Investigating the Existence of Barker Codes

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Greg Coxson, TSC and other people

Overview

- Introduction
- Problem statement
- Areas of work
- Results
- Conclusion
- Future work

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Physical problem

Accurately identify an object of interest using sequences of radar pulses.

Our approaches

- Continuous model (lunatic fringe)
- Discrete model

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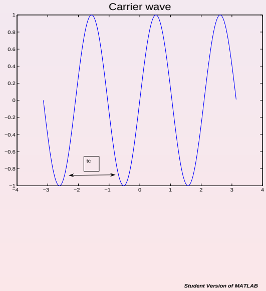
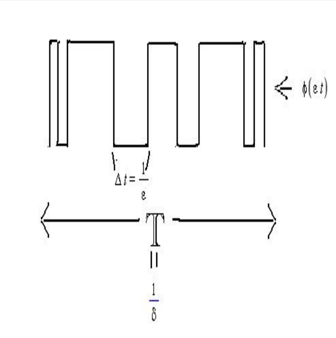
Continuous model





Defining the pulse

- pulse: $f(t) = e^{i(t+\phi(\epsilon t))}A(\delta t)$
- return pulse: $g(t) = f^*(t - \frac{\tau_s}{\delta})$
- convolution: $I(\tau) = \int_{-\infty}^{\infty} f(t - \frac{\tau}{\delta})g(t)dt$

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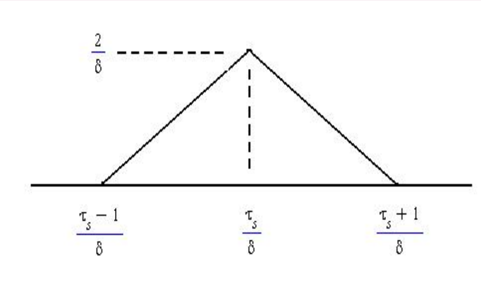
Why unmodulated pulses don't work

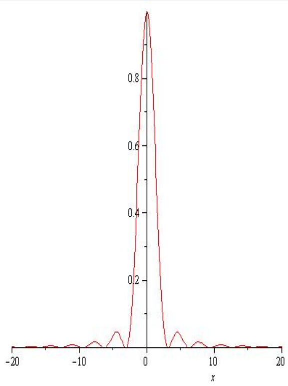
Suppose that $\phi(\epsilon t) = 0$. So

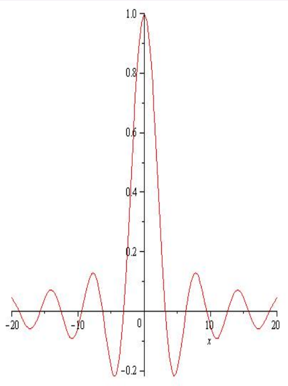
$$I(\tau) = \int_{-\infty}^{\infty} e^{i(t-\frac{\tau}{\delta})} A\left((t - \frac{\tau}{\delta})\delta\right) e^{-i(t-\frac{\tau_s}{\delta})} A\left((t - \frac{\tau_s}{\delta})\delta\right) dt$$

$$= \int_{\frac{\tau_s - 1}{\delta}}^{\frac{\tau_s + 1}{\delta}} A(\delta(t - \frac{\tau_s}{\delta}))A(\delta(t - \frac{\tau}{\delta}))dt$$



Some pretty pictures





Ellis's brilliant insight

Let $f(x)$ be the characteristic function from $-\mu$ to μ. Then

$$\widehat{f}(\omega) = \frac{\sin(\mu\omega)}{\omega} =: g(\omega).$$

$$\begin{aligned} \widehat{g}(x) &= f(x) \\ \widehat{g^2}(x) &= (f * f)(x) \\ &\vdots \end{aligned}$$

