CFD Modeling of Abrasive-Fluid Jet for Precision Machining of Most Materials from Macro to Micro Scales

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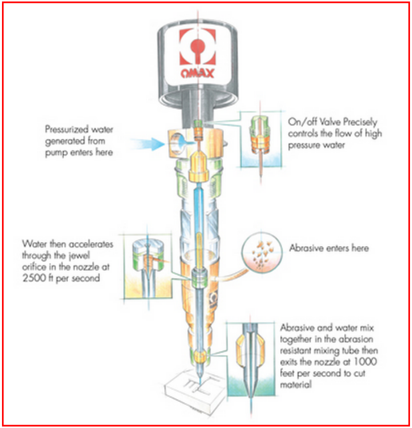
29th Annual MPI Workshop, June 17–21, 2013

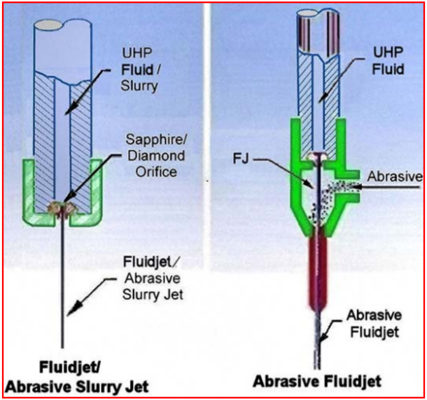
How does an Abrasive WaterJet (*AWJ*) Work?

- Machine for precision cutting
- Gravity takes garnet particles to mixing chamber
- Jet stream of water mixes with air and garnet
- Focusing tube accelerates particles



Advantages of the AWJ

- Cuts most materials
- Produces much less wear on the diamond orifice
- Longer operational lifespan
  e.g. 80 hours vs. less than 1 hour before maintenance



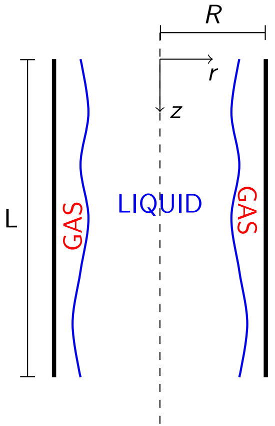
Objectives

- Understand the fluid dynamics of the AWJ device
- See which parameters may affect:
  - Cutting efficiency
  - Wear on parts
- Identify desirable design changes

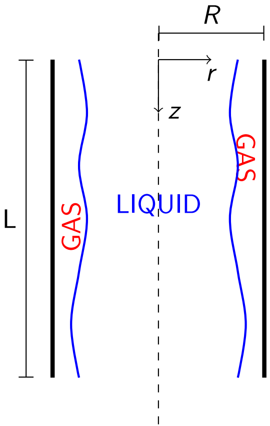

a. Upstream half


b. Downstream half

Continuum Model: Water and Air



- Interface is described by $h(z,t)$.
- Incompressible Euler equations in the liquid.
- Compressible Euler equations in the gas
- Ideal gas equation of state.
- Assume axial symmetry
- Radial velocity is continuous over the interface, the jump in normal stress is balanced by capillary forces and kinematic boundary condition.



Liquid

$$\nabla \cdot \mathbf{u}^{(1)} = 0, \qquad (1)$$

$$\rho_\ell \left\{ \mathbf{u}_t^{(1)} + \mathbf{u}^{(1)} \cdot \nabla \mathbf{u}^{(1)} \right\} + \nabla p^{(1)} = 0 \qquad (2)$$

