Chapter 14
The MODEL Procedure

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Chapter 14 The MODEL Procedure

Overview

The MODEL procedure analyzes models in which the relationships among the variables comprise a system of one or more nonlinear equations. Primary uses of the MODEL procedure are estimation, simulation, and forecasting of nonlinear simultaneous equation models.

PROC MODEL features include

- SAS programming statements to define simultaneous systems of nonlinear equations
- tools to analyze the structure of the simultaneous equation system
- ARIMA, PDL, and other dynamic modeling capabilities
- tools to specify and estimate the error covariance structure
- tools to estimate and solve ordinary differential equations
- the following methods for parameter estimation:
  - Ordinary Least Squares (OLS)
  - Two-Stage Least Squares (2SLS)
  - Seemingly Unrelated Regression (SUR) and iterative SUR (ITSUR)
  - Three-Stage Least Squares (3SLS) and iterative 3SLS (IT3SLS)
  - Generalized Method of Moments (GMM)
  - Full Information Maximum Likelihood (FIML)
- simulation and forecasting capabilities
- Monte Carlo simulation
- goal seeking solutions

A system of equations can be nonlinear in the parameters, nonlinear in the observed variables, or nonlinear in both the parameters and the variables. *Nonlinear* in the parameters means that the mathematical relationship between the variables and parameters is not required to have a linear form. (A linear model is a special case of a nonlinear model.) A general nonlinear system of equations can be written as

$$\begin{array}{ll} q_1 & (y_{1,t}, y_{2,t}, \ldots, y_{g,t}, x_{1,t}, x_{2,t}, \ldots, x_{m,t}, \theta_1, \theta_2, \ldots, \theta_p) = \epsilon_{1,t} \\ q_2 & (y_{1,t}, y_{2,t}, \ldots, y_{g,t}, x_{1,t}, x_{2,t}, \ldots, x_{m,t}, \theta_1, \theta_2, \ldots, \theta_p) = \epsilon_{2,t} \\ \vdots & \\ q_g & (y_{1,t}, y_{2,t}, \ldots, y_{g,t}, x_{1,t}, x_{2,t}, \ldots, x_{m,t}, \theta_1, \theta_2, \ldots, \theta_p) = \epsilon_{g,t} \end{array}$$

where $y_{i,t}$ is an endogenous variable, $x_{i,t}$ is an exogenous variable, θ_i is a parameter, and ϵ_i is the unknown error. The subscript t represents time or some index to the data. In econometrics literature, the observed variables are either *endogenous* (dependent) variables or *exogenous* (independent) variables. This system can be written more succinctly in vector form as

$$\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) = \epsilon_t$$

This system of equations is in *general form* because the error term is by itself on one side of the equality. Systems can also be written in *normalized form* by placing the endogenous variable on one side of the equality, with each equation defining a predicted value for a unique endogenous variable. A normalized form equation system can be written in vector notation as

$$\mathbf{y}_t = \mathbf{f}(\mathbf{y}_t, \mathbf{x}_t, \theta) + \epsilon_t.$$

PROC MODEL handles equations written in both forms.

Econometric models often explain the current values of the endogenous variables as functions of past values of exogenous and endogenous variables. These past values are referred to as *lagged* values, and the variable x_{t-i} is called lag *i* of the variable x_t. Using lagged variables, you can create a *dynamic*, or time dependent, model. In the preceding model systems, the lagged exogenous and endogenous variables are included as part of the exogenous variables.

If the data are time series, so that *t* indexes time (see Chapter 2, "Working with Time Series Data," for more information on time series), it is possible that ϵ_t depends on ϵ_{t-i} or, more generally, the ϵ_t's are not identically and independently distributed. If the errors of a model system are autocorrelated, the standard error of the estimates of the parameters of the system will be inflated.

Sometimes the ϵ_i's are not identically distributed because the variance of ϵ is not constant. This is known as *heteroscedasticity*. Heteroscedasticity in an estimated model can also inflate the standard error of the estimates of the parameters. Using a weighted estimation can sometimes eliminate this problem. Alternately, a variance model such as GARCH or EGARCH can be estimated to correct for heteroscedasticity. If the proper weighting scheme and the form of the error model is difficult to determine, generalized methods of moments (GMM) estimation can be used to determine parameter estimates that are asymptotically more efficient than the OLS parameter estimates.

Other problems may also arise when estimating systems of equations. Consider the system of equations:

$$\begin{aligned} y_{1,t} &= \theta_1 + (\theta_2 + \theta_3\theta_4^t)^{-1} + \theta_5 y_{2,t} + \epsilon_{1,t} \\ y_{2,t} &= \theta_6 + (\theta_7 + \theta_8\theta_9^t)^{-1} + \theta_{10} y_{1,t} + \epsilon_{2,t} \end{aligned}$$

which is nonlinear in its parameters and cannot be estimated with linear regression. This system of equations represents a rudimentary predator-prey process with y_1 as the prey and y_2 as the predator (the second term in both equations is a logistics curve). The two equations must be estimated simultaneously because of the cross dependency of y's. Nonlinear ordinary least-squares estimation of these equations will produce biased and inconsistent parameter estimates. This is called *simultaneous equation bias*.

One method to remove simultaneous equation bias, in the linear case, is to replace the endogenous variables on the right-hand side of the equations with predicted values that are uncorrelated with the error terms. These predicted values can be obtained through a preliminary, or "first stage," *instrumental variable regression*. *Instrumental variables*, which are uncorrelated with the error term, are used as regressors to model the predicted values. The parameter estimates are obtained by a second regression using the predicted values of the regressors. This process is called *two-stage least squares*.

In the nonlinear case, nonlinear ordinary least-squares estimation is performed iteratively using a linearization of the model with respect to the parameters. The instrumental solution to simultaneous equation bias in the nonlinear case is the same as the linear case except the linearization of the model with respect to the parameters is predicted by the instrumental regression. Nonlinear two-stage least squares is one of several instrumental variables methods available in the MODEL procedure to handle simultaneous equation bias.

When you have a system of several regression equations, the random errors of the equations can be correlated. In this case, the large-sample efficiency of the estimation can be improved by using a joint generalized least-squares method that takes the cross-equation correlations into account. If the equations are not simultaneous (no dependent regressors), then *seemingly unrelated regression* (SUR) can be used. The SUR method requires an estimate of the cross-equation error covariance matrix, Σ. The usual approach is to first fit the equations using OLS, compute an estimate $\hat{\Sigma}$ from the OLS residuals, and then perform the SUR estimation based on $\hat{\Sigma}$. The MODEL procedure estimates Σ by default, or you can supply your own estimate of Σ.

If the equation system is simultaneous, you can combine the 2SLS and SUR methods to take into account both simultaneous equation bias and cross-equation correlation of the errors. This is called *three-stage least squares* or 3SLS.

A different approach to the simultaneous equation bias problem is the full information maximum likelihood, or FIML, estimation method. FIML does not require instrumental variables, but it assumes that the equation errors have a multivariate normal distribution. 2SLS and 3SLS estimation do not assume a particular distribution for the errors.

Once a nonlinear model has been estimated, it can be used to obtain forecasts. If the model is linear in the variables you want to forecast, a simple linear solve can generate the forecasts. If the system is nonlinear, an iterative procedure must be used. The preceding example system is linear in its endogenous variables. The MODEL procedure's SOLVE statement is used to forecast nonlinear models.

One of the main purposes of creating models is to obtain an understanding of the relationship among the variables. There are usually only a few variables in a model you can control (for example, the amount of money spent on advertising). Often you want to determine how to change the variables under your control to obtain some target goal. This process is called *goal seeking*. PROC MODEL allows you to solve for any subset of the variables in a system of equations given values for the remaining variables.

The nonlinearity of a model creates two problems with the forecasts: the forecast errors are not normally distributed with zero mean, and no formula exits to calculate the forecast confidence intervals. PROC MODEL provides Monte Carlo techniques, which, when used with the covariance of the parameters and error covariance matrix, can produce approximate error bounds on the forecasts.

Getting Started

This section introduces the MODEL procedure and shows how to use PROC MODEL for several kinds of nonlinear regression analysis and nonlinear systems simulation problems.

Nonlinear Regression Analysis

One of the most important uses of PROC MODEL is to estimate unknown parameters in a nonlinear model. A simple nonlinear model has the form:

$$y = f(\mathbf{x}, \theta) + \epsilon$$

where $\mathbf{x}$ is a vector of exogenous variables. To estimate unknown parameters using PROC MODEL, do the following:

1. Use the DATA= option in a PROC MODEL statement to specify the input SAS data set containing $\mathbf{y}$ and $\mathbf{x}$, the observed values of the variables.
2. Write the equation for the model using SAS programming statements, including all parameters and arithmetic operators but leaving off the unobserved error component, ϵ.
3. Use a FIT statement to fit the model equation to the input data to determine the unknown parameters, θ.

An Example

The SASHELP library contains the data set CITIMON, which contains the variable LHUR, the monthly unemployment figures, and the variable IP, the monthly industrial production index. You suspect that the unemployment rates are inversely proportional to the industrial production index. Assume that these variables are related by the following nonlinear equation:

$$lhur = \frac{1}{a \cdot \mathrm{ip} + b} + c + \epsilon$$

In this equation *a*, *b*, and *c* are unknown coefficients and ϵ is an unobserved random error.

The following statements illustrate how to use PROC MODEL to estimate values for *a*, *b*, and *c* from the data in SASHELP.CITIMON.

```
proc model data=sashelp.citimon;
   lhur = 1/(a * ip + b) + c;
   fit lhur;
run;
```

Notice that the model equation is written as a SAS assignment statement. The variable LHUR is assumed to be the dependent variable because it is named in the FIT statement and is on the left-hand side of the assignment.

PROC MODEL determines that LHUR and IP are observed variables because they are in the input data set. A, B, and C are treated as unknown parameters to be estimated from the data because they are not in the input data set. If the data set contained a variable named A, B, or C, you would need to explicitly declare the parameters with a PARMS statement.

In response to the FIT statement, PROC MODEL estimates values for A, B, and C using nonlinear least squares and prints the results. The first part of the output is a "Model Summary" table, shown in Figure 14.1.

```
                                The MODEL Procedure

                                   Model Summary

                             Model Variables         1
                             Parameters              3
                             Equations               1
                             Number of Statements    1


                               Model Variables  LHUR
                                    Parameters  a b c
                                     Equations  LHUR
```

Figure 14.1. Model Summary Report

This table details the size of the model, including the number of programming statements defining the model, and lists the dependent variables (LHUR in this case), the unknown parameters (A, B, and C), and the model equations. In this case the equation is named for the dependent variable, LHUR.

PROC MODEL then prints a summary of the estimation problem, as shown in Figure 14.2.

```
                                The MODEL Procedure

                            The Equation to Estimate is

                               LHUR =  F(a, b, c(1))
```

Figure 14.2. Estimation Problem Report

The notation used in the summary of the estimation problem indicates that LHUR is a function of A, B, and C, which are to be estimated by fitting the function to the data. If the partial derivative of the equation with respect to a parameter is a simple variable or constant, the derivative is shown in parentheses after the parameter name. In this case, the derivative with respect to the intercept C is 1. The derivatives with respect to A and B are complex expressions and so are not shown.

Next, PROC MODEL prints an estimation summary as shown in Figure 14.3.

```
                        The MODEL Procedure
                       OLS Estimation Summary

                          Data Set Options

                      DATA=    SASHELP.CITIMON


                        Minimization Summary

                  Parameters Estimated            3
                  Method                      Gauss
                  Iterations                     10


                     Final Convergence Criteria

                     R                  0.000737
                     PPC(b)             0.003943
                     RPC(b)              0.00968
                     Object             4.784E-6
                     Trace(S)           0.533325
                     Objective Value    0.522214


                        Observations Processed

                            Read       145
                            Solved     145
                            Used       144
                            Missing      1
```

Figure 14.3. Estimation Summary Report

The estimation summary provides information on the iterative process used to compute the estimates. The heading "OLS Estimation Summary" indicates that the nonlinear ordinary least-squares (OLS) estimation method is used. This table indicates that all 3 parameters were estimated successfully using 144 nonmissing observations from the data set SASHELP.CITIMON. Calculating the estimates required 10 iterations of the GAUSS method. Various measures of how well the iterative process converged are also shown. For example, the "RPC(B)" value 0.00968 means that on the final iteration the largest relative change in any estimate was for parameter B, which changed by .968 percent. See the section "Convergence Criteria" later in this chapter for details.

PROC MODEL then prints the estimation results. The first part of this table is the summary of residual errors, shown in Figure 14.4.

```
                        The MODEL Procedure

               Nonlinear OLS Summary of Residual Errors

                  DF      DF                                        Adj
Equation       Model   Error        SSE        MSE   R-Square      R-Sq

LHUR               3     141    75.1989     0.5333     0.7472    0.7436
```

Figure 14.4. Summary of Residual Errors Report

This table lists the sum of squared errors (SSE), the mean square error (MSE), the root mean square error (Root MSE), and the R^2 and adjusted R^2 statistics. The R^2 value of .7472 means that the estimated model explains approximately 75 percent more of the variability in LHUR than a mean model explains.

Following the summary of residual errors is the parameter estimates table, shown in Figure 14.5.

```
                              The MODEL Procedure

                       Nonlinear OLS Parameter Estimates

                                      Approx                        Approx
          Parameter       Estimate     Std Err     t Value      Pr > |t|

          a               0.009046     0.00343        2.63        0.0094
          b               -0.57059      0.2617       -2.18        0.0309
          c               3.337151      0.7297        4.57        <.0001
```

Figure 14.5. Parameter Estimates

Because the model is nonlinear, the standard error of the estimate, the t value, and its significance level are only approximate. These values are computed using asymptotic formulas that are correct for large sample sizes but only approximately correct for smaller samples. Thus, you should use caution in interpreting these statistics for nonlinear models, especially for small sample sizes. For linear models, these results are exact and are the same as standard linear regression.

The last part of the output produced by the FIT statement is shown in Figure 14.6.

```
                              The MODEL Procedure

               Number of Observations     Statistics for System

               Used               144    Objective         0.5222
               Missing              1    Objective*N      75.1989
```

Figure 14.6. System Summary Statistics

This table lists the objective value for the estimation of the nonlinear system, which is a weighted system mean square error. This statistic can be used for testing cross-equation restrictions in multi-equation regression problems. See the section "Restrictions and Bounds on Parameters" for details. Since there is only a single equation in this case, the objective value is the same as the residual MSE for LHUR except that the objective value does not include a degrees of freedom correction. This can be seen in the fact that "Objective*N" equals the residual SSE, 75.1989. N is 144, the number of observations used.

Convergence and Starting Values

Computing parameter estimates for nonlinear equations requires an iterative process. Starting with an initial guess for the parameter values, PROC MODEL tries different parameter values until the objective function of the estimation method is minimized. (The objective function of the estimation method is sometimes called the *fitting function*.) This process does not always succeed, and whether it does succeed depends greatly on the starting values used. By default, PROC MODEL uses the starting value .0001 for all parameters.

Consequently, in order to use PROC MODEL to achieve convergence of parameter estimates, you need to know two things: how to recognize convergence failure by interpreting diagnostic output, and how to specify reasonable starting values. The MODEL procedure includes alternate iterative techniques and grid search capabilities to aid in finding estimates. See the section "Troubleshooting Convergence Problems" for more details.

Nonlinear Systems Regression

If a model has more than one endogenous variable, several facts need to be considered in the choice of an estimation method. If the model has endogenous regressors, then an instrumental variables method such as 2SLS or 3SLS can be used to avoid simultaneous equation bias. Instrumental variables must be provided to use these methods. A discussion of possible choices for instrumental variables is provided in the "Choice of Instruments" section in this chapter.

The following is an example of the use of 2SLS and the INSTRUMENTS statement:

```
proc model data=test2 ;
   exogenous x1 x2;
   parms a1 a2 b2 2.5 c2 55 d1;

   y1 = a1 * y2 + b2 * x1 * x1 + d1;
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2 + d1;

   fit y1 y2 / 2sls;
   instruments b2 c2 _exog_;
run;
```

The estimation method selected is added after the slash (/) on the FIT statement. The INSTRUMENTS statement follows the FIT statement and in this case selects all the exogenous variables as instruments with the _EXOG_ keyword. The parameters B2 and C2 on the instruments list request that the derivatives with respect to B2 and C2 be additional instruments.

Full information maximum likelihood (FIML) can also be used to avoid simultaneous equation bias. FIML is computationally more expensive than an instrumental variables method and assumes that the errors are normally distributed. On the other hand, FIML does not require the specification of instruments. FIML is selected with the FIML option on the FIT statement.

The preceding example is estimated with FIML using the following statements:

```
proc model data=test2 ;
   exogenous x1 x2;
   parms a1 a2 b2 2.5 c2 55 d1;

   y1 = a1 * y2 + b2 * x1 * x1 + d1;
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2 + d1;

   fit y1 y2 / fiml;
run;
```

General Form Models

The single equation example shown in the preceding section was written in normalized form and specified as an assignment of the regression function to the dependent variable LHUR. However, sometimes it is impossible or inconvenient to write a nonlinear model in normalized form.

To write a general form equation, give the equation a name with the prefix "EQ." . This EQ.-prefixed variable represents the equation error. Write the equation as an assignment to this variable.

For example, suppose you have the following nonlinear model relating the variables *x* and *y*:

$$\epsilon = a + b\ln(cy + dx)$$

Naming this equation ‘one’, you can fit this model with the following statements:

```
proc model data=xydata;
   eq.one = a + b * log( c * y + d * x );
   fit one;
run;
```

The use of the EQ. prefix tells PROC MODEL that the variable is an error term and that it should not expect actual values for the variable ONE in the input data set.

Demand and Supply Models

General form specifications are often useful when you have several equations for the same dependent variable. This is common in demand and supply models, where both the demand equation and the supply equation are written as predictions for quantity as functions of price.

For example, consider the following demand and supply system:

$$\text{(demand)} \qquad \text{quantity} = \alpha_1 + \alpha_2 \text{ price} + \alpha_3 \text{ income} + \epsilon_1$$

$$\text{(supply)} \qquad \text{quantity} = \beta_1 + \beta_2 \text{ price} + \epsilon_2$$

Assume the *quantity* of interest is the amount of energy consumed in the U.S.; the *price* is the price of gasoline, and the *income* variable is the consumer debt. When the market is at equilibrium, these equations determine the market price and the equilibrium quantity. These equations are written in general form as

$$\epsilon_1 = quantity - (\alpha_1 + \alpha_2\ price + \alpha_3\ income)$$

$$\epsilon_2 = quantity - (\beta_1 + \beta_2\ price)$$

Note that the endogenous variables *quantity* and *price* depend on two error terms so that OLS should not be used. The following example uses three-stage least-squares estimation.

