The Problem with Pleats

Pall Group

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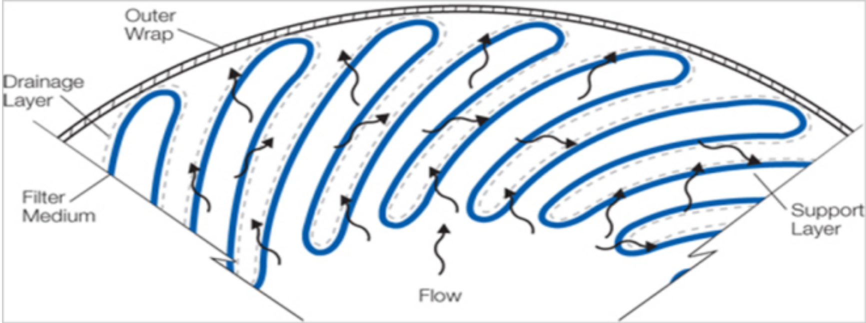
Outline

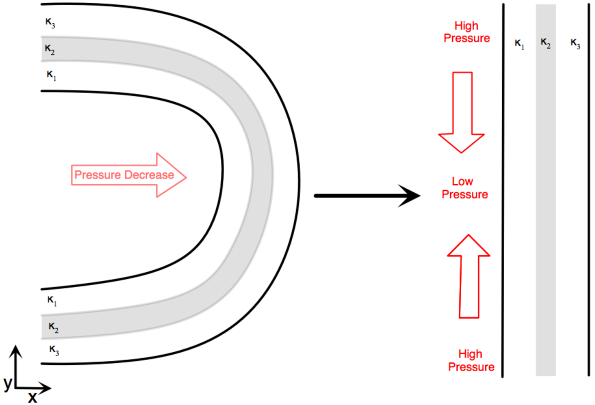
Background



Outer Wrap
Drainage Layer
Filter Medium
Flow
Support Layer

Outline

Formulation



Simulating the bent filter using a flat filter with varying pressure and permeability.

Form of Analytical Solution (Constant κ_i)

Assuming the fluid satisfies Darcy's Law with the following boundary conditions:

$$P(0,y) = P_0 + \Delta P_0 \cos(ly)$$
$$P(H3,y) = P_F + \Delta P_F \cos(ly)$$

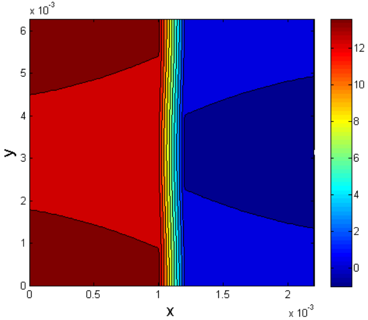
Using the form,

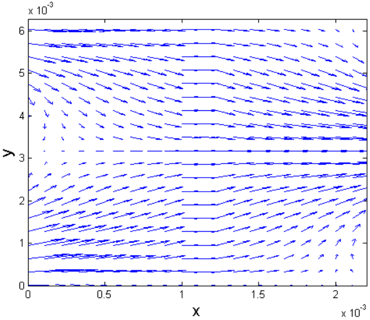
$$P(x,y) = a_1 x + a_2 + (A_1 e^{lx} + A_2 e^{-lx}) \cos(ly)$$

where a_1, a_2, A_1, A_2 are piecewise constants chosen to satisfy both the boundary conditions along with ensuring continuity of both pressure and fluid velocity at the interfaces.

Numerical Results

The following plots are the pressure and velocity field.