Solving for the phase modulation ϕ

Recall $I(\tau) = \int_{-\infty}^{\infty} f(t - \frac{\tau}{\delta}) f^*(t - \frac{\tau_s}{\delta}) dt$. Using Fourier transform gives

$$\frac{\delta}{e^{i\tau_s}} \widehat{g^{2n}} = (\widehat{f})^2$$

$$e^{i(t+\phi(\epsilon t))} A(\delta t) = f(t) = \sqrt{\frac{\delta}{e^{i\tau_s}} \int_{-\infty}^{\infty} \widehat{g^{2n}}(\omega) e^{-it\omega} d\omega}$$

Transition to discrete case

Terminology and Definitions

- Let **x** denote a sequence $(x_1, x_2 \ldots, x_N)$
- Where $x_i = \pm 1$
- The response of the matched filter is the autocorrelation of x: $ACF_x(k) = x * \bar{x} = \sum_{i=1}^{N-k} x_i x_{i+k}$
- For any k, $-(N-1) \leq k \leq N-1$
- Where $ACF_x(0) = N$ and $k \neq N$ are sidelobes
- Energy$=\sum_k ACF_x(k)^2$
- Merit factor $F = N^2/(2 * Energy)$ "ratio of central to sidelobe energy of long, low autocorrelation sequences"

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The Merit Factor of Long Low Autocorrelation Binary Sequence by Golay

- **Motivation for studying this paper**: Golay outlines a method to calculate an upper bound for the merit factor for any large N. If his arguments are plausible there are no Barker codes of length greater than length 13

Outlining Golay's Approach

- Consider a long binary sequence and its autocorrelation as independent random variables
- Approximate the distribution of the autocorrelation by the Gaussian density function
- Consider all autocorrelation functions of the same length $N/2$ and calculate their merit factor $F_1 = 9.82$ using the stationary approximation

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Outlining Golay's Approach(con't)

- Split the autocorrelation into two groups($N/4$ and $3N/4$) recalculate F $F_2 = 11.1613$
- Continue splitting and redefining the merit factor for m stages
- Continuing to the continuous case he finds an optimal bound at $F_{\infty}^{opt} = 12.32$
- There are two equations for the upper bound of sequence, general and skew symmetric respectively

$$F_g^{opt} = \frac{12.3248}{(8\pi N)^3/2N} \tag{1}$$

$$F_s^{opt} = \frac{12.3248}{(\pi^3 N^3)/4 + \frac{12.3248}{N}} \tag{2}$$

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Highest known merit factor from Golay's paper

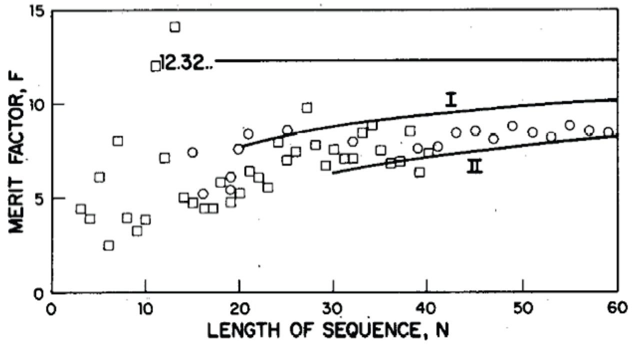
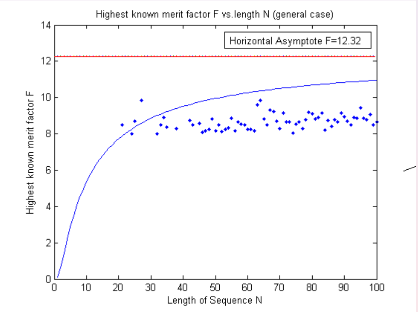
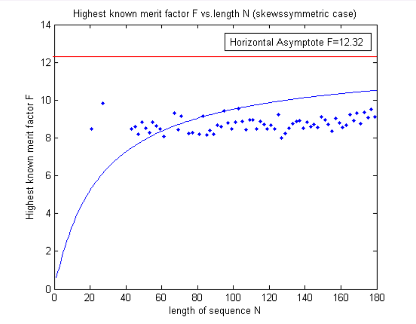


Fig. 1. Highest known merit factions and theoretical curves.