Data for this model is obtained from the SASHELP.CITIMON data set.

```
title1 ’Supply-Demand Model using General-form Equations’;
proc model data=sashelp.citimon;
   endogenous eegp eec;
   exogenous exvus cciutc;
   parameters a1 a2 a3 b1 b2 ;
   label eegp   = ’Gasoline Retail Price’
         eec    = ’Energy Consumption’
         cciutc = ’Consumer Debt’;

   /* -------- Supply equation ------------ */
   eq.supply = eec - (a1 + a2 * eegp + a3 * cciutc);

   /* -------- Demand equation ------------ */
   eq.demand = eec - (b1 + b2 * eegp );

   /* -------- Instrumental variables -------*/
   lageegp = lag(eegp); lag2eegp=lag2(eegp);

   /* -------- Estimate parameters --------- */
   fit supply demand / n3sls fsrsq;
   instruments _EXOG_ lageegp lag2eegp;
run;
```

The FIT statement specifies the two equations to estimate and the method of estimation, N3SLS. Note that ‘3SLS’ is an alias for N3SLS. The option FSRSQ is selected to get a report of the first stage R^2 to determine the acceptability of the selected instruments.

Since three-stage least squares is an instrumental variables method, instruments are specified with the INSTRUMENTS statement. The instruments selected are all the exogenous variables, selected with the _EXOG_ option, and two lags of the variable EEGP, LAGEEGP and LAG2EEGP.

The data set CITIMON has four observations that generate missing values because values for either EEGP, EEC, or CCIUTC are missing. This is revealed in the "Observations Processed" output shown in Figure 14.7. Missing values are also generated when the equations cannot be computed for a given observation. Missing observations are not used in the estimation.

```
                Supply-Demand Model using General-form Equations

                              The MODEL Procedure
                            3SLS Estimation Summary

                             Observations Processed

                                 Read       145
                                 Solved     143
                                 First        3
                                 Last       145
                                 Used       139
                                 Missing      4
                                 Lagged       2
```

Figure 14.7. Supply-Demand Observations Processed

The lags used to create the instruments also reduce the number of observations used. In this case, the first 2 observations were used to fill the lags of EEGP.

The data set has a total of 145 observations, of which 4 generated missing values and 2 were used to fill lags, which left 139 observations for the estimation. In the estimation summary, in Figure 14.8, the total degrees of freedom for the model and error is 139.

```
                Supply-Demand Model using General-form Equations

                              The MODEL Procedure

                  Nonlinear 3SLS Summary of Residual Errors

                 DF     DF                                                Adj
Equation      Model  Error       SSE       MSE  Root MSE  R-Square     R-Sq

supply            3    136   39.5791    0.2910    0.5395
demand            2    137   43.2677    0.3158    0.5620


                       Nonlinear 3SLS Parameter Estimates

                                                                         1st
                              Approx                    Approx         Stage
  Parameter       Estimate     Std Err    t Value     Pr > |t|    R-Square

  a1               6.82196      0.3788      18.01       <.0001      1.0000
  a2              -0.00614     0.00303      -2.02       0.0450      0.9617
  a3                  9E-7   3.165E-7       2.84       0.0051      1.0000
  b1               7.30952      0.3799      19.24       <.0001      1.0000
  b2              -0.00853     0.00328      -2.60       0.0103      0.9617
```

Figure 14.8. Supply-Demand Parameter Estimates

One disadvantage of specifying equations in general form is that there are no actual values associated with the equation, so the R^2 statistic cannot be computed.

Solving Simultaneous Nonlinear Equation Systems

You can use a SOLVE statement to solve the nonlinear equation system for some variables when the values of other variables are given.

Consider the demand and supply model shown in the preceding example. The following statement computes equilibrium price (EEGP) and quantity (EEC) values for given observed cost (CCIUTC) values and stores them in the output data set EQUILIB.

```
title1 'Supply-Demand Model using General-form Equations';
proc model data=sashelp.citimon;
   endogenous eegp eec;
   exogenous exvus cciutc;
   parameters a1 a2 a3 b1 b2 ;
   label eegp   = 'Gasoline Retail Price'
         eec    = 'Energy Consumption'
         cciutc = 'Consumer Debt';

   /* -------- Supply equation ------------- */
   eq.supply = eec - (a1 + a2 * eegp + a3 * cciutc);

   /* -------- Demand equation ------------- */
   eq.demand = eec - (b1 + b2 * eegp );

   /* -------- Instrumental variables -------*/
   lageegp = lag(eegp); lag2eegp=lag2(eegp);

   /* -------- Estimate parameters --------- */
   instruments _EXOG_ lageegp lag2eegp;
   fit supply demand / n3sls ;
   solve eegp eec / out=equilib;
run;
```

As a second example, suppose you want to compute points of intersection between the square root function and hyperbolas of the form $a + b/x$. That is, solve the system:

$$\begin{aligned}
&\text{(square root)} \qquad && y = \sqrt{x} \\
&\text{(hyperbola)} \qquad && y = a + \frac{b}{x}
\end{aligned}$$

The following statements read parameters for several hyperbolas in the input data set TEST and solve the nonlinear equations. The SOLVEPRINT option on the SOLVE statement prints the solution values. The ID statement is used to include the values of A and B in the output of the SOLVEPRINT option.

```
data test;
  input a b @@;
  datalines;
  0 1   1 1   1 2
;

proc model data=test;
   eq.sqrt      = sqrt(x) - y;
   eq.hyperbola = a + b / x - y;
   solve x y / solveprint;
   id a b;
run;
```

The printed output produced by this example consists of a model summary report, a listing of the solution values for each observation, and a solution summary report. The model summary for this example is shown in Figure 14.9.

```
                Supply-Demand Model using General-form Equations

                              The MODEL Procedure

                                 Model Summary

                           Model Variables         2
                           ID Variables            2
                           Equations               2
                           Number of Statements    2


                        Model Variables  x y
                              Equations  sqrt hyperbola
```

Figure 14.9. Model Summary Report

The output produced by the SOLVEPRINT option is shown in Figure 14.10.

```
                                The MODEL Procedure
                              Simultaneous Simulation

Observation  1  a                  0  b      1.0000  eq.hyperbola   0.000000
                Iterations        17  CC   0.000000


                                 Solution Values

                                    x              y

                             1.000000       1.000000


Observation  2  a             1.0000  b      1.0000  eq.hyperbola   0.000000
                Iterations         5  CC   0.000000


                                 Solution Values

                                    x              y

                             2.147899       1.465571


Observation  3  a             1.0000  b      2.0000  eq.hyperbola   0.000000
                Iterations         4  CC   0.000000


                                 Solution Values

                                    x              y

                             2.875130       1.695621
```

Figure 14.10. Solution Values for Each Observation

For each observation, a heading line is printed that lists the values of the ID variables for the observation and information on the iterative process used to compute the solution. Following the heading line for the observation, the solution values are printed.

The heading line shows the solution method used (Newton's method by default), the number of iterations required, and the convergence measure, labeled CC=. This convergence measure indicates the maximum error by which solution values fail to satisfy the equations. When this error is small enough (as determined by the CONVERGE= option), the iterations terminate. The equation with the largest error is indicated in parentheses. For example, for observation 3 the HYPERBOLA equation has an error of 4.42×10^{-13} while the error of the SQRT equation is even smaller.

The last part of the SOLVE statement output is the solution summary report shown in Figure 14.11. This report summarizes the iteration history and the model solved.

```
                     The MODEL Procedure
                   Simultaneous Simulation

                       Data Set Options

                        DATA=    TEST


                       Solution Summary

                Variables Solved             2
                Implicit Equations           2
                Solution Method         NEWTON
                CONVERGE=                 1E-8
                Maximum CC           9.176E-9
                Maximum Iterations          17
                Total Iterations            26
                Average Iterations    8.666667


                    Observations Processed

                         Read      3
                         Solved    3


           Variables Solved For    x y
           Equations Solved        sqrt hyperbola
```

Figure 14.11. Solution Summary Report

Monte Carlo Simulation

The RANDOM= option is used to request Monte Carlo (or stochastic) simulation to generate confidence intervals for a forecast. The confidence intervals are implied by the model's relationship to the the implicit random error term ϵ and the parameters.

The Monte Carlo simulation generates a random set of additive error values, one for each observation and each equation, and computes one set of perturbations of the parameters. These new parameters, along with the additive error terms, are then used to compute a new forecast that satisfies this new simultaneous system. Then a new set of additive error values and parameter perturbations is computed, and the process is repeated the requested number of times.

Consider the following exchange rate model for the U.S. dollar with the German mark and the Japanese yen:

$$rate\_jp = a_1 + b_1 im\_jp + c_1 di\_jp;$$

$$rate\_wg = a_2 + b_2 im\_wg + c_1 di\_wg;$$

where *rate_jp* and *rate_wg* are the exchange rate of the Japanese yen and the German mark versus the U.S. dollar respectively; *im_jp* and *im_wg* are the imports from Japan and Germany in 1984 dollars respectively; and *di_jp* and *di_wg* are the differences in inflation rate of Japan and the U.S., and Germany and the U.S. respectively. The

Monte Carlo capabilities of the MODEL procedure are used to generate error bounds on a forecast using this model.

```
proc model data=exchange;
   endo im_jp im_wg;
   exo di_jp di_wg;
   parms a1 a2 b1 b2 c1 c2;
   label rate_jp = 'Exchange Rate of Yen/$'
         rate_wg = 'Exchange Rate of Gm/$'
         im_jp = 'Imports to US from Japan in 1984 $'
         im_wg = 'Imports to US from WG in 1984 $'
         di_jp = 'Difference in Inflation Rates US-JP'
         di_wg = 'Difference in Inflation Rates US-WG';

   rate_jp = a1 + b1*im_jp + c1*di_jp;
   rate_wg = a2 + b2*im_wg + c2*di_wg;

            /* Fit the EXCHANGE data */
   fit rate_jp rate_wg / sur outest=xch_est outcov outs=s;

            /* Solve using the WHATIF data set */
   solve rate_jp rate_wg / data=whatif estdata=xch_est sdata=s
          random=100 seed=123 out=monte forecast;
   id yr;
   range yr=1986;
run;
```

Data for the EXCHANGE data set was obtained from the Department of Commerce and the yearly "Economic Report of the President."

First, the parameters are estimated using SUR selected by the SUR option on the FIT statement. The OUTEST= option is used to create the XCH_EST data set which contains the estimates of the parameters. The OUTCOV option adds the covariance matrix of the parameters to the XCH_EST data set. The OUTS= option is used to save the covariance of the equation error in the data set S.

Next, Monte Carlo simulation is requested using the RANDOM= option on the SOLVE statement. The data set WHATIF, shown below, is used to drive the forecasts. The ESTDATA= option reads in the XCH_EST data set which contains the parameter estimates and covariance matrix. Because the parameter covariance matrix is included, perturbations of the parameters are performed. The SDATA= option causes the Monte Carlo simulation to use the equation error covariance in the S data set to perturb the equation errors. The SEED= option selects the number 123 as seed value for the random number generator. The output of the Monte Carlo simulation is written to the data set MONTE selected by the OUT= option.

```
/* data for simulation */
data whatif;
   input yr rate_jp rate_wg imn_jp imn_wg emp_us emp_jp
      emp_wg  prod_us prod_jp prod_wg cpi_us cpi_jp cpi_wg;
   label cpi_us = 'US CPI 1982-1984 = 100'
```

```
            cpi_jp = 'JP CPI 1982-1984 = 100'
            cpi_wg = 'WG CPI 1982-1984 = 100';
      im_jp = imn_jp/cpi_us;
      im_wg = imn_wg/cpi_us;
      ius = 100*(cpi_us-(lag(cpi_us)))/(lag(cpi_us));
      ijp = 100*(cpi_jp-(lag(cpi_jp)))/(lag(cpi_jp));
      iwg = 100*(cpi_wg-(lag(cpi_wg)))/(lag(cpi_wg));
      di_jp = ius - ijp;
      di_wg = ius - iwg;
   datalines;
   1980 226.63 1.8175 30714 11693 103.3 101.3 100.4 101.7
        125.4 109.8  .824  .909  .868
   1981 220.63 2.2631 35000 11000 102.8 102.2  97.9 104.6
        126.3 112.8  .909  .954  .922
   1982 249.06 2.4280 40000 12000  95.8 101.4  95.0 107.1
        146.8 113.3  .965  .980  .970
   1983 237.55 2.5539 45000 13100  94.4 103.4  91.1 111.6
        152.8 116.8  .996  .999 1.003
   1984 237.45 2.8454 50000 14300  99.0 105.8  90.4 118.5
        152.2 124.7 1.039 1.021 1.027
   1985 238.47 2.9419 55000 15600  98.1 107.6  91.3 124.2
        161.1 128.5 1.076 1.042 1.048
   1986    .      .   60000 17000  96.8 107.3  92.7 128.8
        163.8 130.7 1.096 1.049 1.047
   1987    .      .   65000 18500  97.1 106.1  92.8 132.0
        176.5 129.9 1.136 1.050 1.049
   1988    .      .   70000 20000  99.6 108.8  92.7 136.2
        190.0 135.9 1.183 1.057 1.063
   ;
```

To generate a confidence interval plot for the forecast, use PROC UNIVARIATE to generate percentile bounds and use PROC GPLOT to plot the graph. The following SAS statements produce the graph in Figure 14.12.

```
   proc sort data=monte;
      by yr;
   run;

   proc univariate data=monte noprint;
      by yr;
      var rate_jp rate_wg;
      output out=bounds mean=mean p5=p5 p95=p95;
   run;

   title "Monte Carlo Generated Confidence
              Intervals on a Forecast";
   proc gplot data=bounds;
      plot mean*yr p5*yr p95*yr /overlay;
      symbol1 i=join value=triangle;
      symbol2 i=join value=square l=4;
      symbol3 i=join value=square l=4;
   run;
```

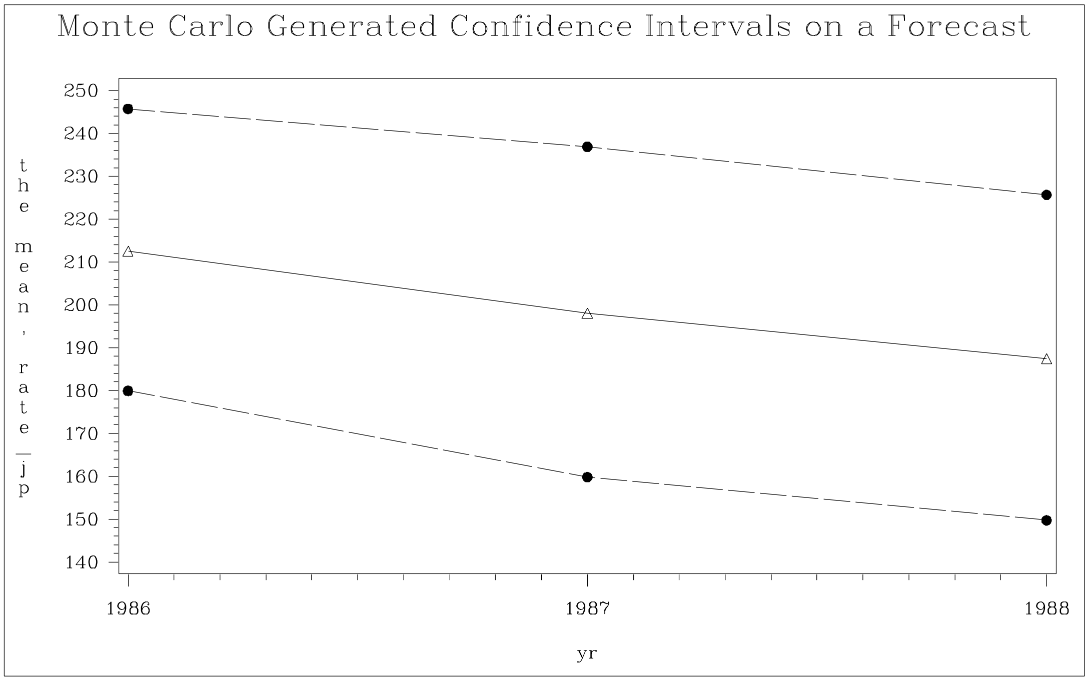


Figure 14.12. Monte Carlo Confidence Interval Plot

Syntax

The following statements can be used with the MODEL procedure:

PROC MODEL *options***;**
ABORT ;
ARRAY *arrayname variables* . . . **;**
ATTRIB *variable-list attribute-list [variable-list attribute-list]***;**
BOUNDS *bound1, bound2* . . . **;**
BY *variables***;**
CALL *name [(expression [, expression . . .])]* **;**
CONTROL *variable [value]* . . . **;**
DELETE ;
DO *[variable = expression [TO expression] [BY expression]*
*[***,** *expression* **TO** *expression] [* **BY** *expression] . . .]*
[**WHILE** *expression] [* **UNTIL** *expression]* **;**
END ;
DROP *variable* . . . **;**
ENDOGENOUS *variable [initial values]* . . . **;**
ESTIMATE *item [, item . . .] [,/ options]* **;**
EXOGENOUS *variable [initial values]* . . . **;**
FIT *equations [* **PARMS***=(parameter values . . .)]*
START*=(parameter values . . .)*
[**DROP***=(parameters)] [/ options]***;**
FORMAT *variables [format] [* **DEFAULT** *= default-format]***;**
GOTO *statement_label* **;**

ID *variables*;
IF *expression* ;
IF *expression* **THEN** *programming_statement* ;
    ELSE *programming_statement* ;
variable **=** *expression* ;
variable **+** *expression* ;
INCLUDE *model files* . . . ;
INSTRUMENTS *[instruments]* [**_EXOG_**]
        [**EXCLUDE**=*(parameters)*] [/ *options*] ;
KEEP *variable* . . . ;
LABEL *variable* =*'label'* . . . ;
LENGTH *variables* [**$**] *length* . . . [**DEFAULT**=*length*];
LINK *statement_label* ;
OUTVARS *variable* . . . ;
PARAMETERS *variable [value] variable [value]* . . . ;
PUT *print_item* . . . [@] [@@] ;
RANGE *variable [= first]* [**TO** *last*];
RENAME *old-name =new-name* . . . [*old-name=new-name*];
RESET *options*;
RESTRICT *restriction1 [, restriction2 . . .]* ;
RETAIN *variables values [variables values. . .]* ;
RETURN ;
SOLVE *variables* [**SATISFY=***(equations)*] [/ *options*] ;
SUBSTR*(variable, index, length) = expression* ;
SELECT *[(expression)]* ;
    OTHERWISE *programming_statement* ;
STOP ;
TEST [**"***name***"**] *test1 [, test2 . . .]* [**,/** *options*] ;
VAR *variable [initial values]* . . . ;
WEIGHT *variable*;
WHEN *(expression) programming_statement* ;

Functional Summary

The statements and options in the MODEL procedure are summarized in the following table.