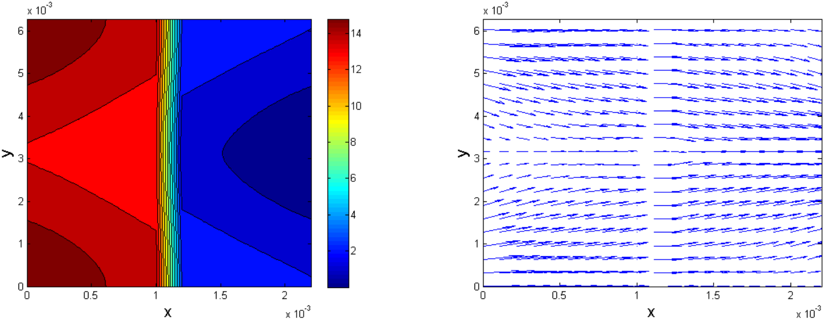




As expected, the flow travels from high pressure to low pressure and the concentration of the flow is through the tip.

Changing Porosity

We now allow the permeability, κ_2, to change in y, with the minimum value at the tip of the pleat,

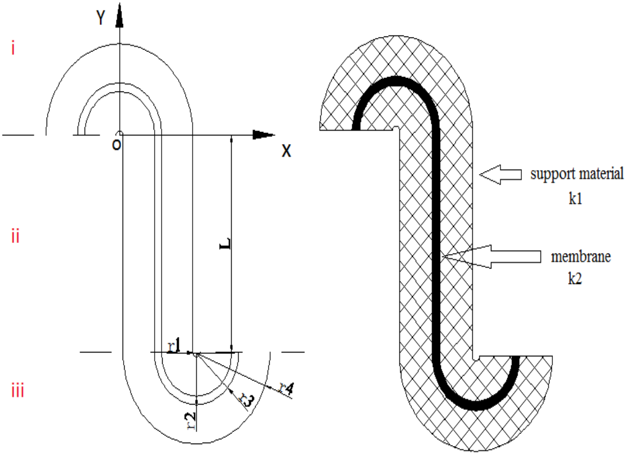


We notice the pressure and velocity are similar to the case where k_2 is constant, however the fluid flow is not as concentrated at the tip as in the prior case.

Outline

Goal:

Find the pressure through an 'S'-like domain using a conformal mapping approach.



- A conformal mapping is an easy way to reduce the problem to an easier domain

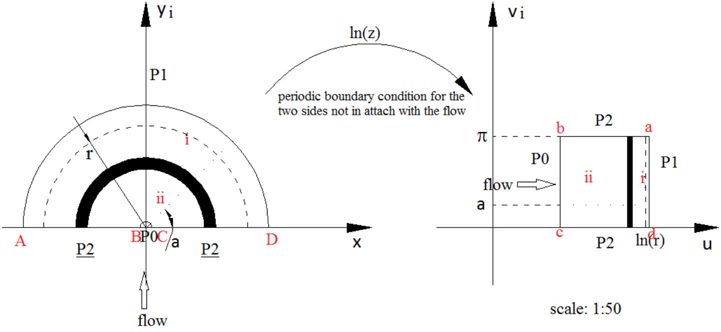
  -**especially** when pressure is non-trivial
- $\nabla \cdot (k\nabla P) = 0$: when k is constant, P is harmonic.

Process

- Broke up the domain into three sections:
  -the top semi-circle, the lower semi-circle, and the middle rectangle connecting the semi-circles
- Applied a conformal mapping to each semi-circle
- Calculated the pressure in the new domain and then mapped back to original domain
- Match the three sub-domains using the continuity of both pressure and the normal derivative along the boundaries

First Conformal Mapping: Top Semi-Circle

$$f(z) = \ln(z) \quad \text{for } z \in \mathbb{C}$$
$$u = \ln\sqrt{x^2 + y^2}$$
$$v = \arctan\left(\frac{y}{x}\right) + \frac{\pi}{2}$$



- In the simplest case, we let P_0 and P_1 be constant.
- Pressure in the rectangular domain gives us,

$$P' = A_i{}' u + B_i{}'.$$

- Pressure mapped back to original domain gives us:

$$P = A_i \ln \sqrt{x^2 + y^2} + B_i,$$

for $i = 1, 2, 3$ for each layer in the domain. Pressure on the boundary ($y = 0$):

$$\underline{P_2}^1 = A_i \ln(x) + B_i$$

- Normal derivative on the boundary:

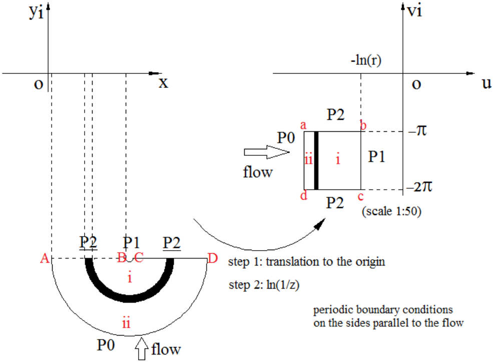
$$\frac{\partial \underline{P_2}^1}{\partial n} = 0$$

Second Conformal Mapping: Bottom Semi-Circle

$$f(\tilde{z}) = \ln\left(\frac{1}{\tilde{z}}\right) \qquad \text{for } \tilde{z} = z - r_4 - r_1 + iL$$

$$u = -\ln\sqrt{(x - r_4 - r_1)^2 + (y + L)^2}$$

$$v = -\arctan\left(\frac{y + L}{x - r_4 - r_1}\right) - \frac{3\pi}{2}$$



- Pressure mapped back to original domain:

$$P = -C_i \ln \sqrt{(x - r_4 - r_1)^2 + (y + L)^2} + D_i$$

for $i = 1, 2, 3$ for each layer in the domain.

- Pressure on the boundary ($y = -L$):

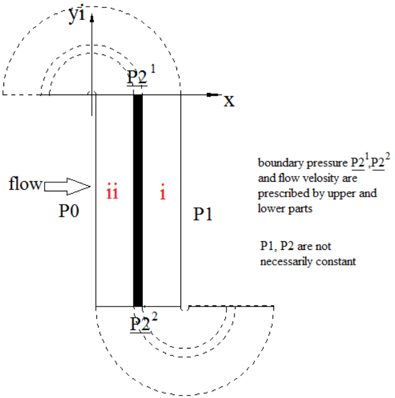
$$\underline{P_2}^2 = -C_i \ln(x - r_4 - r_1) + D_i$$

- Normal derivative on the boundary:

$$\frac{\partial \underline{P_2}^2}{\partial n} = 0$$

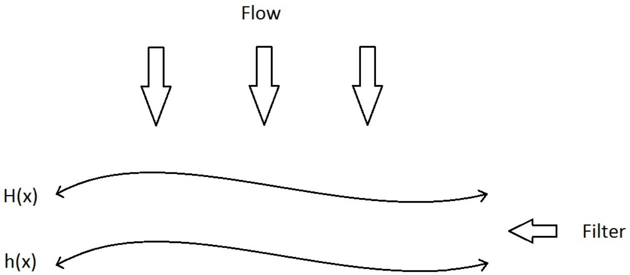
Middle Sub-domain

We use this middle sub-domain to match the two semi-circles using the continuity of pressure and normal derivative along the boundaries.



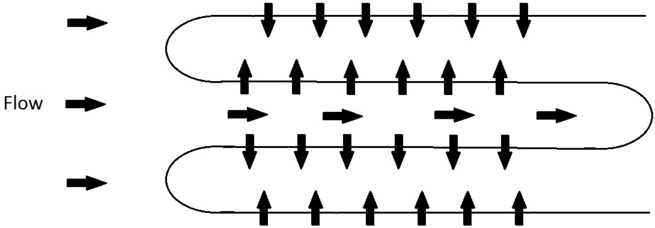
Outline

- Solve Laplace in curvi-linear coordinate system.
- H and h are periodic functions such that $H(x) > h(x)$.