Highest known merit factor



(a) general case



(b) skewsymmetric case

Figure: Highest known merit factor F vs. length N

Curve Fitting Approach

- Calculating binary sequences for length less than 40 can be done by exhausting all possible sequences
- For strings greater than 40 other techniques involving curve fitting and selecting random sequences help explore higher merit factors
- One technique is to integrate the tail of the distribution from 0 to 1/8 to find the number sequences with merit greater than 8 or 0 to 1/9 to find sequences with merit factor greater than 9

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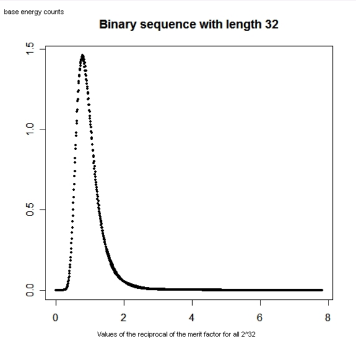
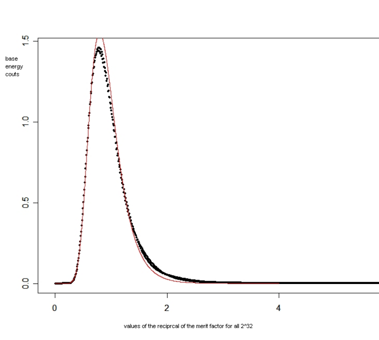
Two fits of the distribution curve



(a) Full distribution of $1/F$ at $N = 32$



(b) The least square

Experimental Techniques

- As previously mentioned, for small sequences of length N all combinations can be exhausted but for large N sequences must be randomly generated.
- Is it possible to reduce the number of sequences we have to check by looking at the merit factors of a given sequence and then analyzing the ratios of 1's and -1's

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- As previously mentioned, for small sequences of length N all combinations can be exhausted but for large N sequences must be randomly generated.
- Is it possible to reduce the number of sequences we have to check by looking at the merit factors of a given sequence and then analyzing the ratios of 1's and -1's

The idea

- Start with a sequence of length N with all elements in the sequence being 1
- Randomly choose an element in the sequence and flip the sign
- If the merit factor of the new sequence goes up, we keep this sequence. We proceed with the new sequence with some probability p. Where $p = exp(-\alpha * (ACF_2 - ACF_1))$
- Goal: To find sequences of large N that have high merit and low PSL

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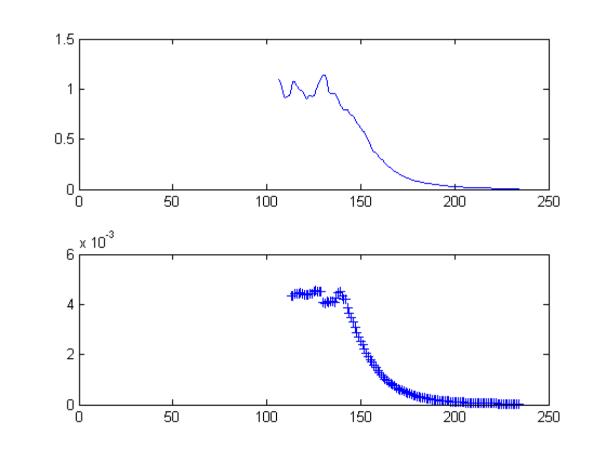

-
- Length 233, approximately 3000 time steps
- number of 1's-125(Found in Ron Ferguson's data)
- number of 1's-125(Found in our data)

Conclusions

- There are a variety of different perspectives on what makes a "good" Barker type pulse
- Considering continuous phase modulation we may find a barker-like pulse for a given convolution function with high and narrow peak and arbitrarily small sidelobes
- In order to discover higher merit factors, new techniques in the discrete and continuous cases need to be developed to handle larger sequences

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References

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