The Problem with Pleats

Pall Group

MPI Workshop, June 21, 2013

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1. Problem Set Up

2. 2D Flow in the Flat Filter with Support; Darcy’s Law

3. Conformal Map

4. Differential Geometry

5. Filter Buckling; A Worst Case Scenario

6. Idealized flow in pleated membrane: How geometry affects particle capture

7. Flow in a Pleated Segment with Fixed Pressure
Background
Outline

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Simulating the bent filter using a flat filter with varying pressure and permeability.
Assuming the fluid satisfies Darcy’s Law with the following boundary conditions:

\[ P(0, y) = P_0 + \Delta P_0 \cos(ly) \]
\[ P(H3, y) = P_F + \Delta P_F \cos(ly) \]

Using the form,

\[ P(x, y) = a_1 x + a_2 + (A_1 e^{lx} + A_2 e^{-lx}) \cos(ly) \]

where \( a_1, a_2, A_1, A_2 \) are piecewise constants chosen to satisfy both the boundary conditions along with ensuring continuity of both pressure and fluid velocity at the interfaces.
The following plots are the pressure and velocity field.

As expected, the flow travels from high pressure to low pressure and the concentration of the flow is through the tip.
Changing Porosity

We now allow the permeability, $\kappa_2$, to change in $y$, with the minimum value at the tip of the pleat,

We notice the pressure and velocity are similar to the case where $k_2$ is constant, however the fluid flow is not as concentrated at the tip as in the prior case.
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Goal:
Find the pressure through an ‘S’-like domain using a conformal mapping approach.
A conformal mapping is an easy way to reduce the problem to an easier domain
- especially when pressure is non-trivial
\[ \nabla \cdot (k \nabla P) = 0: \text{ when } k \text{ is constant, } P \text{ is harmonic.} \]
Process

- Broke up the domain into three sections:
  - the top semi-circle, the lower semi-circle, and the middle rectangle connecting the semi-circles
- Applied a conformal mapping to each semi-circle
- Calculated the pressure in the new domain and then mapped back to original domain
- Match the three sub-domains using the continuity of both pressure and the normal derivative along the boundaries
First Conformal Mapping: Top Semi-Circle

\[ f(z) = \ln(z) \quad \text{for } z \in \mathbb{C} \]

\[ u = \ln \sqrt{x^2 + y^2} \]

\[ \nu = \arctan \left( \frac{y}{x} \right) + \frac{\pi}{2} \]

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• In the simplest case, we let $P_0$ and $P_1$ be constant.

• Pressure in the rectangular domain gives us,

$P' = A_i' u + B_i'$.  

• Pressure mapped back to original domain gives us:

$P = A_i \ln \sqrt{x^2 + y^2} + B_i$,  

for $i = 1, 2, 3$ for each layer in the domain. Pressure on the boundary ($y = 0$):

$P_{21} = A_i \ln(x) + B_i$

• Normal derivative on the boundary:

$\frac{\partial P_{21}}{\partial n} = 0$
Second Conformal Mapping: Bottom Semi-Circle

\[ f(\tilde{z}) = \ln \left( \frac{1}{\tilde{z}} \right) \quad \text{for} \quad \tilde{z} = z - r_4 - r_1 + iL \]

\[ u = -\ln \sqrt{(x - r_4 - r_1)^2 + (y + L)^2} \]

\[ v = -\arctan \left( \frac{y + L}{x - r_4 - r_1} \right) - \frac{3\pi}{2} \]
Pressure mapped back to original domain:

\[ P = -C_i \ln \sqrt{(x - r_4 - r_1)^2 + (y + L)^2 + D_i} \]

for \( i = 1, 2, 3 \) for each layer in the domain.

Pressure on the boundary \((y = -L)\):

\[ P_2^2 = -C_i \ln(x - r_4 - r_1) + D_i \]

Normal derivative on the boundary:

\[ \frac{\partial P_2^2}{\partial n} = 0 \]
Middle Sub-domain
We use this middle sub-domain to match the two semi-circles using the continuity of pressure and normal derivative along the boundaries.

boundary pressure $P_2^1, P_2^2$ and flow velocity are prescribed by upper and lower parts

$P_1, P_2$ are not necessarily constant
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• Solve Laplace in curvi-linear coordinate system.
• \( H \) and \( h \) are periodic functions such that \( H(x) > h(x) \).
Boundary conditions were determined by physical constraints

\[
\frac{dP}{dy} = AQ(x), \quad \frac{d^2P}{dy^2} = -2Av, \quad \frac{dQ}{dy} = -2v
\]
Pressure change in the x direction is quadratic
The boundary conditions for pressure take the form

\[ P_{in}(x) = (P_2 - P_1)x^2 - P_1, \quad P_1 > P_2 \]

\[ P_{out}(x) = (P_4 - P_3)x^2 - P_3, \quad P_3 > P_4 \]
Finite Difference Result

P1=2, P2=.2, P3=.5, P4=0
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Does the permeability of the filter change due to bending?