Gas

$$\rho_t^{(2)} + \nabla \cdot \left\{ \rho^{(2)}\, \mathbf{u}^{(2)} \right\} = 0\ , \qquad (3)$$

$$\rho^{(2)} \left\{ \mathbf{u}_t^{(2)} + \mathbf{u}^{(2)} \cdot \nabla \mathbf{u}^{(2)} \right\} + \nabla p^{(2)} = 0\ , \qquad (4)$$

$$p^{(2)} = R \rho^{(2)} T \qquad (5)$$

Boundary Conditions

$$r = 0: \quad u^{(1)} = 0 \ , \ \lim_{r \to 0} r w_r^{(1)} = 0$$

$$r = R: \ r = R: \quad u^{(2)} = 0$$

Conditions at free surface

$$u^{(1)} = u^{(2)}$$

$$p^{(1)} - p^{(2)} = \sigma \left\{ \frac{1}{h\sqrt{1 + h_z^2}} - \frac{h_{zz}}{\left(1 + h_z^2\right)^{3/2}} \right\}$$

$$h_t + w^{(1)} h_z = u^{(1)} \ .$$

Nondimensionalization

Identifying the aspect ratio of the tube as a small parameter: $\epsilon = R/L$, writing $We = \sigma R/(\rho_\ell W^2)$, and $Ma = W^2/c^2$, and performing perturbation expansion yields:

$$w_t^{(1)} + w^{(1)} w_z^{(1)} = p_z^{(2)} + We \left\{ \epsilon^2 h_{zz} - \frac{1}{h} \right\}_z \ , \tag{6}$$

$$u^{(1)} = -\frac{r}{2} w_z^{(1)} \tag{7}$$

$$u^{(2)} = -\frac{r^2 - 1}{2r} \left\{ \frac{\rho_t + \left(\rho w^{(2)}\right)_z}{\rho} \right\} \tag{8}$$

$$0 = \rho_t + \left(\rho w^{(2)}\right)_z + \frac{h^2}{1 - h^2} \rho w_z^{(1)} \tag{9}$$

$$0 = \left(h^2\right)_t + \left(h^2 w^{(1)}\right)_z \tag{10}$$

$$Map^{(2)} = \rho \tag{11}$$

Linear Stability Analysis

Applying a small perturbation from a constant base state:

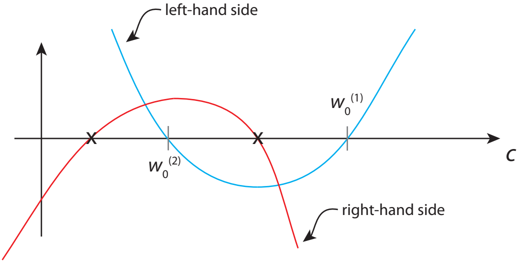
$$\begin{pmatrix} w^{(1)} \\ w^{(2)} \\ \rho \\ h \end{pmatrix}(z,t) = \begin{pmatrix} w_0^{(1)} \\ w_0^{(2)} \\ \rho_0 \\ h_0 \end{pmatrix} + \epsilon e^{\sigma t + ikz} \begin{pmatrix} \widetilde{w^{(1)}} \\ \widetilde{w^{(2)}} \\ \tilde{\rho} \\ \tilde{h} \end{pmatrix}$$

$$\begin{pmatrix} -ikw_0^{(1)} & 0 & \frac{ik}{Ma} & We\left(ikh_0 - i\epsilon^2 k^3\right) \\ 0 & -ikw_0^{(2)} & -\frac{ik}{\rho_0 Ma} & 0 \\ -\frac{ikh_0^2}{1-h_0^2}\rho_0 & -ik\rho_0 & -ikw_0^{(2)} & 0 \\ -\frac{ikh_0}{2} & 0 & 0 & -ikw_0^{(1)} \end{pmatrix} \begin{pmatrix} \widetilde{w^{(1)}} \\ \widetilde{w^{(2)}} \\ \tilde{\rho} \\ \tilde{h} \end{pmatrix} = \sigma \begin{pmatrix} \widetilde{w^{(1)}} \\ \widetilde{w^{(2)}} \\ \tilde{\rho} \\ \tilde{h} \end{pmatrix}$$

Results

Letting $\sigma = ikc$, $c_1 = c - w_0^{(1)}$ and $c_2 = c - w_0^{(2)}$, then the following equation must be satisfied:

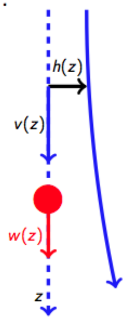
$$\frac{2c_1 c_2 h_0^2 \rho_0}{1 - h_0^2} = (1 - c_2^2 Ma)(2c_1^2 + We - \epsilon^2 Wek^2 h_0)$$



- The plug flow is linearly unstable.
- The jet will eventually break up, even without particles.