Description	Statement	Option
Data Set Options		
specify the input data set for the variables	FIT, SOLVE	DATA=
specify the input data set for parameters	FIT, SOLVE	ESTDATA=
specify the method for handling missing values	FIT	MISSING=
specify the input data set for parameters	MODEL	PARMSDATA=

Description	Statement	Option
specify the output data set for residual, predicted, or actual values	FIT	OUT=
specify the output data set for solution mode results	SOLVE	OUT=
write the actual values to OUT= data set	FIT	OUTACTUAL
select all output options	FIT	OUTALL
write the covariance matrix of the estimates	FIT	OUTCOV
write the parameter estimates to a data set	FIT	OUTEST=
write the parameter estimates to a data set	MODEL	OUTPARMS=
write the observations used to start the lags	SOLVE	OUTLAGS
write the predicted values to the OUT= data set	FIT	OUTPREDICT
write the residual values to the OUT= data set	FIT	OUTRESID
write the covariance matrix of the equation errors to a data set	FIT	OUTS=
write the **S** matrix used in the objective function definition to a data set	FIT	OUTSUSED=
write the estimate of the variance matrix of the moment generating function	FIT	OUTV=
read the covariance matrix of the equation errors	FIT, SOLVE	SDATA=
read the covariance matrix for GMM and ITGMM	FIT	VDATA=
specify the name of the time variable	FIT, SOLVE, MODEL	TIME=
select the estimation type to read	FIT, SOLVE	TYPE=
General ESTIMATE Statement Options		
specify the name of the data set in which the estimate of the functions of the parameters are to be written	ESTIMATE	OUTEST=
write the covariance matrix of the functions of the parameters to the OUTEST= data set	ESTIMATE	OUTCOV
print the covariance matrix of the functions of the parameters	ESTIMATE	COVB
print the correlation matrix of the functions of the parameters	ESTIMATE	CORRB
Printing Options for FIT Tasks		
print the modified Breusch-Pagan test for heteroscedasticity	FIT	BREUSCH
print collinearity diagnostics	FIT	COLLIN
print the correlation matrices	FIT	CORR
print the correlation matrix of the parameters	FIT	CORRB

Description	Statement	Option
print the correlation matrix of the residuals	FIT	CORRS
print the covariance matrices	FIT	COV
print the covariance matrix of the parameters	FIT	COVB
print the covariance matrix of the residuals	FIT	COVS
print Durbin-Watson *d* statistics	FIT	DW
print first-stage R^2 statistics	FIT	FSRSQ
print Godfrey's tests for autocorrelated residuals for each equation	FIT	GODFREY
print tests of normality of the model residuals	FIT	NORMAL
specify all the printing options	FIT	PRINTALL
print White's test for heteroscedasticity	FIT	WHITE
Options to Control FIT Iteration Output		
print the inverse of the crossproducts Jacobian matrix	FIT	I
print a summary iteration listing	FIT	ITPRINT
print a detailed iteration listing	FIT	ITDETAILS
print the crossproduct Jacobian matrix	FIT	XPX
specify all the iteration printing-control options	FIT	ITALL
Options to Control the Minimization Process		
specify the convergence criteria	FIT	CONVERGE=
select the Hessian approximation used for FIML	FIT	HESSIAN=
specifies the local truncation error bound for the integration	FIT, SOLVE, MODEL	LTEBOUND=
specify the maximum number of iterations allowed	FIT	MAXITER=
specify the maximum number of subiterations allowed	FIT	MAXSUBITER=
select the iterative minimization method to use	FIT	METHOD=
specifies the smallest allowed time step to be used in the integration	FIT, SOLVE, MODEL	MINTIMESTEP=
modify the iterations for estimation methods that iterate the **S** matrix or the **V** matrix	FIT	NESTIT
specify the smallest pivot value	MODEL, FIT, SOLVE	SINGULAR
specify the number of minimization iterations to perform at each grid point	FIT	STARTITER=
specify a weight variable	WEIGHT	

Description	Statement	Option
Options to Read and Write Model Files		
read a model from one or more input model files	INCLUDE	MODEL=
suppress the default output of the model file	MODEL, RESET	NOSTORE
specify the name of an output model file	MODEL, RESET	OUTMODEL=
delete the current model	RESET	PURGE
Options to List or Analyze the Structure of the Model		
print a dependency structure of a model	MODEL	BLOCK
print a graph of the dependency structure of a model	MODEL	GRAPH
print the model program and variable lists	MODEL	LIST
print the derivative tables and compiled model program code	MODEL	LISTCODE
print a dependency list	MODEL	LISTDEP
print a table of derivatives	MODEL	LISTDER
print a cross-reference of the variables	MODEL	XREF
General Printing Control Options		
expand parts of the printed output	FIT, SOLVE	DETAILS
print a message for each statement as it is executed	FIT, SOLVE	FLOW
select the maximum number of execution errors that can be printed	FIT, SOLVE	MAXERRORS=
select the number of decimal places shown in the printed output	FIT, SOLVE	NDEC=
suppress the normal printed output	FIT, SOLVE	NOPRINT
specify all the noniteration printing options	FIT, SOLVE	PRINTALL
print the result of each operation as it is executed	FIT, SOLVE	TRACE
request a comprehensive memory usage summary	FIT, SOLVE, MODEL, RESET	MEMORYUSE
turns off the NOPRINT option	RESET	PRINT
Statements that Declare Variables		
associate a name with a list of variables and constants	ARRAY	
declare a variable to have a fixed value	CONTROL	
declare a variable to be a dependent or endogenous variable	ENDOGENOUS	
declare a variable to be an independent or exogenous variable	EXOGENOUS	
specify identifying variables	ID	

Description	Statement	Option
assign a label to a variable	LABEL	
select additional variables to be output	OUTVARS	
declare a variable to be a parameter	PARAMETERS	
force a variable to hold its value from a previous observation	RETAIN	
declare a model variable	VAR	
declare an instrumental variable	INSTRUMENTS	
omit the default intercept term in the instruments list	INSTRUMENTS	NOINT
General FIT Statement Options		
omit parameters from the estimation	FIT	DROP=
associate a *variable* with an initial value as a parameter or a constant	FIT	INITIAL=
bypass OLS to get initial parameter estimates for GMM, ITGMM, or FIML	FIT	NOOLS
bypass 2SLS to get initial parameter estimates for GMM, ITGMM, or FIML	FIT	NO2SLS
specify the parameters to estimate	FIT	PARMS=
request confidence intervals on estimated parameters	FIT	PRL=
select a grid search	FIT	START=
Options to Control the Estimation Method Used		
specify nonlinear ordinary least squares	FIT	OLS
specify iterated nonlinear ordinary least squares	FIT	ITOLS
specify seemingly unrelated regression	FIT	SUR
specify iterated seemingly unrelated regression	FIT	ITSUR
specify two-stage least squares	FIT	2SLS
specify iterated two-stage least squares	FIT	IT2SLS
specify three-stage least squares	FIT	3SLS
specify iterated three-stage least squares	FIT	IT3SLS
specify full information maximum likelihood	FIT	FIML
select the variance-covariance estimator used for FIML	FIT	COVBEST=
specify generalized method of moments	FIT	GMM
specify the kernel for GMM and ITGMM	FIT	KERNEL=
specify iterated generalized method of moments	FIT	ITGMM
specify the denominator for computing variances and covariances	FIT	VARDEF=

Description	Statement	Option
Solution Mode Options		
select a subset of the model equations	SOLVE	SATISFY=
solve only for missing variables	SOLVE	FORECAST
solve for all solution variables	SOLVE	SIMULATE
Solution Mode Options: Lag Processing		
use solved values in the lag functions	SOLVE	DYNAMIC
use actual values in the lag functions	SOLVE	STATIC
produce successive forecasts to a fixed forecast horizon	SOLVE	NAHEAD=
select the observation to start dynamic solutions	SOLVE	START=
Solution Mode Options: Numerical Methods		
specify the maximum number of iterations allowed	SOLVE	MAXITER=
specify the maximum number of subiterations allowed	SOLVE	MAXSUBITER=
specify the convergence criteria	SOLVE	CONVERGE=
compute a simultaneous solution using a Jacobi-like iteration	SOLVE	JACOBI
compute a simultaneous solution using a Gauss-Seidel-like iteration	SOLVE	SEIDEL
compute a simultaneous solution using Newton's method	SOLVE	NEWTON
compute a nonsimultaneous solution	SOLVE	SINGLE
Monte Carlo Simulation Options		
specify psuedo or quasi-random number generator	SOLVE	QUASI=
repeat the solution multiple times	SOLVE	RANDOM=
initialize the pseudo-random number generator	SOLVE	SEED=
Solution Mode Printing Options		
print between data points integration values for the DERT. variables and the auxiliary variables	FIT, SOLVE, MODEL	INTGPRINT
print the solution approximation and equation errors	SOLVE	ITPRINT

Description	Statement	Option
print the solution values and residuals at each observation	SOLVE	SOLVEPRINT
print various summary statistics	SOLVE	STATS
print tables of Theil inequality coefficients	SOLVE	THEIL
specify all printing control options	SOLVE	PRINTALL
General TEST Statement Options		
specify that a Wald test be computed	TEST	WALD
specify that a Lagrange multiplier test be computed	TEST	LM
specify that a likelihood ratio test be computed	TEST	LR
requests all three types of tests	TEST	ALL
specify the name of an output SAS data set that contains the test results	TEST	OUT=
Miscellaneous Statements		
specify the range of observations to be used	RANGE	
subset the data set with *by* variables	BY	

PROC MODEL Statement

PROC MODEL *options;*

The following options can be specified in the PROC MODEL statement. All of the nonassignment options (the options that do not accept a value after an equal sign) can have NO prefixed to the option name in the RESET statement to turn the option off. The default case is not explicitly indicated in the discussion that follows. Thus, for example, the option DETAILS is documented in the following, but NODETAILS is not documented since it is the default. Also, the NOSTORE option is documented because STORE is the default.

Data Set Options

DATA= *SAS-data-set*

names the input data set. Variables in the model program are looked up in the DATA= data set and, if found, their attributes (type, length, label, format) are set to be the same as those in the input data set (if not previously defined otherwise). The values for the variables in the program are read from the input data set when the model is estimated or simulated by FIT and SOLVE statements.

OUTPARMS= *SAS-data-set*

writes the parameter estimates to a SAS data set. See "Output Data Sets" for details.

PARMSDATA= *SAS-data-set*

names the SAS data set that contains the parameter estimates. See "Input Data Sets" for details.

Options to Read and Write Model Files

MODEL= *model-name*
MODEL= *(model-list)*

reads the model from one or more input model files created by previous PROC MODEL executions. Model files are written by the OUTMODEL= option.

NOSTORE

suppresses the default output of the model file. This option is only applicable when FIT or SOLVE statements are not used, the MODEL= option is not used, and when a model is specified.

OUTMODEL= *model-name*

specifies the name of an output model file to which the model is to be written. Model files are stored as members of a SAS catalog, with the type MODEL.

V5MODEL= *model-name*

reads model files written by Version 5 of SAS/ETS software.

Options to List or Analyze the Structure of the Model

These options produce reports on the structure of the model or list the programming statements defining the models. These options are automatically reset (turned off) after the reports are printed. To turn these options back on after a RUN statement has been entered, use the RESET statement or specify the options on a FIT or SOLVE statement.

BLOCK

prints an analysis of the structure of the model given by the assignments to model variables appearing in the model program. This analysis includes a classification of model variables into endogenous (dependent) and exogenous (independent) groups based on the presence of the variable on the left-hand side of an assignment statement. The endogenous variables are grouped into simultaneously determined blocks. The dependency structure of the simultaneous blocks and exogenous variables is also printed. The BLOCK option cannot analyze dependencies implied by general form equations.

GRAPH

prints the graph of the dependency structure of the model. The GRAPH option also invokes the BLOCK option and produces a graphical display of the information listed by the BLOCK option.

LIST

prints the model program and variable lists, including the statements added by PROC MODEL and macros.

LISTALL
selects the LIST, LISTDEP, LISTDER, and LISTCODE options.

LISTCODE
prints the derivative tables and compiled model program code. LISTCODE is a debugging feature and is not normally needed.

LISTDEP
prints a report that lists for each variable in the model program the variables that depend on it and that it depends on. These lists are given separately for current-period values and for lagged values of the variables.

The information displayed is the same as that used to construct the BLOCK report but differs in that the information is listed for all variables (including parameters, control variables, and program variables), not just the model variables. Classification into endogenous and exogenous groups and analysis of simultaneous structure is not done by the LISTDEP report.

LISTDER
prints a table of derivatives for FIT and SOLVE tasks. (The LISTDER option is only applicable for the default NEWTON method for SOLVE tasks.) The derivatives table shows each nonzero derivative computed for the problem. The derivative listed can be a constant, a variable in the model program, or a special derivative variable created to hold the result of the derivative expression. This option is turned on by the LISTCODE and PRINTALL options.

XREF
prints a cross-reference of the variables in the model program showing where each variable was referenced or given a value. The XREF option is normally used in conjunction with the LIST option. A more detailed description is given in the "Diagnostics and Debugging" section.

General Printing Control Options

DETAILS
specifies the detailed printout. Parts of the printed output are expanded when the DETAILS option is specified.

FLOW
prints a message for each statement in the model program as it is executed. This debugging option is needed very rarely and produces voluminous output.

MAXERRORS= *n*
specifies the maximum number of execution errors that can be printed. The default is MAXERRORS=50.

NDEC= *n*
specifies the precision of the format that PROC MODEL uses when printing various numbers. The default is NDEC=3, which means that PROC MODEL attempts to print values using the D format but ensures that at least three significant digits are shown. If the NDEC= value is greater than nine, the BEST. format is used. The smallest value allowed is NDEC=2.

The NDEC= option affects the format of most, but not all, of the floating point numbers that PROC MODEL can print. For some values (such as parameter estimates), a precision limit one or two digits greater than the NDEC= value is used. This option does not apply to the precision of the variables in the output data set.

NOPRINT
suppresses the normal printed output but does not suppress error listings. Using any other print option turns the NOPRINT option off. The PRINT option can be used with the RESET statement to turn off NOPRINT.

PRINTALL
turns on all the printing-control options. The options set by PRINTALL are DETAILS; the model information options LIST, LISTDEP, LISTDER, XREF, BLOCK, and GRAPH; the FIT task printing options FSRSQ, COVB, CORRB, COVS, CORRS, DW, and COLLIN; and the SOLVE task printing options STATS, THEIL, SOLVEPRINT, and ITPRINT.

TRACE
prints the result of each operation in each statement in the model program as it is executed, in addition to the information printed by the FLOW option. This debugging option is needed very rarely and produces voluminous output.

MEMORYUSE
prints a report of the memory required for the various parts of the analysis.

FIT Task Options

The following options are used in the FIT statement (parameter estimation) and can also be used in the PROC MODEL statement: COLLIN, CONVERGE=, CORR, CORRB, CORRS, COVB, COVBEST=, COVS, DW, FIML, FSRSQ, GMM, HESSIAN=, I, INTGPRINT, ITALL, ITDETAILS, ITGMM, ITPRINT, ITOLS, ITSUR, IT2SLS, IT3SLS, KERNEL=, LTEBOUND=, MAXITER=, MAXSUBITER=, METHOD=, MINTIMESTEP=, NESTIT, N2SLS, N3SLS, OLS, OUTPREDICT, OUTRESID, OUTACTUAL, OUTLAGS, OUTERRORS, OUTALL, OUTCOV, SINGULAR=, STARTITER=, SUR, TIME=, VARDEF, and XPX. See "FIT Statement Syntax" later in this chapter for a description of these options.

When used in the PROC MODEL or RESET statement, these are default options for subsequent FIT statements. For example, the statement

```
proc model n2sls ... ;
```

makes two-stage least squares the default parameter estimation method for FIT statements that do not specify an estimation method.

SOLVE Task Options

The following options used in the SOLVE statement can also be used in the PROC MODEL statement: CONVERGE=, DYNAMIC, FORECAST, INTGPRINT, ITPRINT, JACOBI, LTEBOUND=, MAXITER=, MAXSUBITER=, MINTIMESTEP=, NAHEAD=, NEWTON, OUTPREDICT, OUTRESID, OUTACTUAL, OUTLAGS, OUTERRORS, OUTALL, SEED=, SEIDEL, SIMULATE, SINGLE, SINGULAR=, SOLVEPRINT, START=, STATIC, STATS, THEIL, TIME=, and TYPE=. See "SOLVE Statement Syntax" later in this chapter for a description of these options.

When used in the PROC MODEL or RESET statement, these options provide default values for subsequent SOLVE statements.

BOUNDS Statement

BOUNDS *bound1 [, bound2 ...] ;*

The BOUNDS statement imposes simple boundary constraints on the parameter estimates. BOUNDS statement constraints refer to the parameters estimated by the associated FIT statement (that is, to either the preceding FIT statement or, in the absence of a preceding FIT statement, to the following FIT statement). You can specify any number of BOUNDS statements.

Each *bound* is composed of parameters and constants and inequality operators:

item operator item [operator item [operator item . . .]]

Each *item* is a constant, the name of an estimated parameter, or a list of parameter names. Each *operator* is ’<’, ’>’, ’<=’, or ’>=’.

You can use both the BOUNDS statement and the RESTRICT statement to impose boundary constraints; however, the BOUNDS statement provides a simpler syntax for specifying these kinds of constraints. See "The RESTRICT Statement" for information on the computational details of estimation with inequality restrictions.

Lagrange multipliers are reported for all the active boundary constraints. In the printed output and in the OUTEST= data set, the Lagrange multiplier estimates are identified with the names BOUND1, BOUND2, and so forth. The probability of the Lagrange multipliers are computed using a beta distribution (LaMotte 1994). To give the constraints more descriptive names, use the RESTRICT statement instead of the BOUNDS statement.

