	4.04	4.13

	$q(0.95, k, \nu)$								
	$k$								
$\nu$	2	3	4	5	6	7	8	9	10
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07
2	6.08	8.33	9.80	10.88	11.73	12.43	13.03	13.54	13.99
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49
7	3.34	4.16	4.68	5.06	5.36	5.61	5.81	5.99	6.15
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.26	5.39
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07
19	2.96	3.59	3.98	4.25	4.47	4.64	4.79	4.92	5.04
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01
25	2.91	3.52	3.89	4.15	4.36	4.53	4.67	4.79	4.90
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82
35	2.87	3.46	3.81	4.07	4.26	4.42	4.55	4.67	4.77
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73
45	2.85	3.43	3.77	4.02	4.21	4.36	4.49	4.61	4.70
50	2.84	3.42	3.76	4.00	4.19	4.34	4.47	4.58	4.68
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65
70	2.82	3.39	3.72	3.96	4.14	4.29	4.42	4.53	4.62
80	2.81	3.38	3.71	3.95	4.13	4.28	4.40	4.51	4.60
90	2.81	3.37	3.70	3.94	4.12	4.27	4.39	4.50	4.59
100	2.81	3.36	3.70	3.93	4.11	4.26	4.38	4.48	4.58
$\infty$	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.48

	$q(0.99, k, \nu)$								
	$k$								
$\nu$	2	3	4	5	6	7	8	9	10
1	89.98	135.04	164.25	185.56	202.19	215.74	227.13	236.93	245.50
2	14.03	19.02	22.29	24.72	26.63	28.20	29.53	30.68	31.69
3	8.26	10.62	12.17	13.32	14.24	15.00	15.64	16.20	16.69
4	6.51	8.12	9.17	9.96	10.58	11.10	11.54	11.93	12.26
5	5.70	6.98	7.81	8.42	8.91	9.32	9.67	9.97	10.24
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10
7	4.95	5.92	6.55	7.02	7.39	7.70	7.98	8.21	8.43
8	4.74	5.64	6.20	6.63	6.97	7.24	7.48	7.69	7.88
9	4.60	5.43	5.96	6.35	6.66	6.92	7.14	7.33	7.50
10	4.48	5.27	5.77	6.14	6.43	6.67	6.88	7.05	7.21
11	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99
12	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.68	6.82
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67
14	4.21	4.89	5.32	5.63	5.88	6.09	6.26	6.41	6.55
15	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44
16	4.13	4.79	5.19	5.49	5.72	5.91	6.08	6.22	6.35
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20
19	4.05	4.67	5.05	5.33	5.55	5.74	5.89	6.02	6.14
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09
25	3.94	4.53	4.88	5.14	5.35	5.51	5.65	5.78	5.89
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76
35	3.85	4.40	4.74	4.98	5.17	5.32	5.45	5.57	5.67
40	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60
45	3.80	4.34	4.66	4.89	5.07	5.22	5.34	5.45	5.55
50	3.79	4.32	4.63	4.86	5.04	5.18	5.31	5.41	5.51
60	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45
70	3.74	4.26	4.57	4.79	4.96	5.10	5.21	5.31	5.40
80	3.73	4.24	4.55	4.76	4.93	5.07	5.18	5.28	5.37
90	3.72	4.23	4.53	4.74	4.91	5.05	5.16	5.26	5.35
100	3.71	4.22	4.52	4.73	4.90	5.03	5.14	5.24	5.33
$\infty$	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16

## 1.10 Probabilities of the Spearman Correlation

Tabled are values of  $p^+ = P(r_s \geq r_s^*)$ , for independent variates and potential observed values  $r_s^*$  of the Spearman correlation. Probabilities are presented for  $n = 3, 4, \dots, 10$  and the  $r_s^*$  values are selected to approximate common significance levels. Values of  $p_- = P(r_s \leq -r_s^*)$  may be obtained as  $P(r_s \geq -r_s^*)$ , and values of  $p^\pm = P(|r_s| \geq |r_s^*|)$  may be obtained as  $p^\pm = 2 \min\{p_-, p^+\}$ . For  $n > 10$ , an approximate test of independence may be obtained from the fact that  $r_s \sqrt{\frac{n-2}{1-r_s^2}}$  has approximately a  $t_{n-2}$  distribution under the assumption of independence.