Continuum Model: Water and Particles

- Variables
  - $v(z)$: velocity of the fluid
  - $w(z)$: velocity of the particle
  - $h(z)$: the radius of the cross section of the water jet at position z
  - $N(z)$: # of particles per unit length
- Parameters
  - ρ: density of the fluid
  - $\bar{\rho}$: density of the particle
  - a: radius of one particle
  - C_D: coefficient of drag



- D: drag

$$D(N) = \frac{1}{2} C_D \rho (\pi a^2) N = D_o N$$

- Boundary conditions

$$v(0) = v_o \quad h(0) = h_o \quad w(0) = \delta \ll v_o \quad N(0)w(0) = Q$$

Governing Equations

Conservation of Mass (fluid)

$$\frac{d}{dz}\left(\rho v h^2\right) = 0$$

Conservation of Mass (particles)

$$\frac{d}{dz}(\bar{\rho} N w) = 0$$

Conservation of Momentum (fluid)

$$\frac{d}{dz}\left(\rho v^2 \pi h^2\right) = D_o N(w - v)|w - v|$$

Conservation of Momentum (particles)

$$\frac{d}{dz}\left[\bar{\rho}\left(\frac{4}{3}\pi a^3\right) N w^2\right] = -D_o N(w - v)|w - v|$$

Governing Equations (Dimensionless)

- Scales & dimensionless group

$$L = \frac{2 v_o h_o^2}{C_D a^2 Q} \quad \lambda = \frac{3}{4} \frac{v_o h_o^2}{a^3 Q} \frac{\rho}{\bar{\rho}}$$

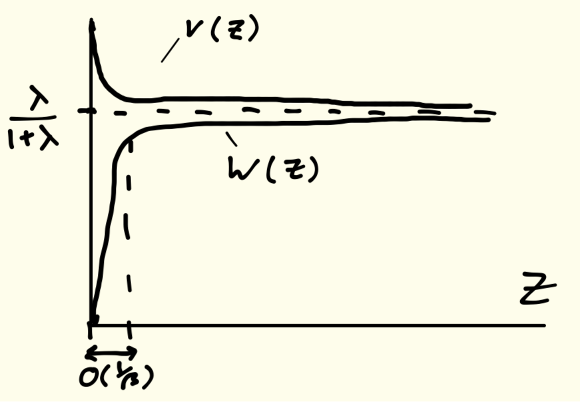
- Dimensionless equations

$$\begin{aligned} \left(v\, h^2\right)' &= 0 \\ \left(v^2\, h^2\right)' &= -N(w-v)^2 \\ \left(N w^2\right)' &= \lambda N (w-v)^2 \\ (N w)' &= 0 \end{aligned}$$

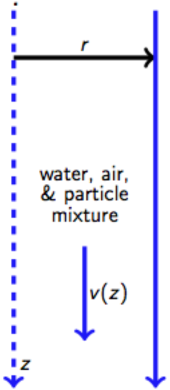
with boundary conditions

$$v(0) = 1 \qquad w(0) = \delta \qquad h(0) = 1 \qquad N(0) w(0) = 1$$

Solution Sketch of the Velocities



Continuum Model: Water, Air and Particles



Continuum Model: Water, Air and Particles

1D steady model

	water	air	particles
volume fraction	α	β	$1-\alpha-\beta$
velocity	v_α	v_β	v_p
density	ρ_α	ρ_β	ρ_p
pressure	P_α	P_β	

- Basic assumptions
  - Steady flow
  - Water is inviscid & incompressible
  - Air is inviscid & compressible