The following BOUNDS statement constrains the estimates of the parameters A and B and the ten parameters P1 through P10 to be between zero and one. This example illustrates the use of parameter lists to specify boundary constraints.

```
bounds 0 < a b p1-p10 < 1;
```

The following is an example of the use of the BOUNDS statement:

```
title ’Holzman Function (1969), Himmelblau No. 21, N=3’;
data zero;
   do i = 1 to 99;
      output;
   end;
run;
```

```
proc model data=zero ;
   parms x1= 100 x2= 12.5 x3=  3;
   bounds .1 <= x1 <= 100,
          0 <= x2 <=  25.6,
          0 <= x3 <=   5;

   t = 2 / 3;
   u = 25 + (-50 * log(0.01 * i )) ** t;
   v = (u - x2) ** x3;
   w = exp(-v / x1);
   eq.foo = -.01 * i + w;

   fit foo / method=marquardt;
run;
```

```
             Holzman Function (1969), Himmelblau No. 21, N=3

                           The MODEL Procedure

                    Nonlinear OLS Parameter Estimates

                                    Approx                     Approx
        Parameter       Estimate     Std Err     t Value     Pr > |t|

        x1              49.99999           0        .           .
        x2                    25           0        .           .
        x3                   1.5           0        .           .


             Number of Observations     Statistics for System

             Used                 99    Objective      5.455E-18
             Missing               0    Objective*N      5.4E-16
```

Figure 14.13. Output from Bounded Estimation

BY Statement

BY *variables;*

A BY statement is used with the FIT statement to obtain separate estimates for observations in groups defined by the BY variables. Note that if an output model file is written, using the OUTMODEL= option, the parameter values stored are those from the last BY group processed. To save parameter estimates for each BY group, use the OUTEST= option in the FIT statement.

A BY statement is used with the SOLVE statement to obtain solutions for observations in groups defined by the BY variables. The BY statement only applies to the DATA= data set.

BY group processing is done separately for the FIT and the SOLVE tasks. It is not possible to use the BY statement to estimate and solve a model for each instance of a BY variable. If BY group processing is done for the FIT and the SOLVE tasks, the parameters obtained from the last BY group processed by the FIT statement are used by the SOLVE statement for all of the BY groups.

CONTROL Statement

CONTROL *variable [value] ... ;*

The CONTROL statement declares control variables and specifies their values. A control variable is like a parameter except that it has a fixed value and is not estimated from the data. You can use control variables for constants in model equations that you may want to change in different solution cases. You can use control variables to vary the program logic. Unlike the retained variables, these values are fixed across iterations.

ENDOGENOUS Statement

ENDOGENOUS *variable [initial-values] ... ;*

The ENDOGENOUS statement declares model variables and identifies them as endogenous. You can declare model variables with an ENDOGENOUS statement instead of with a VAR statement to help document the model or to indicate the default solution variables. The variables declared endogenous are solved when a SOLVE statement does not indicate which variables to solve. Valid abbreviations for the ENDOGENOUS statement are ENDOG and ENDO.

The ENDOGENOUS statement optionally provides initial values for lagged dependent variables. See "Lag Logic" in the "Functions Across Time" section for more information.

ESTIMATE Statement

ESTIMATE *item [, item ...] [,/ options] ;*

The ESTIMATE statement computes estimates of functions of the parameters.

The ESTIMATE statement refers to the parameters estimated by the associated FIT statement (that is, to either the preceding FIT statement or, in the absence of a preceding FIT statement, to the following FIT statement). You can use any number of ESTIMATE statements.

If you specify options on the ESTIMATE statement, a comma is required before the "/" character separating the test expressions from the options, since the "/" character can also be used within test expressions to indicate division. Each *item* is written as an optional name followed by an expression,

["name"] expression

where *"name"* is a string used to identify the estimate in the printed output and in the OUTEST= data set.

Expressions can be composed of parameter names, arithmetic operators, functions, and constants. Comparison operators (such as "=" or "<") and logical operators (such as “&”) cannot be used in ESTIMATE statement expressions. Parameters named in ESTIMATE expressions must be among the parameters estimated by the associated FIT statement.

You can use the following options in the ESTIMATE statement:

OUTEST=

specifies the name of the data set in which the estimate of the functions of the parameters are to be written. The format for this data set is identical to the OUTEST= data set for the FIT statement.

If you specify a *name* in the ESTIMATE statement, that name is used as the parameter name for the estimate in the OUTEST= data set. If no *name* is provided and the expression is just a symbol, the symbol name is used; otherwise, the string "_Estimate #" is used, where "#" is the variable number in the OUTEST= data set.

OUTCOV

writes the covariance matrix of the functions of the parameters to the OUTEST= data set in addition to the parameter estimates.

COVB

prints the covariance matrix of the functions of the parameters.

CORRB

prints the correlation matrix of the functions of the parameters.

The following is an example of the use of the ESTIMATE statement in a segmented model:

```
data a;
   input y x @@;
   datalines;
   .46 1  .47  2 .57  3 .61  4 .62  5 .68  6 .69  7
   .78 8  .70  9 .74 10 .77 11 .78 12 .74 13 .80 13
   .80 15 .78 16
   ;

title ’Segmented Model -- Quadratic with Plateau’;
proc model data=a;

   x0 = -.5 * b / c;

   if x < x0 then y = a + b*x + c*x*x;
   else           y = a + b*x0 + c*x0*x0;

   fit y start=( a .45 b .5 c -.0025 );
```

```
      estimate 'Join point' x0 ,
               'plateau' a + b*x0 + c*x0**2 ;
   run;
```

```
                   Segmented Model -- Quadratic with Plateau

                              The MODEL Procedure

                           Nonlinear OLS  Estimates

                               Approx                 Approx
Term            Estimate     Std Err    t Value     Pr > |t|    Label

Join point       12.7504      1.2785       9.97       <.0001    x0
plateau         0.777516      0.0123      63.10       <.0001    a + b*x0 + c*x0**2
```

Figure 14.14. ESTIMATE Statement Output

EXOGENOUS Statement

EXOGENOUS *variable [initial-values] ... ;*

The EXOGENOUS statement declares model variables and identifies them as exogenous. You can declare model variables with an EXOGENOUS statement instead of with a VAR statement to help document the model or to indicate the default instrumental variables. The variables declared exogenous are used as instruments when an instrumental variables estimation method is requested (such as N2SLS or N3SLS) and an INSTRUMENTS statement is not used. Valid abbreviations for the EXOGENOUS statement are EXOG and EXO.

The EXOGENOUS statement optionally provides initial values for lagged exogenous variables. See "Lag Logic" in the "Functions Across Time" section for more information.

FIT Statement

FIT *[equations] [* **PARMS=***(parameter [values] ...)]*
[**START=***(parameter values ...)]*
[**DROP=***(parameter ...)]*
[**INITIAL=***(variable = [parameter | constant] ...)*
[/ options] **;**

The FIT statement estimates model parameters by fitting the model equations to input data and optionally selects the equations to be fit. If the list of equations is omitted, all model equations containing parameters are fit.

The following options can be used in the FIT statement.

DROP= *(parameters ...)*
specifies that the named parameters not be estimated. All the parameters in the equations fit are estimated except those listed in the DROP= option. The dropped parameters retain their previous values and are not changed by the estimation.

INITIAL= *(variable = [parameter | constant] ...)*
associates a *variable* with an initial value as a *parameter* or a *constant* .

NOOLS
NO2SLS
specifies bipassing OLS or 2SLS to get initial parameter estimates for GMM, ITGMM, or FIML. This is important for certian models that are poorly defined in OLS or 2SLS or if good initial parameter values are already provided. Note that for GMM, the V matrix is created using the initial values specified and this may not be consistently estimated.

PARMS= *(parameters [values] ...)*
selects a subset of the parameters for estimation. When the PARMS= option is used, only the named parameters are estimated. Any parameters not specified in the PARMS= list retain their previous values and are not changed by the estimation.

PRL= WALD | LR | BOTH
requests confidence intervals on estimated parameters. By default the PRL option produces 95% likelihood ratio confidence limits. The coverage of the confidence interval is controlled by the ALPHA= option in the FIT statement.

START= *(parameter values ...)*
supplies starting values for the parameter estimates. If the START= option specifies more than one starting value for one or more parameters, a grid search is performed over all combinations of the values, and the best combination is used to start the iterations. For more information, see the STARTITER= option.

Options to Control the Estimation Method Used

COVBEST=GLS | CROSS | FDA
specifies the variance-covariance estimator used for FIML. COVBEST=GLS selects the generalized least-squares estimator. COVBEST=CROSS selects the crossproducts estimator. COVBEST=FDA selects the inverse of the finite difference approximation to the Hessian. The default is COVBEST=CROSS.

FIML
specifies full information maximum likelihood estimation.

GMM
specifies generalized method of moments estimation.

ITGMM
specifies iterated generalized method of moments estimation.

ITOLS
specifies iterated ordinary least-squares estimation. This is the same as OLS unless there are cross-equation parameter restrictions.

ITSUR
specifies iterated seemingly unrelated regression estimation

IT2SLS
specifies iterated two-stage least-squares estimation. This is the same as 2SLS unless there are cross-equation parameter restrictions.

IT3SLS
specifies iterated three-stage least-squares estimation.

KERNEL=(PARZEN | BART | QS, *[c]*, *[e]* **)**
KERNEL=PARZEN | BART | QS
specifies the kernel to be used for GMM and ITGMM. PARZEN selects the Parzen kernel, BART selects the Bartlett kernel, and QS selects the Quadratic Spectral kernel. $e \geq 0$ and $c \geq 0$ are used to compute the bandwidth parameter. The default is KERNEL=(PARZEN, 1, 0.2). See the "Estimation Methods" section for more details.

N2SLS | 2SLS
specifies nonlinear two-stage least-squares estimation. This is the default when an INSTRUMENTS statement is used.

N3SLS | 3SLS
specifies nonlinear three-stage least-squares estimation.

OLS
specifies ordinary least-squares estimation. This is the default when no INSTRUMENTS statement is used.

SUR
specifies seemingly unrelated regression estimation.

VARDEF=N | WGT | DF | WDF
specifies the denominator to be used in computing variances and covariances. VARDEF=N specifies that the number of nonmissing observations be used. VARDEF=WGT specifies that the sum of the weights be used. VARDEF=DF specifies that the number of nonmissing observations minus the model degrees of freedom (number of parameters) be used. VARDEF=WDF specifies that the sum of the weights minus the model degrees of freedom be used. The default is VARDEF=DF. VARDEF=N is used for FIML estimation.

Data Set Options

DATA= *SAS-data-set*
specifies the input data set. Values for the variables in the program are read from this data set. If the DATA= option is not specified on the FIT statement, the data set specified by the DATA= option on the PROC MODEL statement is used.

ESTDATA= *SAS-data-set*
specifies a data set whose first observation provides initial values for some or all of the parameters.

MISSING= PAIRWISE | DELETE

The option MISSING=PAIRWISE specifies that missing values are tracked on an equation-by-equation basis. The MISSING=DELETE option specifies that the entire observation is omitted from the analysis when any equation has a missing predicted or actual value for the equation. The default is MISSING=DELETE.

OUT= *SAS-data-set*

names the SAS data set to contain the residuals, predicted values, or actual values from each estimation. Only the residuals are output by default.

OUTACTUAL

writes the actual values of the endogenous variables of the estimation to the OUT= data set. This option is applicable only if the OUT= option is specified.

OUTALL

selects the OUTACTUAL, OUTERRORS, OUTLAGS, OUTPREDICT, and OUT-RESID options.

OUTCOV
COVOUT

writes the covariance matrix of the estimates to the OUTEST= data set in addition to the parameter estimates. The OUTCOV option is applicable only if the OUTEST= option is also specified.

OUTEST= *SAS-data-set*

names the SAS data set to contain the parameter estimates and optionally the covariance of the estimates.

OUTLAGS

writes the observations used to start the lags to the OUT= data set. This option is applicable only if the OUT= option is specified.

OUTPREDICT

writes the predicted values to the OUT= data set. This option is applicable only if OUT= is specified.

OUTRESID

writes the residual values computed from the parameter estimates to the OUT= data set. The OUTRESID option is the default if neither OUTPREDICT nor OUTACTUAL is specified. This option is applicable only if the OUT= option is specified.

OUTS= *SAS-data-set*

names the SAS data set to contain the estimated covariance matrix of the equation errors. This is the covariance of the residuals computed from the parameter estimates.

OUTSUSED= *SAS-data-set*

names the SAS data set to contain the S matrix used in the objective function definition. The OUTSUSED= data set is the same as the OUTS= data set for the methods that iterate the S matrix.

OUTV= *SAS-data-set*

names the SAS data set to contain the estimate of the variance matrix for GMM and ITGMM.

SDATA= *SAS-data-set*
specifies a data set that provides the covariance matrix of the equation errors. The matrix read from the SDATA= data set is used for the equation covariance matrix (S matrix) in the estimation. (The SDATA= S matrix is used to provide only the initial estimate of **S** for the methods that iterate the S matrix.)

TIME= *name*
specifies the name of the time variable. This variable must be in the data set.

TYPE= *name*
specifies the estimation type to read from the SDATA= and ESTDATA= data sets. The name specified in the TYPE= option is compared to the _TYPE_ variable in the ESTDATA= and SDATA= data sets to select observations to use in constructing the covariance matrices. When the TYPE= option is omitted, the last estimation type in the data set is used. Valid values are the estimation methods used in PROC MODEL.

VDATA= *SAS-data-set*
specifies a data set containing a variance matrix for GMM and ITGMM estimation.

Printing Options for FIT Tasks

BREUSCH= *(variable-list)*
specifies the modified Breusch-Pagan test, where *variable-list* is a list of variables used to model the error variance.

COLLIN
prints collinearity diagnostics for the Jacobian crossproducts matrix (XPX) after the parameters have converged. Collinearity diagnostics are also automatically printed if the estimation fails to converge.

CORR
prints the correlation matrices of the residuals and parameters. Using CORR is the same as using both CORRB and CORRS.

CORRB
prints the correlation matrix of the parameter estimates.

CORRS
prints the correlation matrix of the residuals.

COV
prints the covariance matrices of the residuals and parameters. Specifying COV is the same as specifying both COVB and COVS.

COVB
prints the covariance matrix of the parameter estimates.

COVS
prints the covariance matrix of the residuals.

DW
prints Durbin-Watson *d* statistics, which measure autocorrelation of the residuals. When the residual series is interrupted by missing observations, the Durbin-Watson statistic calculated is *d*primesym as suggested by Savin and White (1978). This is the

usual Durbin-Watson computed by ignoring the gaps. Savin and White show that it has the same null distribution as the DW with no gaps in the series and can be used to test for autocorrelation using the standard tables. The Durbin-Watson statistic is not valid for models containing lagged endogenous variables.

FSRSQ

prints the first-stage R^2 statistics for instrumental estimation methods. These R^2s measure the proportion of the variance retained when the Jacobian columns associated with the parameters are projected through the instruments space.

GODFREY
GODFREY= *n*

performs Godfrey's tests for autocorrelated residuals for each equation, where *n* is the maximum autoregressive order, and specifies that Godfrey's tests be computed for lags 1 through *n*. The default number of lags is one.

NORMAL

performs tests of normality of the model residuals.

PRINTALL

specifies the printing options COLLIN, CORRB, CORRS, COVB, COVS, DETAILS, DW, and FSRSQ.

WHITE

specifies White's test.

Options to control iteration output

Details of the output produced are discussed in the section "Iteration History".

I

prints the inverse of the crossproducts Jacobian matrix at each iteration.

ITALL

specifies all iteration printing-control options (I, ITDETAILS, ITPRINT, and XPX). ITALL also prints the crossproducts matrix (labeled CROSS), the parameter change vector, and the estimate of the cross-equation covariance of residuals matrix at each iteration.

ITDETAILS

prints a detailed iteration listing. This includes the ITPRINT information and additional statistics.

ITPRINT

prints the parameter estimates, objective function value, and convergence criteria at each iteration.

XPX

prints the crossproducts Jacobian matrix at each iteration.

Options to Control the Minimization Process

The following options may be helpful when you experience a convergence problem:

CONVERGE= *value1*
CONVERGE= *(value1, value2)*
specifies the convergence criteria. The convergence measure must be less than *value1* before convergence is assumed. *value2* is the convergence criterion for the **S** and **V** matrices for **S** and **V** iterated methods. *value2* defaults to *value1*. See "The Convergence Criteria" for details. The default value is CONVERGE=.001.

HESSIAN= CROSS | GLS | FDA
specifies the Hessian approximation used for FIML. HESSIAN=CROSS selects the crossproducts approximation to the Hessian, HESSIAN=GLS selects the generalized least-squares approximation to the Hessian, and HESSIAN=FDA selects the finite difference approximation to the Hessian. HESSIAN=GLS is the default.

LTEBOUND= *n*
specifies the local truncation error bound for the integration. This option is ignored if no ODE's are specified.

MAXITER= *n*
specifies the maximum number of iterations allowed. The default is MAXITER=100.

MAXSUBITER= *n*
specifies the maximum number of subiterations allowed for an iteration. For the GAUSS method, the MAXSUBITER= option limits the number of step halvings. For the MARQUARDT method, the MAXSUBITER= option limits the number of times λ can be increased. The default is MAXSUBITER=30. See "Minimization Methods" for details.

METHOD= GAUSS | MARQUARDT
specifies the iterative minimization method to use. METHOD=GAUSS specifies the Gauss-Newton method, and METHOD=MARQUARDT specifies the Marquardt-Levenberg method. The default is METHOD=GAUSS. See "Minimization Methods" for details.

MINTIMESTEP= *n*
specifies the smallest allowed time step to be used in the integration. This option is ignored if no ODE's are specified.

NESTIT
changes the way the iterations are performed for estimation methods that iterate the estimate of the equation covariance (**S** matrix). The NESTIT option is relevant only for the methods that iterate the estimate of the covariance matrix (ITGMM, ITOLS, ITSUR, IT2SLS, IT3SLS). See "Details on the Covariance of Equation Errors" for an explanation of NESTIT.

SINGULAR= *value*
specifies the smallest pivot value allowed. The default 1.0E-12.

STARTITER= *n*

specifies the number of minimization iterations to perform at each grid point. The default is STARTITER=0, which implies that no minimization is performed at the grid points. See "Using the STARTITER option" for more details.

Other Options

Other options that can be used on the FIT statement include the following that list and analyze the model: BLOCK, GRAPH, LIST, LISTCODE, LISTDEP, LISTDER, and XREF. The following printing control options are also available: DETAILS, FLOW, INTGPRINT, MAXERRORS=, NOPRINT, PRINTALL, and TRACE. For complete descriptions of these options, see the discussion of the PROC MODEL statement options earlier in this chapter.

ID Statement

ID *variables;*

The ID statement specifies variables to identify observations in error messages or other listings and in the OUT= data set. The ID variables are normally SAS date or datetime variables. If more than one ID variable is used, the first variable is used to identify the observations; the remaining variables are added to the OUT= data set.