n	$r_s^*$	$P(r_s \geq r_s^*)$
3	1.000	0.167
4	0.800	0.167
	1.000	0.042
5	1.000	0.008
	0.900	0.042
	0.800	0.067
	0.700	0.117
6	0.942	0.008
	0.885	0.017
	0.828	0.029
	0.771	0.051
	0.657	0.088
7	0.600	0.121
	0.928	0.003
	0.892	0.006
	0.857	0.012
	0.785	0.024
	0.714	0.044
	0.750	0.033
0.678	0.055	
	0.571	0.100

n	$r_s^*$	$P(r_s \geq r_s^*)$
8	0.857	0.005
	0.833	0.008
	0.809	0.011
	0.738	0.023
	0.714	0.029
	0.642	0.048
	0.619	0.057
	0.523	0.098
9	0.500	0.108
	0.816	0.005
	0.783	0.009
	0.766	0.011
	0.683	0.025
	0.600	0.048
10	0.583	0.054
	0.483	0.097
	0.466	0.106
	0.781	0.005
	0.733	0.010
	0.648	0.024
	0.636	0.027
	0.563	0.048
	0.551	0.052
	0.454	0.095
	0.442	0.102

## 1.11 Control Chart Constants

Number in Subgroup n	Constants		
	$c_4$	$d_2$	$d_3$
2	0.7979	1.1284	0.8525
3	0.8862	1.6926	0.8884
4	0.9213	2.0588	0.8798
5	0.9400	2.3259	0.8641
6	0.9515	2.5344	0.8480
7	0.9594	2.7044	0.8332
8	0.9650	2.8472	0.8198
9	0.9693	2.9700	0.8078
10	0.9727	3.0775	0.7971
11	0.9754	3.1729	0.7873
12	0.9776	3.2585	0.7785
13	0.9794	3.3360	0.7704
14	0.9810	3.4068	0.7630
15	0.9823	3.4718	0.7562
16	0.9835	3.5320	0.7499
17	0.9845	3.5879	0.7441
18	0.9854	3.6401	0.7386
19	0.9862	3.6890	0.7335
20	0.9869	3.7350	0.7287
21	0.9876	3.7783	0.7242
22	0.9882	3.8194	0.7199
23	0.9887	3.8583	0.7159
24	0.9892	3.8953	0.7121
25	0.9896	3.9306	0.7084

### 1.12 Lower Tail Probabilities for the Wilcoxon Signed Rank Statistic

Tabled are the quantities  $p_- = P(W \leq w^*)$ , computed under the assumption that  $H_0$  is true. The upper tail probabilities  $p^+ = P(W \geq w^*)$  can be obtained from these by using the relation