Governing Equations

Conservation of Mass

$$\frac{d}{dz}\left(\rho_\alpha \alpha v_\alpha\right) = 0$$
$$\frac{d}{dz}\left(\rho_\beta \beta v_\beta\right) = 0$$
$$\frac{d}{dz}\left(\rho_p (1-\alpha-\beta) v_p\right) = 0$$

Conservation of Momentum

$$\frac{d}{dz}\left(\rho_\alpha \alpha v_\alpha^2\right) = -\alpha\frac{dP_\alpha}{dz} - k_{\alpha\beta}\left(v_\alpha - v_\beta\right)^2 - k_{\alpha p}\left(v_\alpha - v_p\right)^2 - A\alpha v_\alpha^2$$
$$\frac{d}{dz}\left(\rho_\beta \beta v_\beta^2\right) = -\beta\frac{dP_\beta}{dz} + k_{\alpha\beta}\left(v_\alpha - v_\beta\right)^2 - k_{\beta p}\left(v_\beta - v_p\right)^2 - B\beta v_\beta^2$$
$$\frac{d}{dz}\left(\rho_p (1-\alpha-\beta) v_p^2\right) = k_{\alpha p}\left(v_\alpha - v_p\right)^2 + k_{\beta p}\left(v_\beta - v_p\right)^2 - C(1-\alpha-\beta) v_p^2$$

Equations of State

$$P_\alpha = P_\beta \qquad P_\beta = \rho_\beta R T$$

Reduced Equations

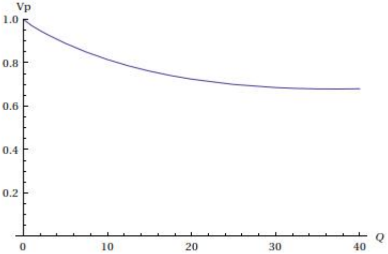
- Boundary conditions

$$v_\alpha(0) = V_o \quad v_\beta(0) = U_o \quad v_p(0) = \delta Q$$
$$\alpha(0) = \alpha_o \quad \beta(0) = \beta_o \quad P_\beta(0) = P_\alpha(0) = P_A - \frac{1}{2}\rho_\alpha U_o^2$$

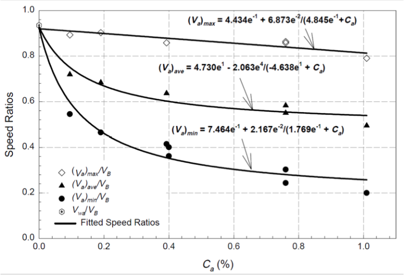
- Assumptions
  - Drag between air and particles is negligible
  - Drag from the wall on gas phase is negligible
- Reduced momentum equations

$$\begin{aligned} \rho_\alpha V_o v_\alpha' &= -\alpha P' - k_{\alpha\beta}(v_\alpha - v_\beta)^2 - k_{\alpha p}(v_\alpha - v_p)^2 - A\alpha v_\alpha^2 \\ \beta_o \rho_o U_o v_\beta' &= -\beta P' + k_{\alpha\beta}(v_\alpha - v_\beta)^2 \\ \rho_p(1 - \alpha_o - \beta_o)\delta Q v_p' &= k_{\alpha p}(v_\alpha - v_p)^2 - C(1 - \alpha_o - \beta_o)\delta Q v_p \end{aligned}$$

- Plot of the speed of the particles as a function of Q



- Plot of the speed of the particles as a function of concentration



Particle Dynamics Models

Goal of these models is follow the path of single particle as it moves through a fluid.

- We want a model that can explain the damage on the sides of the mixing tube

a. Upstream half

b. Downstream half

Particle Dynamics Models

Model Asumptions

- uniform fluid velocity
- fixed tube width
- spherical particle
- particle starts with some velocity obtained in the mixing chamber
- particles are sliding not rotating.