INCLUDE Statement

INCLUDE *model-names ... ;*

The INCLUDE statement reads model files and inserts their contents into the current model. However, instead of replacing the current model as the RESET MODEL= option does, the contents of included model files are inserted into the model program at the position that the INCLUDE statement appears.

INSTRUMENTS Statement

The INSTRUMENTS statement specifies the instrumental variables to be used in the N2SLS, N3SLS, IT2SLS, IT3SLS, GMM, and ITGMM estimation methods. There are two forms of the INSTRUMENTS statement:

INSTRUMENTS *variables [* **_EXOG_** *] ;*

INSTRUMENTS *[instruments] [* **_EXOG_** *]*

[**EXCLUDE***=(parameters)] [/ options]* **;**

The first form of the INSTRUMENTS statement is used only before a FIT statement and defines the default instruments list. The items specified as instruments can be variables or the special keyword _EXOG_. _EXOG_ indicates that all the model variables declared EXOGENOUS are to be added to the instruments list.

The second form of the INSTRUMENTS statement is used only after the FIT statement and before the next RUN statement. The items specified as instruments for the second form can be variables, names of parameters to be estimated, or the special keyword _EXOG_. If you specify the name of a parameter in the instruments list, the partial derivatives of the equations with respect to the parameter (that is, the columns of the Jacobian matrix associated with the parameter) are used as instruments. The parameter itself is not used as an instrument. These partial derivatives should not depend on any of the parameters to be estimated. Only the names of parameters to be estimated can be specified.

EXCLUDE= *(parameters)*
specifies that the derivatives of the equations with respect to all of the parameters to be estimated, except the parameters listed in the EXCLUDE list, be used as instruments, in addition to the other instruments specified. If you use the EXCLUDE= option, you should be sure that the derivatives with respect to the non-excluded parameters in the estimation are independent of the endogenous variables and not functions of the parameters estimated.

The following option is specified on the INSTRUMENTS statement following a slash (/):

NOINTERCEPT
NOINT
excludes the constant of 1.0 (intercept) from the instruments list. An intercept is always included as an instrument unless NOINTERCEPT is specified.

When a FIT statement specifies an instrumental variables estimation method and no INSTRUMENTS statement accompanies the FIT statement, the default instruments are used. If no default instruments list has been specified, all the model variables declared EXOGENOUS are used as instruments.

See "Choice of Instruments" for more details.

LABEL Statement

LABEL *variable='label' ... ;*

The LABEL statement specifies a label of up to 255 characters for parameters and other variables used in the model program. Labels are used to identify parts of the printout of FIT and SOLVE tasks. The labels will be displayed in the output if the LINESIZE= option is large enough.

OUTVARS Statement

OUTVARS *variables;*

The OUTVARS statement specifies additional variables defined in the model program to be output to the OUT= data sets. The OUTVARS statement is not needed unless the variables to be added to the output data set are not referred to by the model, or unless you wish to include parameters or other special variables in the OUT= data set. The OUTVARS statement includes additional variables, whereas the KEEP statement excludes variables.

PARAMETERS Statement

PARAMETERS *variable [value] [variable [value]] ... ;*

The PARAMETERS statement declares the parameters of a model and optionally sets their values. Valid abbreviations are PARMS and PARM.

Each parameter has a single value associated with it, which is the same for all observations. Lagging is not relevant for parameters. If a value is not specified in the PARMS statement (or by the PARMS= option of a FIT statement), the value defaults to 0.0001 for FIT tasks and to a missing value for SOLVE tasks.

RANGE Statement

RANGE *variable [= first] [***TO** *last];*

The RANGE statement specifies the range of observations to be read from the DATA= data set. For FIT tasks, the RANGE statement controls the period of fit for the estimation. For SOLVE tasks, the RANGE statement controls the simulation period or forecast horizon.

The RANGE variable must be a numeric variable in the DATA= data set that identifies the observations, and the data set must be sorted by the RANGE variable. The first observation in the range is identified by *first*, and the last observation is identified by *last*.

PROC MODEL uses the first *l* observations prior to *first* to initialize the lags, where *l* is the maximum number of lags needed to evaluate any of the equations to be fit or solved, or the maximum number of lags needed to compute any of the instruments when an instrumental variables estimation method is used. There should be at least *l* observations in the data set before *first*. If *last* is not specified, all the nonmissing observations starting with *first* are used.

If *first* is omitted, the first *l* observations are used to initialize the lags, and the rest of the data, until *last*, is used. If a RANGE statement is used but both *first* and *last* are omitted, the RANGE statement variable is used to report the range of observations processed.

The RANGE variable should be nonmissing for all observations. Observations containing missing RANGE values are deleted.

The following are examples of RANGE statements:

```
range year = 1971 to 1988;              /* yearly  data  */
range date = ’1feb73’d to ’1nov82’d;    /* monthly data  */
range time = 60.5;                      /* time in years */
range year to 1977;         /* use all years through 1977 */
range date; /* use values of date to report period-of-fit */
```

RESET Statement

RESET *options;*

All of the options of the PROC MODEL statement can be reset by the RESET statement. In addition, the RESET statement supports one additional option:

PURGE
deletes the current model so that a new model can be defined.

When the MODEL= option is used in the RESET statement, the current model is deleted before the new model is read.

RESTRICT Statement

RESTRICT *restriction1 [, restriction2 ...] ;*

The RESTRICT statement is used to impose linear and nonlinear restrictions on the parameter estimates.

RESTRICT statements refer to the parameters estimated by the associated FIT statement (that is, to either the preceding FIT statement or, in the absence of a preceding FIT statement, to the following FIT statement). You can specify any number of RESTRICT statements.

Each *restriction* is written as an optional name, followed by an expression, followed by an equality operator (=) or an inequality operator (<, >, <=, >=), followed by a second expression:

["name"] expression operator expression

The optional *"name"* is a string used to identify the restriction in the printed output and in the OUTEST= data set. The *operator* can be =, <, >, <= , or >=. The operator and second expression are optional, as in the TEST statement (=0).

Restriction expressions can be composed of parameter names, arithmetic operators, functions, and constants. Comparison operators (such as "=" or "<") and logical operators (such as "&") cannot be used in RESTRICT statement expressions. Parameters named in restriction expressions must be among the parameters estimated by the associated FIT statement. Expressions can refer to variables defined in the program.

The restriction expressions can be linear or nonlinear functions of the parameters.

The following is an example of the use of the RESTRICT statement:

```
proc model data=one;
   endogenous y1 y2;
   exogenous x1 x2;
   parms a b c;
   restrict b*(b+c) <= a;

   eq.one = -y1/c + a/x2 + b * x1**2 + c * x2**2;
   eq.two = -y2 * y1 + b * x2**2 - c/(2 * x1);

   fit one two / fiml;
run;
```

SOLVE Statement

SOLVE *[variables] [***SATISFY=** *equations] [***INITIAL=** *(variable=[parameter]] [/options];*

The SOLVE statement specifies that the model be simulated or forecast for input data values and, optionally, selects the variables to be solved. If the list of variables is omitted, all of the model variables declared ENDOGENOUS are solved. If no model variables are declared ENDOGENOUS, then all model variables are solved.

The following specification can be used in the SOLVE statement:

SATISFY= *equation*
SATISFY= *(equations)*
specifies a subset of the model equations that the solution values are to satisfy. If the SATISFY= option is not used, the solution is computed to satisfy all the model equations. Note that the number of equations must equal the number of variables solved.

Data Set Options

DATA= *SAS-data-set*
names the input data set. The model is solved for each observation read from the DATA= data set. If the DATA= option is not specified on the SOLVE statement, the data set specified by the DATA= option on the PROC MODEL statement is used.

ESTDATA= *SAS-data-set*
names a data set whose first observation provides values for some or all of the parameters and whose additional observations (if any) give the covariance matrix of the

parameter estimates. The covariance matrix read from the ESTDATA= data set is used to generate multivariate normal pseudo-random shocks to the model parameters when the RANDOM= option requests Monte Carlo simulation.

OUT= *SAS-data-set*
outputs the predicted (solution) values, residual values, actual values, or equation errors from the solution to a data set. Only the solution values are output by default.

OUTACTUAL
outputs the actual values of the solved variables read from the input data set to the OUT= data set. This option is applicable only if the OUT= option is specified.

OUTALL
specifies the OUTACTUAL, OUTERRORS, OUTLAGS, OUTPREDICT, and OUTRESID options

OUTERRORS
writes the equation errors to the OUT= data set. These values are normally very close to zero when a simultaneous solution is computed; they can be used to double-check the accuracy of the solution process. It is applicable only if the OUT= option is specified.

OUTLAGS
writes the observations used to start the lags to the OUT= data set. This option is applicable only if the OUT= option is specified.

OUTPREDICT
writes the solution values to the OUT= data set. This option is relevant only if the OUT= option is specified. The OUTPREDICT option is the default unless one of the other output options is used.

OUTRESID
writes the residual values computed as the difference of the solution values and the values for the solution variables read from the input data set to the OUT= data set. This option is applicable only if the OUT= option is specified.

PARMSDATA= *SAS-data-set*
specifies a data set that contains the parameter estimates. See the "Input Data Sets" section for more details.

SDATA= *SAS-data-set*
specifies a data set that provides the covariance matrix of the equation errors. The covariance matrix read from the SDATA= data set is used to generate multivariate normal pseudo-random shocks to the equations when the RANDOM= option requests Monte Carlo simulation.

TYPE= *name*
specifies the estimation type. The name specified in the TYPE= option is compared to the _TYPE_ variable in the ESTDATA= and SDATA= data sets to select observations to use in constructing the covariance matrices. When TYPE= is omitted, the last estimation type in the data set is used.

Solution Mode Options: Lag Processing

DYNAMIC

specifies a dynamic solution. In the dynamic solution mode, solved values are used by the lagging functions. DYNAMIC is the default.

NAHEAD= *n*

specifies a simulation of *n*-period-ahead dynamic forecasting. The NAHEAD= option is used to simulate the process of using the model to produce successive forecasts to a fixed forecast horizon, with each forecast using the historical data available at the time the forecast is made.

Note that NAHEAD=1 produces a static (one-step-ahead) solution. NAHEAD=2 produces a solution using one-step-ahead solutions for the first lag (LAG1 functions return static predicted values) and actual values for longer lags. NAHEAD=3 produces a solution using NAHEAD=2 solutions for the first lags, NAHEAD=1 solutions for the second lags, and actual values for longer lags. In general, NAHEAD=*n* solutions use NAHEAD=*n*-1 solutions for LAG1, NAHEAD=*n*-2 solutions for LAG2, and so forth.

START= *s*

specifies static solutions until the *s*th observation and then changes to dynamic solutions. If the START=*s* option is specified, the first observation in the range in which LAG*n* delivers solved predicted values is *s*+*n*, while LAG*n* returns actual values for earlier observations.

STATIC

specifies a static solution. In static solution mode, actual values of the solved variables from the input data set are used by the lagging functions.

Solution Mode Options: Use of Available Data

FORECAST

specifies that the actual value of a solved variable is used as the solution value (instead of the predicted value from the model equations) whenever nonmissing data are available in the input data set. That is, in FORECAST mode, PROC MODEL solves only for those variables that are missing in the input data set.

SIMULATE

specifies that PROC MODEL always solves for all solution variables as a function of the input values of the other variables, even when actual data for some of the solution variables are available in the input data set. SIMULATE is the default.

Solution Mode Options: Numerical Solution Method

JACOBI

computes a simultaneous solution using a Jacobi iteration.

NEWTON

computes a simultaneous solution using Newton's method. When the NEWTON option is selected, the analytic derivatives of the equation errors with respect to the solution variables are computed and memory-efficient sparse matrix techniques are used for factoring the Jacobian matrix.

The NEWTON option can be used to solve both normalized-form and general-form equations and can compute goal-seeking solutions. NEWTON is the default.

SEIDEL
computes a simultaneous solution using a Gauss-Seidel method.

SINGLE
ONEPASS
specifies a single-equation (nonsimultaneous) solution. The model is executed once to compute predicted values for the variables from the actual values of the other endogenous variables. The SINGLE option can only be used for normalized-form equations and cannot be used for goal-seeking solutions.

For more information on these options, see the "Solution Modes" section later in this chapter.

Monte Carlo Simulation Options

QUASI= NONE|SOBOL|FAURE
specifies a psuedo or quasi-random number generator. Two Quasi-random number generators supported by the MODEL procedure, the Sobol sequence (QUASI=SOBOL) and the Faure sequence (QUASI=FAURE). The default is QUASI=NONE which is the psuedo random number generator.

RANDOM= *n*
repeats the solution *n* times for each BY group, with different random perturbations of the equation errors if the SDATA= option is used; with different random perturbations of the parameters if the ESTDATA= option is used and the ESTDATA= data set contains a parameter covariance matrix; and with different values returned from the random-number generator functions, if any are used in the model program. If RANDOM=0, the random-number generator functions always return zero. See "Monte Carlo Simulation" for details. The default is RANDOM=0.

SEED= *n*
specifies an integer to use as the seed in generating pseudo-random numbers to shock the parameters and equations when the ESTDATA= or the SDATA= options are specified. If *n* is negative or zero, the time of day from the computer's clock is used as the seed. The SEED= option is only relevant if the RANDOM= option is used. The default is SEED=0.

Options for Controlling the Numerical Solution Process

The following options are useful when you have difficulty converging to the simultaneous solution.

CONVERGE= *value*
specifies the convergence criterion for the simultaneous solution. Convergence of the solution is judged by comparing the CONVERGE= value to the maximum over the equations of

$$\frac{|\epsilon_i|}{|y_i| + 1E - 6}$$

if it is computable, otherwise

$$|\epsilon_i|$$

where ϵ_i represents the equation error and y_i represents the solution variable corresponding to the *i*th equation for normalized-form equations. The default is CONVERGE=1E-8.

MAXITER= *n*
specifies the maximum number of iterations allowed for computing the simultaneous solution for any observation. The default is MAXITER=50.

INITIAL= *(variable= [parameter])*
specifies starting values for the parameters

MAXSUBITER= *n*
specifies the maximum number of damping subiterations that are performed in solving a nonlinear system when using the NEWTON solution method. Damping is disabled by setting MAXSUBITER=0. The default is MAXSUBITER=10.

Printing Options

INTGPRINT
prints between data points integration values for the DERT. variables and the auxiliary variables. If you specify the DETAILS option, the integrated derivative variables are printed as well.

ITPRINT
prints the solution approximation and equation errors at each iteration for each observation. This option can produce voluminous output.

PRINTALL
specifies the printing control options DETAILS, ITPRINT, SOLVEPRINT, STATS, and THEIL.

SOLVEPRINT
prints the solution values and residuals at each observation

STATS
prints various summary statistics for the solution values

THEIL
prints tables of Theil inequality coefficients and Theil relative change forecast error measures for the solution values. See "Summary Statistics" in the "Details" section for more information.

Other Options

Other options that can be used on the SOLVE statement include the following that list and analyze the model: BLOCK, GRAPH, LIST, LISTCODE, LISTDEP, LISTDER, and XREF. The LTEBOUND= and MINTIMESTEP= options can be used to control the integration process. The following printing-control options are also available: DETAILS, FLOW, MAXERRORS=, NOPRINT, and TRACE. For complete descriptions of these options, see the PROC MODEL and FIT statement options described earlier in this chapter.

TEST Statement

TEST *["name"] test1 [, test2 ...] [,/ options] ;*

The TEST statement performs tests of nonlinear hypotheses on the model parameters.

The TEST statement applies to the parameters estimated by the associated FIT statement (that is, either the preceding FIT statement or, in the absence of a preceding FIT statement, the following FIT statement). You can specify any number of TEST statements.

If you specify options on the TEST statement, a comma is required before the "/" character separating the test expressions from the options, because the "/" character can also be used within test expressions to indicate division.

Each test is written as an expression optionally followed by an equal sign (=) and a second expression:

[expression] [= expression]

Test expressions can be composed of parameter names, arithmetic operators, functions, and constants. Comparison operators (such as "=") and logical operators (such as “&”) cannot be used in TEST statement expressions. Parameters named in test expressions must be among the parameters estimated by the associated FIT statement.

If you specify only one expression in a test, that expression is tested against zero. For example, the following two TEST statements are equivalent:

```
test a + b;

test a + b = 0;
```

When you specify multiple tests on the same TEST statement, a joint test is performed. For example, the following TEST statement tests the joint hypothesis that both A and B are equal to zero.

```
test a, b;
```

To perform separate tests rather than a joint test, use separate TEST statements. For example, the following TEST statements test the two separate hypotheses that A is equal to zero and that B is equal to zero.

```
test a;
test b;
```

You can use the following options in the TEST statement.

WALD
specifies that a Wald test be computed. WALD is the default.

LM
RAO
LAGRANGE
specifies that a Lagrange multiplier test be computed.

LR
LIKE
specifies that a likelihood ratio test be computed.

ALL
requests all three types of tests.

OUT=
specifies the name of an output SAS data set that contains the test results. The format of the OUT= data set produced by the TEST statement is similar to that of the OUTEST= data set produced by the FIT statement.

VAR Statement

VAR *variables [initial_values] ... ;*

The VAR statement declares model variables and optionally provides initial values for the variables' lags. See the "Lag Logic" section for more information.

WEIGHT Statement

WEIGHT *variable;*

The WEIGHT statement specifies a variable to supply weighting values to use for each observation in estimating parameters.

If the weight of an observation is nonpositive, that observation is not used for the estimation. *variable* must be a numeric variable in the input data set.

An alternative weighting method is to use an assignment statement to give values to the special variable _WEIGHT_. The _WEIGHT_ variable must not depend on the parameters being estimated. If both weighting specifications are given, the weights are multiplied together.

Estimation Details

Estimation Methods

Consider the general nonlinear model:

$$\begin{aligned} \epsilon_t &= \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) \\ \mathbf{z}_t &= Z(\mathbf{x}_t) \end{aligned}$$

where $\mathbf{q}{\in}R^g$ is a real vector valued function, of $\mathbf{y}_t{\in}R^g$, $\mathbf{x}_t{\in}R^l$, $\theta{\in}R^p$, *g* is the number of equations, *l* is the number of exogenous variables (lagged endogenous variables are considered exogenous here), *p* is the number of parameters and *t* ranges from 1 to *n*. $\mathbf{z}_t{\in}R^k$ is a vector of instruments. ϵ_t is an unobservable disturbance vector with the following properties:

$$\begin{aligned} E(\epsilon_t) &= 0 \\ E(\epsilon_t \epsilon_t^{'}) &= \Sigma \end{aligned}$$

All of the methods implemented in PROC MODEL aim to minimize an *objective function*. The following table summarizes the objective functions defining the estimators and the corresponding estimator of the covariance of the parameter estimates for each method.