- Boundary conditions were determined by physical constraints

$$\frac{dP}{dy} = AQ(x), \quad \frac{d^2P}{dy^2} = -2Av, \quad \frac{dQ}{dy} = -2v$$

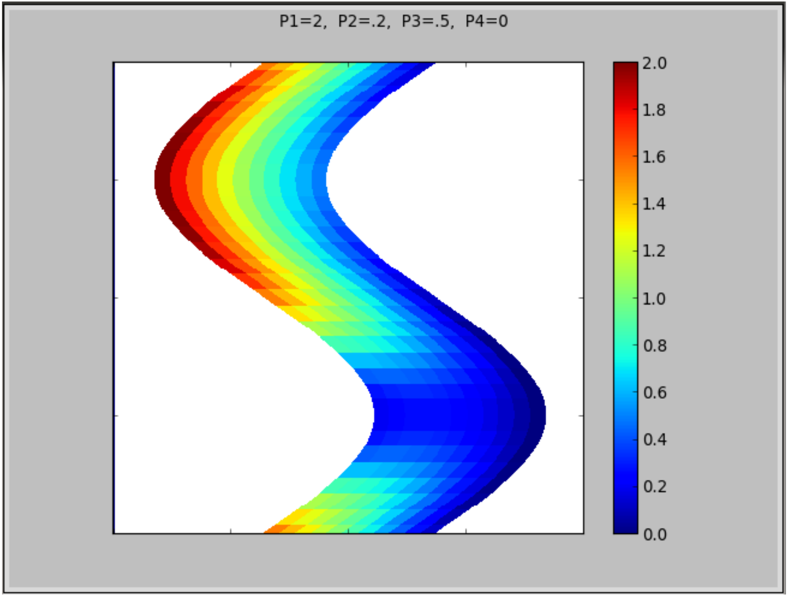


- Pressure change in the x direction is quadratic
- The boundary conditions for pressure take the form

$$P_{in}(x) = (P_2 - P_1)x^2 - P_1, \qquad P_1 > P_2$$

$$P_{out}(x) = (P_4 - P_3)x^2 - P_3, \qquad P_3 > P_4$$

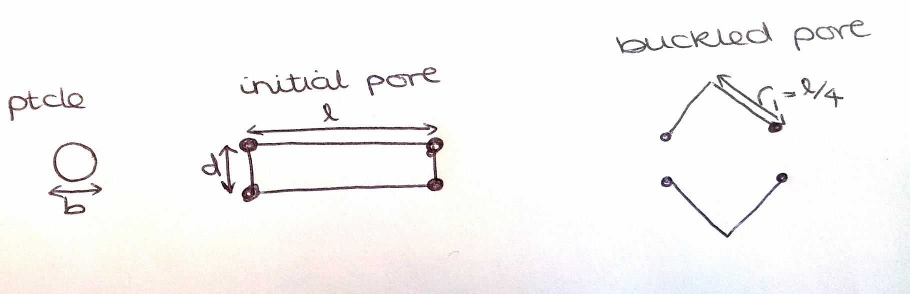
- Finite Difference Result



Outline

Does the permeability of the filter change due to bending?

Under compression



- Calculated strain gives 1 in 3 pores open like this.
- Number density of large pores in first layer $N_1 = \frac{1}{3ld}$.

- Place equivalent second layer on top.
- Number density of large pores through both layers,