Basic Particle Model

Notation

- All equations are dimensionless
- $v_0 = 0.2$ initial particle velocity
- r particle position
- $k = \frac{1}{16}$ is the drag constant
- $C_r = .9$ is the coefficient of restitution with the wall
- $\theta = \frac{\pi}{12}$ is the angle between mixing chamber and tube

Equation of Motion

$$\ddot{r} = -k\|\dot{r} - e_z v\|(\dot{r} - e_z v)$$

Collision Condition

$$|r \cdot e_x| = 1$$

$$\dot{r}^+ \cdot e_x = -C_r \dot{r}^- \cdot e_x$$

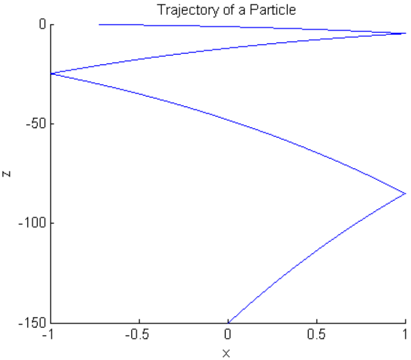
$$\dot{r}^+ \cdot e_z = \dot{r}^- \cdot e_z - \mu(1 + C_r)\,\dot{r}^- \cdot e_x$$

Basic Particle Model

Initial Condtions

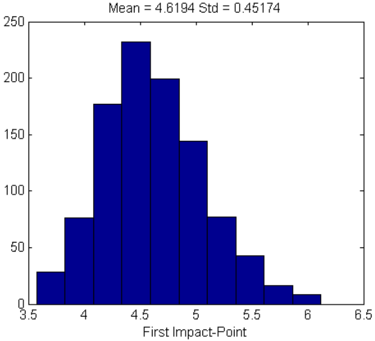
$$r_0 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

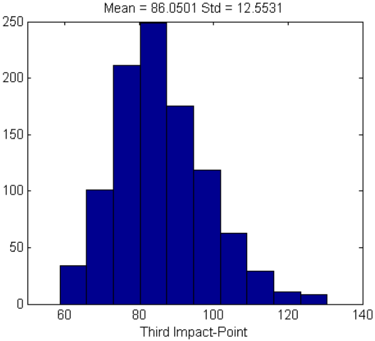
$$\dot{r}_0 = \begin{bmatrix} |v_0| \cos\theta \\ |v_0| \sin\theta \end{bmatrix}$$



Basic Particle Model

v_0 was selected from a normal distribution with $\mu = .2$ and $\sigma = .01$





Deformation Particle Model

Wall Deformation Assumptions

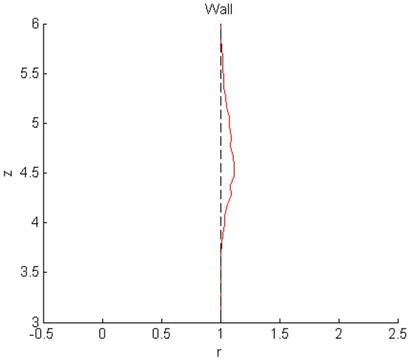
- Damage is due to particles hitting the walls. (No contribution from water)
- Width of deformation is a smaller scale than the tube length
- Wall deforms only in the direction normal to the tube
- Size of the deformation is proportional to the momentum loss in the basic particle model
- Initial particle velocity by the mixing chamber is normally distributed

Deformation Particle Model

Mathematical Approach

- Uniformly discretize wall: z_m are the points along z axis.
- Define w_m as the deformation at point z_m.
- Shoot particle using the basic particle model.
- Compute intersection of the particle trajectory with the deformed wall.
- Deform nearest point by $\Delta w = \epsilon \mu \dot{r} \cdot e_r (1 + C_r)$
- Repeat with another particle trajectory.

Deformation Particle Model



Deformation Particle Model

Model Results

- The curve for the deformation is consistent with the observed result.
- The curve becomes wider with increased standard deviation, accounting for the widening of the deformation along the tube.
- The model provides a means to describe the wear pattern.