Table 14.1. Summary of PROC MODEL Estimation Methods

Method	Instruments	Objective Function	Covariance of θ
OLS	no	$\mathbf{r}'\mathbf{r}/n$	$(\mathbf{X}'(\mathrm{diag}(\mathbf{S})^{-1}{\otimes}\mathbf{I})\mathbf{X})^{-1}$
ITOLS	no	$\mathbf{r}'(\mathrm{diag}(\mathbf{S})^{-1}{\otimes}\mathbf{I})\mathbf{r}/\mathrm{n}$	$(\mathbf{X}'(\mathrm{diag}(\mathbf{S})^{-1}{\otimes}\mathbf{I})\mathbf{X})^{-1}$
SUR	no	$\mathbf{r}'(\mathbf{S}_{\mathrm{OLS}}^{-1}{\otimes}\mathbf{I})\mathbf{r}/n$	$(\mathbf{X}'(\mathbf{S}^{-1}{\otimes}\mathbf{I})\mathbf{X})^{-1}$
ITSUR	no	$\mathbf{r}'(\mathbf{S}^{-1}{\otimes}\mathbf{I})\mathbf{r}/n$	$(\mathbf{X}'(\mathbf{S}^{-1}{\otimes}\mathbf{I})\mathbf{X})^{-1}$
N2SLS	yes	$\mathbf{r}'(\mathbf{I}{\otimes}\mathbf{W})\mathbf{r}/n$	$(\mathbf{X}'(\mathrm{diag}(\mathbf{S})^{-1}{\otimes}\mathbf{W})\mathbf{X})^{-1}$
IT2SLS	yes	$\mathbf{r}'(\mathrm{diag}(\mathbf{S})^{-1}{\otimes}\mathbf{W})\mathbf{r}/\mathrm{n}$	$(\mathbf{X}'(\mathrm{diag}(\mathbf{S})^{-1}{\otimes}\mathbf{W})\mathbf{X})^{-1}$
N3SLS	yes	$\mathbf{r}'(\mathbf{S}_{\mathrm{N2SLS}}^{-1}{\otimes}\mathbf{W})\mathbf{r}/n$	$(\mathbf{X}'(\mathbf{S}^{-1}{\otimes}\mathbf{W})\mathbf{X})^{-1}$
IT3SLS	yes	$\mathbf{r}'(\mathbf{S}^{-1}{\otimes}\mathbf{W})\mathbf{r}/n$	$(\mathbf{X}'(\mathbf{S}^{-1}{\otimes}\mathbf{W})\mathbf{X})^{-1}$
GMM	yes	$[n\mathbf{m}_n(\theta)]'\hat{\mathbf{V}}_{\mathrm{N2SLS}}^{-1}[n\mathbf{m}_n(\theta)]/n$	$[(\mathbf{YX})'\hat{\mathbf{V}}^{-1}(\mathbf{YX})]^{-1}$
ITGMM	yes	$[n\mathbf{m}_n(\theta)]'\hat{\mathbf{V}}^{-1}[n\mathbf{m}_n(\theta)]/n$	$[(\mathbf{YX})'\hat{\mathbf{V}}^{-1}(\mathbf{YX})]^{-1}$
FIML	no	$constant + \frac{n}{2}\ln(\det(\mathbf{S}))$ $- \sum_1^n \ln\lvert(\mathbf{J}_t)\rvert$	$[\hat{\mathbf{Z}}'(\mathbf{S}^{-1}{\otimes}\mathbf{I})\hat{\mathbf{Z}}]^{-1}$

The column labeled "Instruments" identifies the estimation methods that require instruments. The variables used in this table and the remainder of this chapter are defined as follows:

$n =$ is the number of nonmissing observations.

$g =$ is the number of equations.

$k =$ is the number of instrumental variables.

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_g \end{bmatrix} \text{ is the } ng \times 1 \text{ vector of residuals for the } g \text{ equations stacked together.}$$

$$\mathbf{r}_i = \begin{bmatrix} q_i(\mathbf{y}_1, \mathbf{x}_1, \theta) \\ q_i(\mathbf{y}_2, \mathbf{x}_2, \theta) \\ \vdots \\ q_i(\mathbf{y}_n, \mathbf{x}_n, \theta) \end{bmatrix} \text{ is the } n \times 1 \text{ column vector of residuals for the } i\text{th equation.}$$

S	is a $g \times g$ matrix that estimates Σ, the covariances of the errors across equations (referred to as the **S** matrix).
X	is an $ng \times p$ matrix of partial derivatives of the residual with respect to the parameters.
W	is an $n \times n$ matrix, $\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$.
Z	is an $n \times k$ matrix of instruments.
Y	is a $gk \times ng$ matrix of instruments. $\mathbf{Y} = \mathbf{I}_g \otimes \mathbf{Z}'$.
$\hat{\mathbf{Z}}$	$\hat{\mathbf{Z}} = (\hat{Z}_1, \hat{Z}_2, \ldots, \hat{Z}_p)$ is an $ng \times p$ matrix. $\hat{Z}_i$ is a $ng \times 1$ column vector obtained from stacking the columns of $\mathbf{U}\frac{1}{n}\sum_{t=1}^{n}\left(\frac{\partial \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)'}{\partial y_t}\right)^{-1}\frac{\partial^2 \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)'}{\partial y_t \partial \theta_i} - \mathbf{Q}_i$
U	is an $n \times g$ matrix of residual errors. $\mathbf{U} = \epsilon_1, \epsilon_2, \ldots, \epsilon_n{}'$
Q	is the $n \times g$ matrix $\mathbf{q}(\mathbf{y}_1, \mathbf{x}_1, \theta), \mathbf{q}(\mathbf{y}_2, \mathbf{x}_2, \theta), \ldots, \mathbf{q}(\mathbf{y}_n, n, \theta$.
$\mathbf{Q}_i$	is an $n \times g$ matrix $\frac{\partial \mathbf{Q}}{\partial \theta_i}$.
I	is an $n \times n$ identity matrix.
$\mathbf{J}_t$	is $\frac{\partial \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)}{\partial \mathbf{y}_t'}$ which is a $g \times g$ Jacobian matrix.
$\mathbf{m}_n$	is first moment of the crossproduct $\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) \otimes \mathbf{z}_t$. $m_n = \frac{1}{n}\sum_{t=1}^{n} \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) \otimes \mathbf{z}_t$
$\mathbf{z}_t$	is a k column vector of instruments for observation t. $\mathbf{z}_t'$ is also the tth row of **Z**.
$\hat{\mathbf{V}}$	is the $gk \times gk$ matrix representing the variance of the moment functions.
k	is the number of instrumental variables used.
constant	is the constant $\frac{ng}{2}(1 + \ln(2\pi))$.
$\otimes$	is the notation for a Kronecker product.

All vectors are column vectors unless otherwise noted. Other estimates of the covariance matrix for FIML are also available.

Dependent Regressors and Two-Stage Least Squares

Ordinary regression analysis is based on several assumptions. A key assumption is that the independent variables are in fact statistically independent of the unobserved error component of the model. If this assumption is not true–if the regressor varies systematically with the error–then ordinary regression produces inconsistent results. The parameter estimates are *biased*.

Regressors might fail to be independent variables because they are dependent variables in a larger simultaneous system. For this reason, the problem of dependent regressors is often called *simultaneous equation bias*. For example, consider the following two-equation system.

$$y_1 = a_1 + b_1 y_2 + c_1 x_1 + \epsilon_1$$

$$y_2 = a_2 + b_2 y_1 + c_2 x_2 + \epsilon_2$$

In the first equation, y_2 is a dependent, or *endogenous*, variable. As shown by the second equation, y_2 is a function of y_1, which by the first equation is a function of ϵ_1, and therefore y_2 depends on ϵ_1. Likewise, y_1 depends on ϵ_2 and is a dependent regressor in the second equation. This is an example of a *simultaneous equation* system; y_1 and y_2 are a function of all the variables in the system.

Using the ordinary least squares (OLS) estimation method to estimate these equations produces biased estimates. One solution to this problem is to replace y_1 and y_2 on the right-hand side of the equations with predicted values, thus changing the regression problem to the following:

$$y_1 = a_1 + b_1 \hat{y}_2 + c_1 x_1 + \epsilon_1$$

$$y_2 = a_2 + b_2 \hat{y}_1 + c_2 x_2 + \epsilon_2$$

This method requires estimating the predicted values $\hat{y}_1$ and $\hat{y}_2$ through a preliminary, or "first stage," *instrumental regression*. An instrumental regression is a regression of the dependent regressors on a set of *instrumental variables*, which can be any independent variables useful for predicting the dependent regressors. In this example, the equations are linear and the exogenous variables for the whole system are known. Thus, the best choice for instruments (of the variables in the model) are the variables x_1 and x_2.

This method is known as *two-stage least squares* or 2SLS, or more generally as the *instrumental variables method*. The 2SLS method for linear models is discussed in Pindyck (1981, p. 191-192). For nonlinear models this situation is more complex, but the idea is the same. In nonlinear 2SLS, the derivatives of the model with respect to the parameters are replaced with predicted values. See the section "Choice of Instruments" for further discussion of the use of instrumental variables in nonlinear regression.

To perform nonlinear 2SLS estimation with PROC MODEL, specify the instrumental variables with an INSTRUMENTS statement and specify the 2SLS or N2SLS option on the FIT statement. The following statements show how to estimate the first equation in the preceding example with PROC MODEL.

```
proc model data=in;
   y1 = a1 + b1 * y2 + c1 * x1;
   fit y1 / 2sls;
   instruments x1 x2;
run;
```

The 2SLS or instrumental variables estimator can be computed using a first-stage regression on the instrumental variables as described previously. However, PROC MODEL actually uses the equivalent but computationally more appropriate technique of projecting the regression problem into the linear space defined by the instruments. Thus PROC MODEL does not produce any "first stage" results when you use 2SLS. If you specify the FSRSQ option on the FIT statement, PROC MODEL prints "first-stage R^2" statistic for each parameter estimate.

Formally, the $\hat{\theta}$ that minimizes

$$\hat{S}_n = \frac{1}{n}\left(\sum_{t=1}^{n}(\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)\otimes\mathbf{z}_t)\right)'\left(\sum_{t=1}^{n} I\otimes\mathbf{z}_t\mathbf{z}_t'\right)^{-1}\left(\sum_{t=1}^{n}(\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)\otimes\mathbf{z}_t)\right)$$

is the N2SLS estimator of the parameters. The estimate of Σ at the final iteration is used in the covariance of the parameters given in Table 14.1. Refer to Amemiya (1985, p. 250) for details on the properties of nonlinear two-stage least squares.

Seemingly Unrelated Regression

If the regression equations are not simultaneous, so there are no dependent regressors, *seemingly unrelated regression* (SUR) can be used to estimate systems of equations with correlated random errors. The large-sample efficiency of an estimation can be improved if these cross-equation correlations are taken into account. SUR is also known as *joint generalized least squares* or *Zellner regression*. Formally, the $\hat{\theta}$ that minimizes

$$\hat{S}_n = \frac{1}{n}\sum_{t=1}^{n}\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)'\hat{\Sigma}^{-1}\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)$$

is the SUR estimator of the parameters.

The SUR method requires an estimate of the cross-equation covariance matrix, Σ. PROC MODEL first performs an OLS estimation, computes an estimate, $\hat{\Sigma}$, from the OLS residuals, and then performs the SUR estimation based on $\hat{\Sigma}$. The OLS results are not printed unless you specify the OLS option in addition to the SUR option.

You can specify the $\hat{\Sigma}$ to use for SUR by storing the matrix in a SAS data set and naming that data set in the SDATA= option. You can also feed the $\hat{\Sigma}$ computed from the SUR residuals back into the SUR estimation process by specifying the ITSUR option. You can print the estimated covariance matrix $\hat{\Sigma}$ using the COVS option on the FIT statement.

The SUR method requires estimation of the Σ matrix, and this increases the sampling variability of the estimator for small sample sizes. The efficiency gain SUR has over OLS is a large sample property, and you must have a reasonable amount of data to realize this gain. For a more detailed discussion of SUR, refer to Pindyck (1981, p. 331-333).

Three-Stage Least-Squares Estimation

If the equation system is simultaneous, you can combine the 2SLS and SUR methods to take into account both dependent regressors and cross-equation correlation of the errors. This is called *three-stage least squares* (3SLS).

Formally, the $\hat{\theta}$ that minimizes

$$\hat{S}_n = \frac{1}{n}\left(\sum_{t=1}^{n}(\mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)\otimes\mathbf{z}_t)\right)'\left(\sum_{t=1}^{n}(\hat{\Sigma}\otimes\mathbf{z}_t\mathbf{z}_t')\right)^{-1}\left(\sum_{t=1}^{n}(\mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)\otimes\mathbf{z}_t)\right)$$

is the 3SLS estimator of the parameters. For more details on 3SLS, refer to Gallant (1987, p. 435).

Residuals from the 2SLS method are used to estimate the Σ matrix required for 3SLS. The results of the preliminary 2SLS step are not printed unless the 2SLS option is also specified.

To use the three-stage least-squares method, specify an INSTRUMENTS statement and use the 3SLS or N3SLS option on either the PROC MODEL statement or a FIT statement.

Generalized Method of Moments - GMM

For systems of equations with heteroscedastic errors, generalized method of moments (GMM) can be used to obtain efficient estimates of the parameters. See the "Heteroscedasticity" section for alternatives to GMM.

Consider the nonlinear model

$$\begin{aligned}\boldsymbol{\epsilon}_t &= \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)\\ \mathbf{z}_t &= Z(\mathbf{x}_t)\end{aligned}$$

where $\mathbf{z}_t$ is a vector of instruments and ϵ_t is an unobservable disturbance vector that can be serially correlated and nonstationary.

In general, the following orthogonality condition is desired:

$$E(\boldsymbol{\epsilon}_t\otimes\mathbf{z}_t) = 0$$

which states that the expected crossproducts of the unobservable disturbances, $\boldsymbol{\epsilon}_t$, and functions of the observable variables are set to 0. The first moment of the crossproducts is

$$\begin{aligned}\mathbf{m}_n &= \frac{1}{n}\sum_{t=1}^{n}\mathbf{m}(\mathbf{y}_t,\mathbf{x}_t,\theta)\\ \mathbf{m}(\mathbf{y}_t,\mathbf{x}_t,\theta) &= \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)\otimes\mathbf{z}_t\end{aligned}$$

where $\mathbf{m}(\mathbf{y}_t,\mathbf{x}_t,\theta)\in R^{gk}$.

The case where $gk > p$ is considered here, where *p* is the number of parameters.

Estimate the true parameter vector θ^0 by the value of $\hat{\theta}$ that minimizes

$$S(\theta, V) = [n\mathbf{m}_n(\theta)]'V^{-1}[n\mathbf{m}_n(\theta)]/n$$

where

$$V = \text{Cov}\left([\mathrm{n}\mathbf{m}_{\mathrm{n}}(\theta^0)], [\mathrm{n}\mathbf{m}_{\mathrm{n}}(\theta^0)]'\right)$$

The parameter vector that minimizes this objective function is the GMM estimator. GMM estimation is requested on the FIT statement with the GMM option.

The variance of the moment functions, V, can be expressed as

$$\begin{aligned} V &= E\left(\sum_{t=1}^{n} \epsilon_t \otimes \mathbf{z}_t\right)\left(\sum_{s=1}^{n} \epsilon_s \otimes \mathbf{z}_s\right)' \\ &= \sum_{t=1}^{n}\sum_{s=1}^{n} E\left[(\epsilon_t \otimes \mathbf{z}_t)(\epsilon_s \otimes \mathbf{z}_s)'\right] \\ &= nS_n^0 \end{aligned}$$

where S_n^0 is estimated as

$$\hat{S}_n = \frac{1}{n}\sum_{t=1}^{n}\sum_{s=1}^{n}(\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)\otimes\mathbf{z}_t)(\mathbf{q}(\mathbf{y}_s, \mathbf{x}_s, \theta)\otimes\mathbf{z}_s)'$$

Note that $\hat{S}_n$ is a $gk \times gk$ matrix. Because Var $(\hat{S}_n)$ will not decrease with increasing *n* we consider estimators of S_n^0 of the form:

$$\begin{aligned} \hat{S}_n(l(n)) &= \sum_{\tau=-n+1}^{n-1} w\left(\frac{\tau}{l(n)}\right) D\hat{S}_{n,\tau}D \\ \hat{S}_{n,\tau} &= \begin{cases} \sum_{t=1+\tau}^{n} [\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta^{\#})\otimes\mathbf{z}_t][\mathbf{q}(\mathbf{y}_{t-\tau}, \mathbf{x}_{t-\tau}, \theta^{\#})\otimes\mathbf{z}_{t-\tau}]' & \tau \geq 0 \\ (\hat{S}_{n,-\tau})' & \tau < 0 \end{cases} \end{aligned}$$

where $l(n)$ is a scalar function that computes the bandwidth parameter, $w(\cdot)$ is a scalar valued kernel, and the diagonal matrix D is used for a small sample degrees of freedom correction (Gallant 1987). The initial $\theta^{\#}$ used for the estimation of $\hat{S}_n$ is obtained from a 2SLS estimation of the system. The degrees of freedom correction is handled by the VARDEF= option as for the **S** matrix estimation.

The following kernels are supported by PROC MODEL. They are listed with their default bandwidth functions.