$$N_2 = (2(r_1 + r_1 - b))^2/(3ld).$$

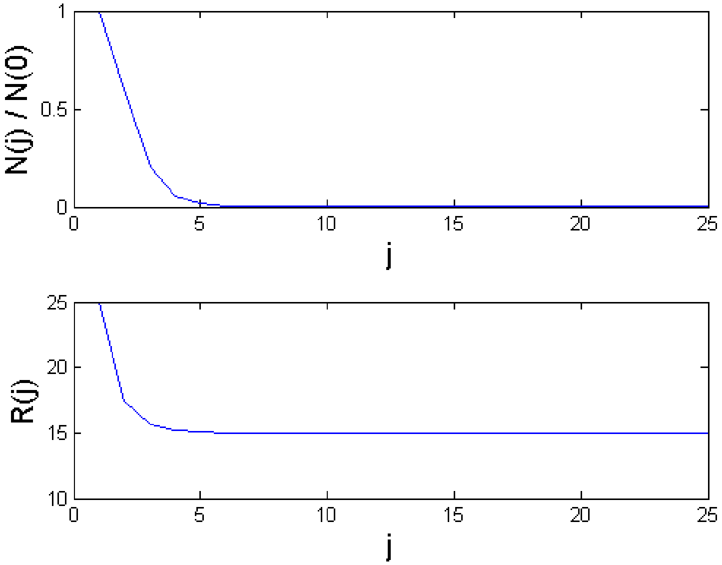
- Large pores have average size

$$r_2 = \frac{1}{4}\left(r_1 + r_1 + b\right).$$

- Composite filter of $m + 1$ equivalent layers gives

$$N_{m+1} = \frac{(r_1 + r_m - b)^2}{3r_1 d} N_m, \qquad r_{m+1} = \frac{r_1 + r_m + b}{4}$$

Results



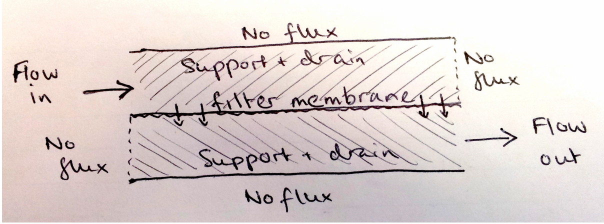
Outline

How could pleated geometry be affecting membrane performance?

- Pall noted that pleated membrane appears to under-perform in terms of particle capture relative to the equivalent flat membrane.
- We model the test scenario in which two membranes of the same area, one flat and the other pleated, are subjected to the same total flux.
- We use an idealized pleat geometry in which the membrane is a thin layer surrounded by porous slabs (the support & drain) and address the question of how particle capture differs in this case and the flat membrane case.

Idealized pleat geometry

Filter membrane lies along x-axis, $0 \leq x \leq S$, with support & drain layers above and below:



Simple model for particle capture (membrane blocking)

- We model the membrane as a simple permeable layer, length S in x-direction, where the number of open (unblocked) pores per unit area, N, is a decreasing function of the local flux per unit area, Q:

$$\frac{dN}{dt} \propto -Q$$

- When a pore is blocked by a particle, its resistance increases by a constant amount.
- Cylindrical pores, radius a, assumed for simplicity (but model easily generalized):

$$\begin{aligned} \text{Resistance of unblocked pore:} &\quad \tfrac{d}{\kappa(a)} \\ \text{Resistance of blocked pore:} &\quad \rho_b + \tfrac{d}{\kappa(a)} \end{aligned}$$

Particle capture (membrane blocking) continued:

- Total flux (per unit area) through membrane is related to pressure drop across membrane by sum of fluxes through unblocked and blocked pores:

$$Q = \Delta P \left(\frac{N}{d/\kappa(a)} + \frac{N_0 - N}{\rho_b + d/\kappa(a)} \right)$$

- Flat membrane case especially simple then: N is spatially homogeneous, decreases linearly in time, and pressure drop across membrane for a given flux Q can be calculated from the above expression.
- Modify this model for the simplified pleat geometry, where ΔP varies spatially, leading to local variations in the flux per unit area, inducing spatial variations in concentration of blocked pores, $N(x, t)$.