Spinning Particle Model

Considering Angular Momentum

- Every time the particle hits a wall, it applies torque and the particle should spin.
- Assumption 1: The change in angular momentum while traveling between two bounces is negligible.
- Assumption 2: The angular momentum imparted by a collision with the wall is proportional to the tangential velocity.
- We solve a set of difference equations given by the following recursion:

A Sequence of Unfortunate Bounces

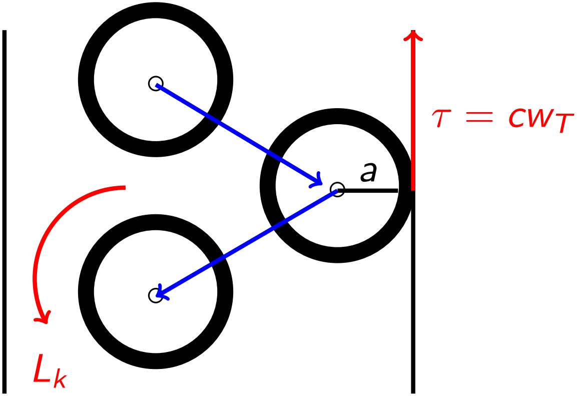
$$L_{k+1} = \left(1 - Ca^2 \frac{\delta t}{I}\right) L_k + (-1)^k Cwa\delta t, \text{ where}$$

- L_i is the angular momentum after the ith bounce.
- a is the radius of the particle.
- δt is the time for which the particle is pressed against the wall.
- I is the moment of inertia of the particle.
- w is the tangential velocity.
- C is the angular drag constant.
- and solution:

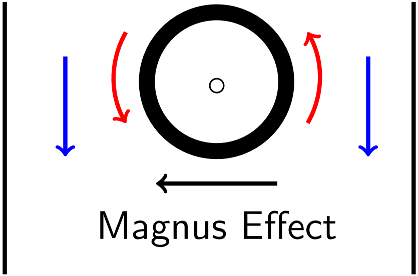
$$L_k = (-1)^k \left[m_0 \left(\frac{ca^2}{I}\delta t - 1 \right)^k + \frac{cwa\delta t}{\frac{ca^2}{I}\delta t - 2} \right]$$

$$\text{for } m_0 = L_0 - \frac{cwl\delta t}{\frac{cl^2}{I} - 2}$$

Spinning Particle Model



Curveball



Spinning Particle Model

The Magnus Effect

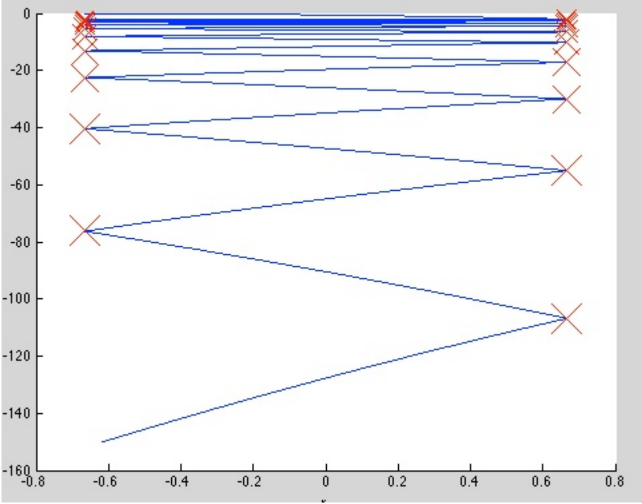
- The effect of the pressure differential on lateral velocity is given by:

$$\text{Lateral acceleration } = \frac{F_k}{m} = \frac{S}{m}(L_k \times v)$$

- So our adjusted model becomes:

$$\ddot{r} = -k\|\dot{r} - e_z v\|(\dot{r} - e_z v) + \frac{F_k}{m} \cdot e_x$$

Effect on the Cylinder's Wall



Conclusion

- Continuum Modeling
  - Interaction between particles & streams strongly alters flow over short segment of mixing tube
  - 2-phase water/air model with particles dynamics required to describe turbulent flow in tube
  - Momentum losses to wall need more study
- Particle Models
  - Wall collision models with & without angular momentum considered
  - Appear to explain wear in mixing tube

Acknowledgements

Thank you for your attention and our sponsors for making this possible!