$$P(W \geq w^*) = P(W \leq n(n+1)/2 - w^*).$$

$n$	$w^*$	$p_-$	$n$	$w^*$	$p_-$
3	0	0.125	13	0.011	
4	1	0.125	12	0.009	
	0	0.062	10	0.005	
5	2	0.094	14	32	0.108
	1	0.062	31	0.097	
	0	0.031	26	0.052	
6	4	0.109	21	0.025	
	2	0.047	16	0.010	
	1	0.031	13	0.005	
	0	0.016	15	37	0.104
7	6	0.109	36	0.094	
	4	0.055	31	0.053	
	2	0.023	30	0.047	
	0	0.008	25	0.024	
8	8	0.098	20	0.011	
	6	0.055	19	0.009	
	4	0.027	16	0.005	
	2	0.012	16	43	0.106
	1	0.008	42	0.096	
	0	0.004	36	0.052	
9	11	0.102	30	0.025	
	8	0.049	24	0.011	
	6	0.027	23	0.009	
	3	0.010	20	0.005	
	1	0.004	17	49	0.103
10	14	0.097	48	0.095	
	11	0.053	41	0.049	
	8	0.024	35	0.025	
	5	0.010	28	0.010	
	3	0.005	24	0.005	
11	18	0.103	18	55	0.098
	14	0.051	47	0.049	
	11	0.027	40	0.024	
	7	0.009	33	0.010	
	5	0.005	28	0.005	
12	22	0.102	19	62	0.098
	18	0.055	54	0.052	
	17	0.046	53	0.048	
	14	0.026	46	0.025	
	10	0.010	38	0.010	
	7	0.005	33	0.005	
13	27	0.108	20	70	0.101
	26	0.095	60	0.049	
	22	0.055	52	0.024	
	21	0.047	43	0.010	
	17	0.024	38	0.005	

### 1.13 Lower Tail Probabilities for the Wilcoxon Rank Sum Statistic

Tabled are the quantities  $p_- = P(V \leq v^*)$ , computed under the assumption that  $H_0$  is true. The lower tail probabilities  $p^* = P(V \geq v^*)$  can be obtained from these by using the relation

$$P(V \geq v^*) = P(V \leq (n_1 + n_2 + 1)(n_1 + n_2)/2 - v^*).$$

$n_1$	$n_2$	$v^*$	$p_-$	$n_1$	$n_2$	$v^*$	$p_-$
2	3	6	0.100	5	5	19	0.048
	4	10	0.067			18	0.028
	5	10	0.048			16	0.008
	6	21	0.036			16	0.008
	7	29	0.056			16	0.008
		28	0.028			15	0.004
	8	37	0.044		6	26	0.041
		36	0.022			25	0.026
3	3	6	0.050			23	0.009
	4	11	0.057			22	0.004
		10	0.029		7	35	0.053
	5	16	0.036			33	0.024
		15	0.018			31	0.009
	6	23	0.048			30	0.005
		22	0.024		8	44	0.047
		21	0.012			42	0.023
	7	31	0.058			40	0.009
		29	0.017			39	0.005
		28	0.008	6	6	28	0.047
	8	39	0.042			26	0.021
		38	0.024			24	0.008
		37	0.012			23	0.004
		36	0.006		7	37	0.051
4	4	12	0.057			35	0.026
		11	0.029			33	0.011
		10	0.014			31	0.004
	5	18	0.056		8	47	0.054
		17	0.032			44	0.021
		16	0.016			42	0.010
		15	0.008			40	0.004
	6	25	0.057	7	7	39	0.049
		23	0.019			37	0.027
		22	0.010			34	0.009
		21	0.005			33	0.006
	7	33	0.055		8	49	0.047
		31	0.021			47	0.027
		30	0.012			44	0.010
		29	0.006			42	0.005
	8	42	0.055	8	8	52	0.052
		40	0.024			49	0.025
		38	0.008			46	0.010
		37	0.004			44	0.005

### 1.14 Critical Constant $k$ for Wilcoxon Signed Rank Confidence Intervals

Sample Size, $n$	Confidence Level, $L$	$k$	Sample Size, $n$	Confidence Level, $L$	$k$	Sample Size, $n$	Confidence Level, $L$	$k$
5	0.938	15	13	0.990	81	20	0.991	173
	0.875	15		0.952	74		0.952	158
6	0.969	21		0.906	70		0.903	150
	0.937	20	14	0.991	93	21	0.990	188
	0.906	19		0.951	84		0.950	172
7	0.984	28		0.896	79		0.897	163
	0.953	26	15	0.990	104	22	0.990	204
	0.891	24		0.952	95		0.950	187
8	0.992	36		0.905	90		0.902	178
	0.945	32	16	0.991	117	23	0.990	221
	0.891	30		0.949	106		0.952	203
9	0.992	44		0.895	100		0.902	193
	0.945	39	17	0.991	130	24	0.990	239
	0.902	37		0.949	118		0.951	219
10	0.990	52		0.902	112		0.899	208
	0.951	47	18	0.990	143	25	0.990	257
	0.895	44		0.952	131		0.952	236
11	0.990	61		0.901	124		0.899	224
	0.946	55	19	0.991	158			
	0.898	52		0.951	144			
12	0.991	71		0.904	137			
	0.948	64						
	0.908	61						