Under compression

- Calculated strain gives 1 in 3 pores open like this.
- Number density of large pores in first layer $N_1 = \frac{1}{3ld}$.
Place equivalent second layer on top.

Number density of large pores through both layers,

\[ N_2 = \frac{(2(r_1 + r_1 - b))^2}{(3ld)}. \]

Large pores have average size

\[ r_2 = \frac{1}{4} (r_1 + r_1 + b). \]

Composite filter of \( m + 1 \) equivalent layers gives

\[ N_{m+1} = \frac{(r_1 + r_m - b)^2}{3r_1d} N_m, \quad r_{m+1} = \frac{r_1 + r_m + b}{4} \]
Results
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How could pleated geometry be affecting membrane performance?

- Pall noted that pleated membrane appears to under-perform in terms of particle capture relative to the equivalent flat membrane.

- We model the test scenario in which two membranes of the same area, one flat and the other pleated, are subjected to the same total flux.

- We use an idealized pleat geometry in which the membrane is a thin layer surrounded by porous slabs (the support & drain) and address the question of how particle capture differs in this case and the flat membrane case.
Idealized pleat geometry

Filter membrane lies along $x$-axis, $0 \leq x \leq S$, with support & drain layers above and below:
We model the membrane as a simple permeable layer, length \( S \) in \( x \)-direction, where the number of open (unblocked) pores per unit area, \( N \), is a decreasing function of the local flux per unit area, \( Q \):

\[
\frac{dN}{dt} \propto -Q
\]

When a pore is blocked by a particle, its resistance increases by a constant amount.

Cylindrical pores, radius \( a \), assumed for simplicity (but model easily generalized):

- Resistance of unblocked pore: \( \frac{d}{\kappa(a)} \)
- Resistance of blocked pore: \( \rho_b + \frac{d}{\kappa(a)} \)
Particle capture (membrane blocking) continued:

- Total flux (per unit area) through membrane is related to pressure drop across membrane by sum of fluxes through unblocked and blocked pores:

\[
Q = \Delta P \left( \frac{N}{d/\kappa(a)} + \frac{N_0 - N}{\rho_b + d/\kappa(a)} \right)
\]

- Flat membrane case especially simple then: \( N \) is spatially homogeneous, decreases linearly in time, and pressure drop across membrane for a given flux \( Q \) can be calculated from the above expression.

- Modify this model for the simplified pleat geometry, where \( \Delta P \) varies spatially, leading to local variations in the flux per unit area, inducing spatial variations in concentration of blocked pores, \( N(x, t) \).
Number of open pores per unit area initially constant: $N_0$.
Membrane permeability therefore spatially uniform initially $\Rightarrow$ solve for fluid flow through the pleat.

Leads to spatially varying pressure drop $\Delta P(x)$, and hence spatially varying flux per unit area, $Q(x)$, satisfying $\int_0^S Q(x) \, dx = Q_{\text{tot}}$ (specified total flux).

Time-evolve $N(x, t)$ according to local membrane flux:

$$\frac{\partial N}{\partial t} \propto -Q(x) \quad \Rightarrow \quad N(x, t) \text{ at next time step}$$

($Q$ actually varying each time step as well).

$N(x, t)$ determines new membrane permeability; solve for fluid flow through pleat again; ETC.
Results: Comparison with flat membrane case

Number density of Unblocked pores

(distance across pleat)

2
2.5
3
3.5
4
4.5
5
5.5

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
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Prescribe a pressure on the surface of a curved three-layer structure and calculate the fluid velocity field.
The flow profile through the flat segment depends on the permeability of the membrane.

new boundary condition to simulate drag in support layers
Conclusions

- Flat filter model agreed with Pall’s experimental results.
- Buckling shouldn’t significantly degrade porosity.
- We have developed a variety of different models and methods for Pall to consider for further research.
- Geometry significantly affects flow and particle capture.
We would like to especially thank WPI, the NSF, and Industry Sponsors.