Bartlett: KERNEL=BART

$$
\begin{aligned}
w(x) &= \begin{cases} 1-|x| & |x| <= 1 \\ 0 & \text{otherwise} \end{cases} \\
l(n) &= \frac{1}{2} n^{1/3}
\end{aligned}
$$

Parzen: KERNEL=PARZEN

$$
\begin{aligned}
w(x) &= \begin{cases} 1-6|x|^2+6|x|^3 & 0 <= |x| <= \frac{1}{2} \\ 2(1-|x|)^3 & \frac{1}{2} <= |x| <= 1 \\ 0 & \text{otherwise} \end{cases} \\
l(n) &= n^{1/5}
\end{aligned}
$$

Quadratic Spectral: KERNEL=QS

$$
\begin{aligned}
w(x) &= \frac{25}{12\pi^2 x^2} \left(\frac{sin(6\pi x/5)}{6\pi x/5} - cos(6\pi x/5) \right) \\
l(n) &= \frac{1}{2} n^{1/5}
\end{aligned}
$$

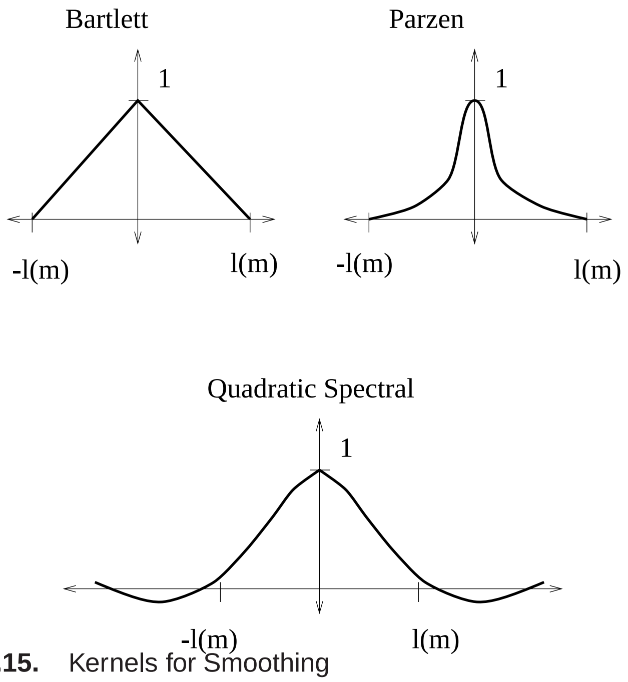


Figure 14.15. Kernels for Smoothing

Details of the properties of these and other kernels are given in Andrews (1991). Kernels are selected with the KERNEL= option; KERNEL=PARZEN is the default. The general form of the KERNEL= option is

```
KERNEL=( PARZEN | QS | BART, c, e )
```

where the $e \geq 0$ and $c \geq 0$ are used to compute the bandwidth parameter as

$$l(n) = cn^e$$

The bias of the standard error estimates increases for large bandwidth parameters. A warning message is produced for bandwidth parameters greater than $n^{\frac{1}{3}}$. For a discussion of the computation of the optimal $l(n)$, refer to Andrews (1991).

The "Newey-West" kernel (Newey (1987)) corresponds to the Bartlett kernel with bandwith parameter $l(n) = L + 1$. That is, if the "lag length" for the Newey-West kernel is L then the corresponding Model procedure syntax is KERNEL=(bart, L+1, 0).

Andrews (1992) has shown that using prewhitening in combination with GMM can improve confidence interval coverage and reduce over rejection of *t*-statistics at the cost of inflating the variance and MSE of the estimator. Prewhitening can be performed using the %AR macros.

For the special case that the errors are not serially correlated, that is

$$E(e_t \otimes \mathbf{z}_t)(e_s \otimes \mathbf{z}_s) = 0 \qquad t \neq s$$

the estimate for S_n^0 reduces to

$$\hat{S}_n = \frac{1}{n} \sum_{t=1}^{n} [\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) \otimes \mathbf{z}_t][\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) \otimes \mathbf{z}_t]'$$

The option KERNEL=(*kernel*,0,) is used to select this type of estimation when using GMM.

Testing Over-Identifying Restrictions

Let *r* be the number of unique instruments times the number of equations. The value *r* represents the number of orthogonality conditions imposed by the GMM method. Under the assumptions of the GMM method, $r - p$ linearly independent combinations of the orthogonality should be close to zero. The GMM estimates are computed by setting these combinations to zero. When *r* exceeds the number of parameters to be estimated, the OBJECTIVE*N, reported at the end of the estimation, is an asymptoticly valid statistic to test the null hypothesis that the over-identifying restrictions of the model are valid. The OBJECTIVE*N is distributed as a chi-square with $r - p$ degrees of freedom (Hansen 1982, p. 1049).

Iterated Generalized Method of Moments - ITGMM

Iterated generalized method of moments is similar to the iterated versions of 2SLS, SUR, and 3SLS. The variance matrix for GMM estimation is re-estimatedg at each iteration with the parameters determined by the GMM estimation. The iteration terminates when the variance matrix for the equation errors change less than the CONVERGE= value. Iterated generalized method of moments is selected by the ITGMM option on the FIT statement. For some indication of the small sample properties of ITGMM, refer to Ferson (1993).

Full Information Maximum Likelihood Estimation - FIML

A different approach to the simultaneous equation bias problem is the full information maximum likelihood (FIML) estimation method (Amemiya 1977).

Compared to the instrumental variables methods (2SLS and 3SLS), the FIML method has these advantages and disadvantages:

- FIML does not require instrumental variables.
- FIML requires that the model include the full equation system, with as many equations as there are endogenous variables. With 2SLS or 3SLS you can estimate some of the equations without specifying the complete system.
- FIML assumes that the equations errors have a multivariate normal distribution. If the errors are not normally distributed, the FIML method may produce poor results. 2SLS and 3SLS do not assume a specific distribution for the errors.
- The FIML method is computationally expensive.

The full information maximum likelihood estimators of θ and σ are the $\hat{\theta}$ and $\hat{\sigma}$ that minimize the negative log likelihood function:

$$\begin{aligned}\mathbf{l}_n(\theta,\sigma) = & \quad \tfrac{ng}{2} \quad \ln(2\pi) - \sum_{t=1}^{n} \ln\left(\left|\frac{\partial \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)}{\partial \mathbf{y}_t'}\right|\right) + \frac{n}{2}\ln\left(|\Sigma(\sigma)|\right) \\ & + \quad \frac{1}{2}\mathrm{tr}\left(\Sigma(\sigma)^{-1}\sum_{\mathrm{t=1}}^{\mathrm{n}} \mathbf{q}(\mathbf{y}_{\mathrm{t}}, \mathbf{x}_{\mathrm{t}}, \theta)\mathbf{q}'(\mathbf{y}_{\mathrm{t}}, \mathbf{x}_{\mathrm{t}}, \theta)\right)\end{aligned}$$

The option FIML requests full information maximum likelihood estimation. If the errors are distributed normally, FIML produces efficient estimators of the parameters. If instrumental variables are not provided the starting values for the estimation are obtained from a SUR estimation. If instrumental variables are provided, then the starting values are obtained from a 3SLS estimation. The negative log likelihood value and the l_2 norm of the gradient of the negative log likelihood function are shown in the estimation summary.

FIML Details

To compute the minimum of $\mathbf{l}_n(\theta,\sigma)$, this function is *concentrated* using the relation

$$\Sigma(\theta) = \frac{1}{n}\sum_{t=1}^{n} \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)\mathbf{q}'(\mathbf{y}_t, \mathbf{x}_t, \theta)$$

This results in the concentrated negative log likelihood function:

$$\mathbf{l}_n(\theta) = \frac{ng}{2}(1+\ln(2\pi)) - \sum_{t=1}^{n}\ln\left|\frac{\partial}{\partial \mathbf{y}_t'}\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)\right| + \frac{n}{2}\ln|\Sigma(\theta|)$$

The gradient of the negative log likelihood function is

$$\frac{\partial}{\partial \theta_i}\mathbf{l}_n(\theta) = \sum_{t=1}^{n} \nabla_i(t)$$

$$\begin{aligned}\nabla_i(t) &= -\mathrm{tr}\left(\left(\frac{\partial \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)}{\partial \mathbf{y}_t'}\right)^{-1}\frac{\partial^2 \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)}{\partial \mathbf{y}_t'\partial\theta_i}\right)\\ &+ \frac{1}{2}\mathrm{tr}\left(\Sigma(\theta)^{-1}\frac{\partial\Sigma(\theta)}{\partial\theta_i}\right.\\ &\qquad \left.\left[I-\Sigma(\theta)^{-1}\mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)\mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)'\right]\right)\\ &+ \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta')\Sigma(\theta)^{-1}\frac{\partial \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)}{\partial\theta_i}\end{aligned}$$

where

$$\frac{\partial\Sigma(\theta)}{\partial\theta_i} = \frac{2}{n}\sum_{t=1}^{n}\mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)\frac{\partial \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)'}{\partial\theta_i}$$

The estimator of the variance-covariance of $\hat{\theta}$ (COVB) for FIML can be selected with the COVBEST= option with the following arguments:

CROSS selects the crossproducts estimator of the covariance matrix (default) (Gallant 1987, p. 473):

$$C = \left(\frac{1}{n}\sum_{t=1}^{n}\nabla(t)\nabla'(t)\right)^{-1}$$

where $\nabla(t) = [\nabla_1(t), \nabla_2(t), \ldots, \nabla_p(t)]'$

GLS selects the generalized least-squares estimator of the covariance matrix. This is computed as (Dagenais 1978)

$$C = [\hat{Z}'(\Sigma(\theta)^{-1}\otimes I)\hat{Z}]^{-1}$$

where $\hat{Z} = (\hat{Z}_1, \hat{Z}_2, \ldots, \hat{Z}_p)$ is $ng \times p$ and each $\hat{Z}_i$ column vector is obtained from stacking the columns of

$$U\frac{1}{n}\sum_{t=1}^{n}\left(\frac{\partial \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)'}{\partial y}\right)^{-1}\frac{\partial^2 \mathbf{q}(\mathbf{y}_t,\mathbf{x}_t,\theta)'}{\partial \mathbf{y}_n'\partial\theta_i} - Q_i$$

U is an $n \times g$ matrix of residuals and q_i is an $n \times g$ matrix $\frac{\partial \mathbf{Q}}{\partial\theta_i}$.

FDA selects the inverse of concentrated likelihood Hessian as an estimator of the covariance matrix. The Hessian is computed numerically, so for a large problem this is computationally expensive.

The HESSIAN= option controls which approximation to the Hessian is used in the minimization procedure. Alternate approximations are used to improve convergence and execution time. The choices are as follows.

CROSS	The crossproducts approximation is used.
GLS	The generalized least-squares approximation is used (default).
FDA	The Hessian is computed numerically by finite differences.

HESSIAN=GLS has better convergence properties in general, but COVBEST=CROSS produces the most pessimistic standard error bounds. When the HESSIAN= option is used, the default estimator of the variance-covariance of $\hat{\theta}$ is the inverse of the Hessian selected.

Properties of the Estimates

All of the methods are consistent. Small sample properties may not be good for nonlinear models. The tests and standard errors reported are based on the convergence of the distribution of the estimates to a normal distribution in large samples.

These nonlinear estimation methods reduce to the corresponding linear systems regression methods if the model is linear. If this is the case, PROC MODEL produces the same estimates as PROC SYSLIN.

Except for GMM, the estimation methods assume that the equation errors for each observation are identically and independently distributed with a 0 mean vector and positive definite covariance matrix Σ consistently estimated by **S**. For FIML, the errors need to be normally distributed. There are no other assumptions concerning the distribution of the errors for the other estimation methods.

The consistency of the parameter estimates relies on the assumption that the **S** matrix is a consistent estimate of Σ. These standard error estimates are asymptotically valid, but for nonlinear models they may not be reliable for small samples.

The **S** matrix used for the calculation of the covariance of the parameter estimates is the best estimate available for the estimation method selected. For **S**-iterated methods this is the most recent estimation of Σ. For OLS and 2SLS, an estimate of the **S** matrix is computed from OLS or 2SLS residuals and used for the calculation of the covariance matrix. For a complete list of the **S** matrix used for the calculation of the covariance of the parameter estimates, see Table 14.1.

Missing Values

An observation is excluded from the estimation if any variable used for FIT tasks is missing, if the weight for the observation is not greater than 0 when weights are used, or if a DELETE statement is executed by the model program. Variables used for FIT tasks include the equation errors for each equation, the instruments, if any, and the derivatives of the equation errors with respect to the parameters estimated. Note that variables can become missing as a result of computational errors or calculations with missing values.

The number of usable observations can change when different parameter values are used; some parameter values can be invalid and cause execution errors for some observations. PROC MODEL keeps track of the number of usable and missing observations at each pass through the data, and if the number of missing observations counted during a pass exceeds the number that was obtained using the previous parameter vector, the pass is terminated and the new parameter vector is considered infeasible.

PROC MODEL never takes a step that produces more missing observations than the current estimate does.

The values used to compute the Durbin-Watson, R^2, and other statistics of fit are from the observations used in calculating the objective function and do not include any observation for which any needed variable was missing (residuals, derivatives, and instruments).

Details on the Covariance of Equation Errors

There are several **S** matrices that can be involved in the various estimation methods and in forming the estimate of the covariance of parameter estimates. These **S** matrices are estimates of Σ, the true covariance of the equation errors. Apart from the choice of instrumental or noninstrumental methods, many of the methods provided by PROC MODEL differ in the way the various **S** matrices are formed and used.

All of the estimation methods result in a final estimate of Σ, which is included in the output if the COVS option is specified. The final **S** matrix of each method provides the initial **S** matrix for any subsequent estimation.

This estimate of the covariance of equation errors is defined as

$$\mathbf{S} = \mathbf{D}(\mathbf{R}'\mathbf{R})\mathbf{D}$$

where $\mathbf{R} = (\mathbf{r}_1, \ldots, \mathbf{r}_g)$ is composed of the equation residuals computed from the current parameter estimates in an $n \times g$ matrix and **D** is a diagonal matrix that depends on the VARDEF= option.

For VARDEF=N, the diagonal elements of **D** are $1/\sqrt{n}$, where *n* is the number of nonmissing observations. For VARDEF=WGT, *n* is replaced with the sum of the weights. For VARDEF=WDF, *n* is replaced with the sum of the weights minus the model degrees of freedom. For the default VARDEF=DF, the *i*th diagonal element of **D** is $1/\sqrt{n - df_i}$, where df_i is the degrees of freedom (number of parameters) for the *i*th equation. Binkley and Nelson (1984) show the importance of using a degrees-of-freedom correction in estimating Σ. Their results indicate that the DF method produces more accurate confidence intervals for N3SLS parameter estimates in the linear case than the alternative approach they tested. VARDEF=N is always used for the computation of the FIML estimates.

For the fixed **S** methods, the OUTSUSED= option writes the **S** matrix used in the estimation to a data set. This **S** matrix is either the estimate of the covariance of equation errors matrix from the preceding estimation, or a prior Σ estimate read in from a data set when the SDATA= option is specified. For the diagonal **S** methods, all of the off-diagonal elements of the **S** matrix are set to 0 for the estimation of the parameters and for the OUTSUSED= data set, but the output data set produced by the OUTS= option will contain the off-diagonal elements. For the OLS and N2SLS methods, there is no previous estimate of the covariance of equation errors matrix, and the option OUTSUSED= will save an identity matrix unless a prior Σ estimate is supplied by the SDATA= option. For FIML the OUTSUSED= data set contains the **S** matrix computed with VARDEF=N. The OUTS= data set contains the **S** matrix computed with the selected VARDEF= option.

If the COVS option is used, the method is not **S**-iterated, and **S** is not an identity, the OUTSUSED= matrix is included in the printed output.

For the methods that iterate the covariance of equation errors matrix, the **S** matrix is iteratively re-estimated from the residuals produced by the current parameter estimates. This **S** matrix estimate iteratively replaces the previous estimate until both the parameter estimates and the estimate of the covariance of equation errors matrix converge. The final OUTS= matrix and OUTSUSED= matrix are thus identical for the **S**-iterated methods.

Nested Iterations

By default, for **S**-iterated methods, the **S** matrix is held constant until the parameters converge once. Then the **S** matrix is re-estimated. One iteration of the parameter estimation algorithm is performed, and the **S** matrix is again re-estimated. This latter process is repeated until convergence of both the parameters and the **S** matrix. Since the objective of the minimization depends on the **S** matrix, this has the effect of chasing a moving target.

When the NESTIT option is specified, iterations are performed to convergence for the structural parameters with a fixed **S** matrix. The **S** matrix is then re-estimated, the parameter iterations are repeated to convergence, and so on until both the parameters and the **S** matrix converge. This has the effect of fixing the objective function for the inner parameter iterations. It is more reliable, but usually more expensive, to nest the iterations.

R^2

For unrestricted linear models with an intercept successfully estimated by OLS, R^2 is always between 0 and 1. However, nonlinear models do not necessarily encompass the dependent mean as a special case and can produce negative R^2 statistics. Negative R^2's can also be produced even for linear models when an estimation method other than OLS is used and no intercept term is in the model.

R^2 is defined for normalized equations as

$$R^2 = 1 - \frac{SSE}{SSA - \bar{y}^2 \times n}$$

where SSA is the sum of the squares of the actual y's and $\bar{y}$ are the actual means. R^2 cannot be computed for models in general form because of the need for an actual Y.

Minimization Methods

PROC MODEL currently supports two methods for minimizing the objective function. These methods are described in the following sections.

GAUSS

The Gauss-Newton parameter-change vector for a system with *g* equations, *n* nonmissing observations, and *p* unknown parameters is

$$\mathbf{\Delta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{r}$$

where Δ is the change vector, **X** is the stacked $ng \times p$ Jacobian matrix of partial derivatives of the residuals with respect to the parameters, and **r** is an $ng \times 1$vector of the stacked residuals. The components of **X** and **r** are weighted by the $\mathbf{S}^{-1}$ matrix. When instrumental methods are used, **X** and **r** are the projections of the Jacobian matrix and residuals vector in the instruments space and not the Jacobian and residuals themselves. In the preceding formula, **S** and W are suppressed. If instrumental variables are used, then the change vector becomes:

$$\mathbf{\Delta} = (\mathbf{X}'(\mathbf{S}^{-1} \otimes \mathbf{W})\mathbf{X})^{-1}\mathbf{X}'(\mathbf{S}^{-1} \otimes \mathbf{W})\mathbf{r}$$

This vector is computed at the end of each iteration. The objective function is then computed at the changed parameter values at the start of the next iteration. If the objective function is not improved by the change, the Δ vector is reduced by one-half and the objective function is re-evaluated. The change vector will be halved up to MAXSUBITER= times until the objective function is improved.

For FIML the $\mathbf{X}'\mathbf{X}$ matrix is substituted with one of three choices for approximations to the Hessian. See the "FIML Estimation" section in this chapter.

MARQUARDT

The Marquardt-Levenberg parameter change vector is

$$\mathbf{\Delta} = (\mathbf{X}'\mathbf{X} + \lambda \text{diag}(\mathbf{X}'\mathbf{X}))^{-1}\mathbf{X}'\mathbf{r}$$

where Δ is the change vector, and **X** and **r** are the same as for the Gauss-Newton method, described in the preceding section. Before the iterations start, λ is set to a small value (1E-6). At each iteration, the objective function is evaluated at the parameters changed by Δ. If the objective function is not improved, λ is increased to 10λ and the step is tried again. λ can be increased up to MAXSUBITER= times to a maximum of 1E15 (whichever comes first) until the objective function is improved. For the start of the next iteration, λ is reduced to max(λ/10,1E-10).