### 1.15 Critical Constant $k$ for Wilcoxon Rank Sum Confidence Intervals

Larger Sample Size	Smaller Sample Size													
	2		3		4		5		6		7			
	Level, $L$	$k$	Level, $L$	$k$	Level, $L$	$k$	Level, $L$	$k$	Level, $L$	$k$	Level, $L$	$k$		
3	0.800	6	0.900	9										
4	0.866	8	0.942	12	0.972	16								
			0.886	11	0.942	15								
5	0.904	10	0.964	15	0.984	20	0.992	25						
			0.928	14	0.936	18	0.944	22						
					0.888	17	0.905	21						
6	0.928	12	0.976	18	0.990	24	0.991	29	0.991	34				
			0.952	17	0.962	22	0.948	26	0.959	31				
			0.904	16	0.886	20	0.918	25	0.907	29				
7	0.944	14	0.984	21	0.988	27	0.990	33	0.992	39	0.989	44		
			0.888	13	0.966	20	0.958	25	0.952	30	0.949	35	0.947	40
8	0.956	16	0.884	18	0.890	23	0.894	28	0.899	33	0.903	38		
			0.988	24	0.992	31	0.989	37	0.992	44	0.991	50		
			0.912	15	0.952	22	0.952	28	0.955	34	0.957	40	0.946	45
9	0.964	18	0.916	21	0.890	26	0.907	32	0.892	37	0.906	43		
			0.928	17	0.990	27	0.988	34	0.988	41	0.992	49	0.992	56
					0.964	25	0.950	31	0.958	38	0.950	44	0.945	50
10	0.970	20	0.900	22	0.894	29	0.888	35	0.912	42	0.909	48		
			0.994	30	0.992	38	0.992	46	0.989	53	0.990	61		
			0.940	19	0.952	27	0.946	34	0.945	41	0.944	48	0.945	55
11	0.878	18	0.924	26	0.894	32	0.901	39	0.907	46	0.891	52		
			0.974	22	0.990	32	0.990	41	0.991	50	0.990	58	0.989	66
			0.948	21	0.962	30	0.944	37	0.948	45	0.952	53	0.956	61
12	0.898	20	0.912	28	0.896	35	0.910	43	0.902	50	0.896	57		
			0.978	24	0.992	35	0.992	45	0.991	54	0.990	63	0.990	72
			0.956	23	0.952	32	0.958	41	0.952	49	0.947	57	0.955	66
	0.912	22	0.898	30	0.896	38	0.896	46	0.898	54	0.900	62		

Larger Sample Size	Smaller Sample Size									
	8		9		10		11		12	
	Level, $L$	$k$	Level, $L$	$k$	Level, $L$	$k$	Level, $L$	$k$	Level, $L$	$k$
8	0.990	56								
	0.950	51								
	0.895	48								
9	0.989	62	0.989	69						
	0.954	57	0.950	63						
10	0.907	54	0.906	60						
	0.991	69	0.990	76	0.991	84				
	0.945	62	0.947	69	0.948	76				
11	0.899	59	0.905	66	0.895	72				
	0.991	75	0.990	83	0.990	91	0.989	99		
	0.949	68	0.954	76	0.949	83	0.953	91		
12	0.909	65	0.905	72	0.901	79	0.899	86		
	0.990	81	0.991	90	0.991	99	0.991	108	0.990	116
	0.953	74	0.951	82	0.950	90	0.949	98	0.948	106
	0.902	70	0.905	78	0.907	86	0.896	93	0.899	101