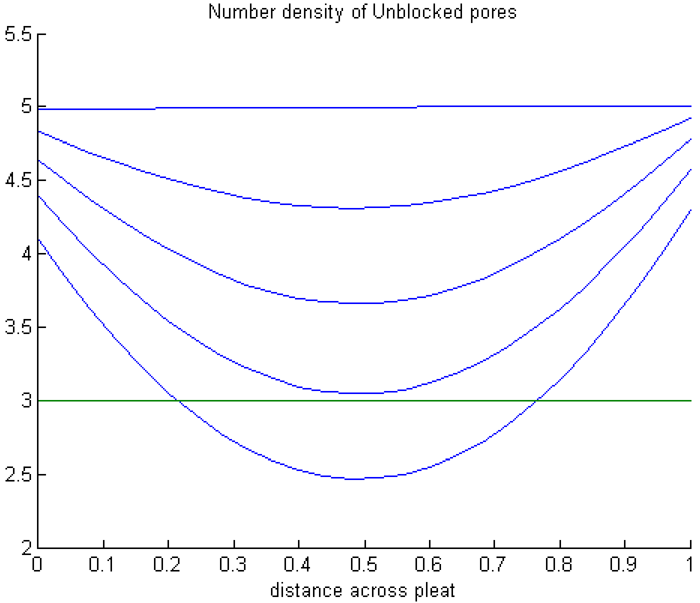
Particle capture in pleated membrane

- Number of open pores per unit area initially constant: N_0.
- Membrane permeability therefore spatially uniform initially $\Rightarrow$ solve for fluid flow through the pleat.
- Leads to spatially varying pressure drop $\Delta P(x)$, and hence spatially varying flux per unit area, $Q(x)$, satisfying $\int_0^S Q(x)\,dx = Q_{\mathrm{tot}}$ (specified total flux).
- Time-evolve $N(x,t)$ according to local membrane flux:

$$\frac{\partial N}{\partial t} \propto -Q(x) \quad \Rightarrow \quad N(x,t) \text{ at next time step}$$

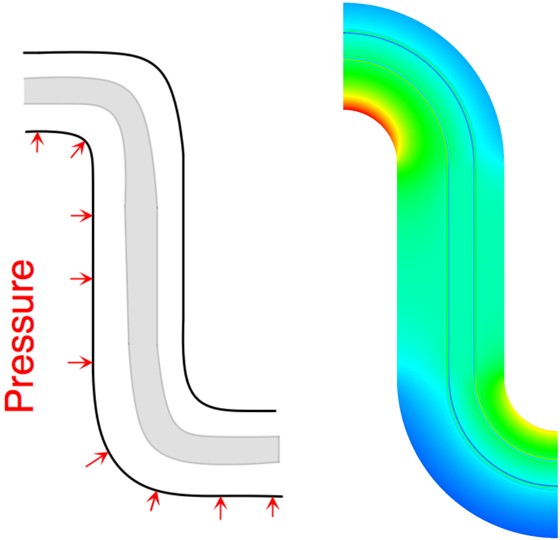
  (Q actually varying each time step as well).
- $N(x,t)$ determines new membrane permeability; solve for fluid flow through pleat again; ETC.

Results: Comparison with flat membrane case



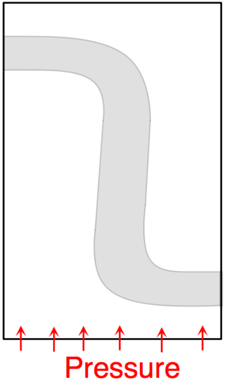
Outline

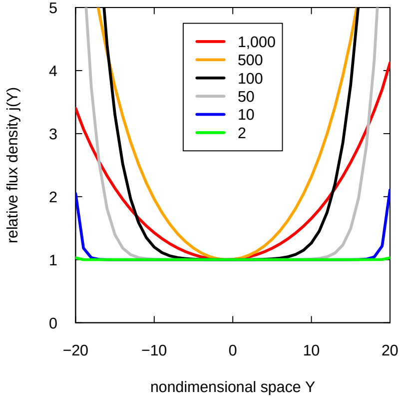
Prescribe a pressure on the surface of a curved three-layer structure and calculate the fluid velocity field.



The flow profile through the flat segment depends on the permeability of the membrane.

new boundary condition to simulate drag in support layers





Conclusions

- Flat filter model agreed with Pall's experimental results.
- Buckling shouldn't significantly degrade porosity.
- We have developed a variety of different models and methods for Pall to consider for further research.
- Geometry significantly affects flow and particle capture.

We would like to especially thank WPI, the NSF, and Industry Sponsors.