Convergence Criteria

There are a number of measures that could be used as convergence or stopping criteria. PROC MODEL computes five convergence measures labeled R, S, PPC, RPC, and OBJECT.

When an estimation technique that iterates estimates of Σ is used (that is, IT3SLS), two convergence criteria are used. The termination values can be specified with the CONVERGE=(*p*,*s*) option on the FIT statement. If the second value, *s*, is not specified, it defaults to *p*. The criterion labeled S (given in the following) controls the convergence of the **S** matrix. When S is less than *s*, the **S** matrix has converged. The criterion labeled R is compared to the *p* value to test convergence of the parameters.

The R convergence measure cannot be computed accurately in the special case of singular residuals (when all the residuals are close to 0) or in the case of a 0 objective value. When either the trace of the **S** matrix computed from the current residuals (trace(S)) or the objective value is less than the value of the SINGULAR= option, convergence is assumed.

The various convergence measures are explained in the following:

R is the primary convergence measure for the parameters. It measures the degree to which the residuals are orthogonal to the Jacobian columns, and it approaches 0 as the gradient of the objective function becomes small. R is defined as the square root of

$$\frac{(r'(S^{-1}\otimes W)X(X'(S^{-1}\otimes W)X)^{-1}X'(S^{-1}\otimes W)r)}{(r'(S^{-1}\otimes W)r)}$$

where **X** is the Jacobian matrix and **r** is the residuals vector. R is similar to the relative offset orthogonality convergence criterion proposed by Bates and Watts (1981).

In the univariate case, the R measure has several equivalent interpretations:

- the cosine of the angle between the residuals vector and the column space of the Jacobian matrix. When this cosine is 0, the residuals are orthogonal to the partial derivatives of the predicted values with respect to the parameters, and the gradient of the objective function is 0.
- the square root of the R^2 for the current linear pseudo-model in the residuals.
- a norm of the gradient of the objective function, where the norming matrix is proportional to the current estimate of the covariance of the parameter estimates. Thus, using R, convergence is judged when the gradient becomes small in this norm.
- the prospective relative change in the objective function value expected from the next GAUSS step, assuming that the current linearization of the model is a good local approximation.

In the multivariate case, R is somewhat more complicated but is designed to go to 0 as the gradient of the objective becomes small and can still be given the previous interpretations for the aggregation of the equations weighted by $\mathbf{S}^{-1}$.

PPC is the prospective parameter change measure. PPC measures the maximum relative change in the parameters implied by the parameter-change vector computed for the next iteration. At the *k*th iteration, PPC is the maximum over the parameters

$$\frac{|\theta_i^{k+1} - \theta_i^k|}{|\theta|_i^k + 1.0e^{-6}}$$

where θ_i^k is the current value of the *i*th parameter and θ_i^{k+1} is the prospective value of this parameter after adding the change vector computed for the next iteration. The parameter with the maximum prospective relative change is printed with the value of PPC, unless the PPC is nearly 0.

RPC is the retrospective parameter change measure. RPC measures the maximum relative change in the parameters from the previous iteration. At the *k*th iteration, RPC is the maximum over *i* of

$$\frac{|\theta_i^k - \theta_i^{k-1}|}{|\theta_i^{k-1} + 1.0e^{-6}|}$$

where θ_i^k is the current value of the *i*th parameter and θ_i^{k-1} is the previous value of this parameter. The name of the parameter with the maximum retrospective relative change is printed with the value of RPC, unless the RPC is nearly 0.

OBJECT measures the relative change in the objective function value between iterations:

$$\frac{|(O^k - O^{k-1}|}{|O^{k-1} + 1.0e^{-6}|}$$

where O^{k-1} is the value of the objective function (O^k) from the previous iteration.

S measures the relative change in the **S** matrix. S is computed as the maximum over *i, j* of

$$\frac{|S_{ij}^k - S_{ij}^{k-1}|}{|S_{ij}^{k-1} + 1.0e^{-6}|}$$

where S^{k-1} is the previous **S** matrix. The S measure is relevant only for estimation methods that iterate the **S** matrix.

An example of the convergence criteria output is as follows:

```
                    The MODEL Procedure
                 IT3SLS Estimation Summary

                   Minimization Summary

             Parameters Estimated           5
             Method                     Gauss
             Iterations                    35


                Final Convergence Criteria

                R                  0.000883
                PPC(d1)            0.000644
                RPC(d1)            0.000815
                Object              0.00004
                Trace(S)           3599.982
                Objective Value    0.435683
                S                  0.000052
```

Figure 14.16. Convergence Criteria Output

This output indicates the total number of iterations required by the Gauss minimization for all the **S** matrices was 35. The "Trace(S)" is the trace (the sum of the diagonal elements) of the **S** matrix computed from the current residuals. This row is labeled MSE if there is only one equation.

Troubleshooting Convergence Problems

As with any nonlinear estimation routine, there is no guarantee that the estimation will be successful for a given model and data. If the equations are linear with respect to the parameters, the parameter estimates always converge in one iteration. The methods that iterate the **S** matrix must iterate further for the **S** matrix to converge. Nonlinear models may not necessarily converge.

Convergence can be expected only with fully identified parameters, adequate data, and starting values sufficiently close to solution estimates.

Convergence and the rate of convergence may depend primarily on the choice of starting values for the estimates. This does not mean that a great deal of effort should be invested in choosing starting values. First, try the default values. If the estimation fails with these starting values, examine the model and data and re-run the estimation using reasonable starting values. It is usually not necessary that the starting values be very good, just that they not be very bad; choose values that seem plausible for the model and data.

An Example of Requiring Starting Values

Suppose you want to regress a variable Y on a variable X assuming that the variables are related by the following nonlinear equation:

$$y = a + bx^c + \epsilon$$

In this equation, Y is linearly related to a power transformation of X. The unknown parameters are *a*, *b*, and *c*. ϵ is an unobserved random error. Some simulated data was generated using the following SAS statements. In this simulation, $a = 10$, $b = 2$, and the use of the SQRT function corresponds to $c = .5$.

```
data test;
  do i = 1 to 20;
     x = 5 * ranuni(1234);
     y = 10 + 2 * sqrt(x) + .5 * rannor(2345);
     output;
     end;
run;
```

The following statements specify the model and give descriptive labels to the model parameters. Then the FIT statement attempts to estimate *a*, *b*, and *c* using the default starting value .0001.

```
proc model data=test;
   y = a + b * x ** c;
   label a = "Intercept"
```

```
            b = "Coefficient of Transformed X"
            c = "Power Transformation Parameter";
      fit y;
   run;
```

PROC MODEL prints model summary and estimation problem summary reports and then prints the output shown in Figure 14.17.

```
                              The MODEL Procedure
                                OLS Estimation

NOTE: The iteration limit is exceeded for OLS.


        ERROR: The parameter estimates failed to converge for OLS after
        100 iterations using CONVERGE=0.001 as the convergence criteria.



                              The MODEL Procedure
                                OLS Estimation

                                                 N
            Iteration N Obs      R Objective Subit        a        b        c
   OLS            100    20 0.9627    3.9678     2 137.3844 -126.536 -0.00213


                      Gauss Method Parameter Change Vector

                           a              b              c

                   -69367.57       69366.51          -1.16
NOTE: The parameter estimation is abandoned. Check your model and data. If the
      model is correct and the input data are appropriate, try rerunning the
      parameter estimation using different starting values for the parameter
      estimates.
PROC MODEL continues as if the parameter estimates had converged.
```

Figure 14.17. Diagnostics for Convergence Failure

By using the default starting values, PROC MODEL was unable to take even the first step in iterating to the solution. The change in the parameters that the Gauss-Newton method computes is very extreme and makes the objective values worse instead of better. Even when this step is shortened by a factor of a million, the objective function is still worse, and PROC MODEL is unable to estimate the model parameters.

The problem is caused by the starting value of C. Using the default starting value C=.0001, the first iteration attempts to compute better values of A and B by what is, in effect, a linear regression of Y on the 10,000th root of X, which is almost the same as the constant 1. Thus the matrix that is inverted to compute the changes is nearly singular and affects the accuracy of the computed parameter changes.

This is also illustrated by the next part of the output, which displays collinearity diagnostics for the crossproducts matrix of the partial derivatives with respect to the parameters, shown in Figure 14.18.

```
                              The MODEL Procedure
                                 OLS Estimation

                            Collinearity Diagnostics

                              Condition    -----Proportion of Variation----
    Number       Eigenvalue      Number            a           b           c

         1         2.376793      1.0000       0.0000      0.0000      0.0000
         2         0.623207      1.9529       0.0000      0.0000      0.0000
         3    1.684616E-12      1187805       1.0000      1.0000      1.0000
```

Figure 14.18. Collinearity Diagnostics

This output shows that the matrix is singular and that the partials of A, B, and C with respect to the residual are collinear at the point $(0.0001, 0.0001, 0.0001)$ in the parameter space. See the section "Linear Dependencies" for a full explanation of the collinearity diagnostics.

The MODEL procedure next prints the note shown in Figure 14.19, which suggests that you try different starting values.

```
                              The MODEL Procedure
                                 OLS Estimation

NOTE: The parameter estimation is abandoned. Check your model and data. If the
      model is correct and the input data are appropriate, try rerunning the
      parameter estimation using different starting values for the parameter
      estimates.
PROC MODEL continues as if the parameter estimates had converged.
```

Figure 14.19. Estimation Failure Note

PROC MODEL then produces the usual printout of results for the nonconverged parameter values. The estimation summary is shown in Figure 14.20. The heading includes the reminder "(Not Converged)."

```
                              The MODEL Procedure
                                OLS Estimation

                            Collinearity Diagnostics

                              Condition    -----Proportion of Variation----
   Number      Eigenvalue        Number           a            b            c

        1        2.376793        1.0000      0.0000       0.0000       0.0000
        2        0.623207        1.9529      0.0000       0.0000       0.0000
        3    1.684616E-12       1187805      1.0000       1.0000       1.0000



                              The MODEL Procedure
                      OLS Estimation Summary (Not Converged)

                              Minimization Summary

                       Parameters Estimated            3
                       Method                      Gauss
                       Iterations                    100
                       Subiterations                 239
                       Average Subiterations        2.39


                          Final Convergence Criteria

                          R                  0.962666
                          PPC(b)             548.1977
                          RPC(b)             540.4224
                          Object             2.633E-6
                          Trace(S)           4.667947
                          Objective Value    3.967755


                             Observations Processed

                                  Read      20
                                  Solved    20
```

Figure 14.20. Nonconverged Estimation Summary

The nonconverged estimation results are shown in Figure 14.21.

```
                              The MODEL Procedure

                   Nonlinear OLS Summary of Residual Errors
                                (Not Converged)
                   DF     DF                                               Adj
  Equation      Model  Error       SSE       MSE  Root MSE  R-Square     R-Sq

  y                 3     17   79.3551    4.6679    2.1605   -1.6812  -1.9966


               Nonlinear OLS Parameter Estimates (Not Converged)

                            Approx                Approx
Parameter      Estimate    Std Err    t Value    Pr > |t|   Label

a              137.3844     263342       0.00      0.9996   Intercept
b              -126.536     263342      -0.00      0.9996   Coefficient of
                                                            Transformed X
c              -0.00213     4.4371      -0.00      0.9996   Power Transformation
                                                            Parameter
```

Figure 14.21. Nonconverged Results

Note that the R^2 statistic is negative. An $R^2 < 0$ results when the residual mean square error for the model is larger than the variance of the dependent variable. Negative R^2 statistics may be produced when either the parameter estimates fail to converge correctly, as in this case, or when the correctly estimated model fits the data very poorly.

Controlling Starting Values

To fit the preceding model you must specify a better starting value for C. Avoid starting values of C that are either very large or close to 0. For starting values of A and B, you can either specify values, use the default, or have PROC MODEL fit starting values for them conditional on the starting value for C.

Starting values are specified with the START= option of the FIT statement or on a PARMS statement. For example, the following statements estimate the model parameters using the starting values A=.0001, B=.0001, and C=5.

```
proc model data=test;
   y = a + b * x ** c;
   label a = "Intercept"
         b = "Coefficient of Transformed X"
         c = "Power Transformation Parameter";
   fit y start=(c=5);
run;
```

Using these starting values, the estimates converge in 16 iterations. The results are shown in Figure 14.22. Note that since the START= option explicitly declares parameters, the parameter C is placed first in the table.

```
                              The MODEL Procedure

                    Nonlinear OLS Summary of Residual Errors

                  DF     DF                                                Adj
  Equation      Model  Error       SSE       MSE  Root MSE  R-Square      R-Sq

  y                 3     17    5.7359    0.3374    0.5809    0.8062    0.7834


                        Nonlinear OLS Parameter Estimates

                             Approx                Approx
Parameter      Estimate    Std Err    t Value    Pr > |t|   Label

c              0.327079     0.2892       1.13      0.2738   Power Transformation
                                                            Parameter
a              8.384311     3.3775       2.48      0.0238   Intercept
b              3.505391     3.4858       1.01      0.3287   Coefficient of
                                                            Transformed X
```

Figure 14.22. Converged Results

Using the STARTITER Option

PROC MODEL can compute starting values for some parameters conditional on starting values you specify for the other parameters. You supply starting values for some parameters and specify the STARTITER option on the FIT statement.

For example, the following statements set C to 1 and compute starting values for A and B by estimating these parameters conditional on the fixed value of C. With C=1 this is equivalent to computing A and B by linear regression on X. A PARMS statement is used to declare the parameters in alphabetical order. The ITPRINT option is used to print the parameter values at each iteration.

```
proc model data=test;
   parms a b c;
   y = a + b * x ** c;
   label a = "Intercept"
         b = "Coefficient of Transformed X"
         c = "Power Transformation Parameter";
   fit y start=(c=1) / startiter itprint;
run;
```

With better starting values, the estimates converge in only 5 iterations. Counting the 2 iterations required to compute the starting values for A and B, this is 5 fewer than the 12 iterations required without the STARTITER option. The iteration history listing is shown in Figure 14.23.

```
                              The MODEL Procedure
                                 OLS Estimation

                                                   N
         Iteration N Obs       R Objective Subit         a         b         c
   GRID          0    20 0.9970      161.9     0  0.00010  0.00010  5.00000
   GRID          1    20 0.0000     0.9675     0 12.29508  0.00108  5.00000


                                                   N
         Iteration N Obs       R Objective Subit         a         b         c
   OLS           0    20 0.6551     0.9675     0 12.29508  0.00108  5.00000
   OLS           1    20 0.6882     0.9558     4 12.26426  0.00201  4.44013
   OLS           2    20 0.6960     0.9490     4 12.25554  0.00251  4.28262
   OLS           3    20 0.7058     0.9428     2 12.24487  0.00323  4.09977
   OLS           4    20 0.7177     0.9380     2 12.23186  0.00430  3.89040
   OLS           5    20 0.7317     0.9354     2 12.21610  0.00592  3.65450
   OLS           6    20 0.7376     0.9289     3 12.20663  0.00715  3.52417
   OLS           7    20 0.7445     0.9223     2 12.19502  0.00887  3.37407
   OLS           8    20 0.7524     0.9162     2 12.18085  0.01130  3.20393
   OLS           9    20 0.7613     0.9106     2 12.16366  0.01477  3.01460
   OLS          10    20 0.7705     0.9058     2 12.14298  0.01975  2.80839
   OLS          11    20 0.7797     0.9015     2 12.11827  0.02690  2.58933
   OLS          12    20 0.7880     0.8971     2 12.08900  0.03712  2.36306
   OLS          13    20 0.7947     0.8916     2 12.05460  0.05152  2.13650
   OLS          14    20 0.7993     0.8835     2 12.01449  0.07139  1.91695
   OLS          15    20 0.8015     0.8717     2 11.96803  0.09808  1.71101
   OLS          16    20 0.8013     0.8551     2 11.91459  0.13284  1.52361
   OLS          17    20 0.7987     0.8335     2 11.85359  0.17666  1.35745
   OLS          18    20 0.8026     0.8311     1 11.71551  0.28373  1.06872
   OLS          19    20 0.7945     0.7935     2 11.57666  0.40366  0.89662
   OLS          20    20 0.7872     0.7607     1 11.29346  0.65999  0.67059
   OLS          21    20 0.7632     0.6885     1 10.81372  1.11483  0.48842
   OLS          22    20 0.6976     0.5587     0  9.54889  2.34556  0.30461
   OLS          23    20 0.0108     0.2868     0  8.44333  3.44826  0.33232
   OLS          24    20 0.0008     0.2868     0  8.39438  3.49500  0.32790


        NOTE: At OLS Iteration 24 CONVERGE=0.001 Criteria Met.
```

Figure 14.23. ITPRINT Listing

The results produced in this case are almost the same as the results shown in Figure 14.22, except that the PARMS statement causes the Parameter Estimates table to be ordered A, B, C instead of C, A, B. They are not exactly the same because the different starting values caused the iterations to converge at a slightly different place. This effect is controlled by changing the convergence criterion with the CONVERGE= option.

By default, the STARTITER option performs one iteration to find starting values for the parameters not given values. In this case the model is linear in A and B, so only one iteration is needed. If A or B were nonlinear, you could specify more than one "starting values" iteration by specifying a number for the STARTITER= option.

Finding Starting Values by Grid Search

PROC MODEL can try various combinations of parameter values and use the combination producing the smallest objective function value as starting values. (For OLS the objective function is the residual mean square.) This is known as a preliminary *grid search*. You can combine the STARTITER option with a grid search.

For example, the following statements try 5 different starting values for C: 10, 5, 2.5, -2.5, -5. For each value of C, values for A and B are estimated. The combination of A, B, and C values producing the smallest residual mean square is then used to start the iterative process.

```
proc model data=test;
   parms a b c;
   y = a + b * x ** c;
   label a = "Intercept"
         b = "Coefficient of Transformed X"
         c = "Power Transformation Parameter";
   fit y start=(c=10 5 2.5 -2.5 -5) / startiter itprint;
run;
```

The iteration history listing is shown in Figure 14.24. Using the best starting values found by the grid search, the OLS estimation only requires 2 iterations. However, since the grid search required 10 iterations, the total iterations in this case is 12.

```
                              The MODEL Procedure
                                OLS Estimation

                                                   N
         Iteration N Obs      R Objective Subit        a        b        c
GRID             0    20 1.0000   26815.5     0  0.00010  0.00010 10.00000
GRID             1    20 0.0000    1.2193     0 12.51792  0.00000 10.00000
GRID             0    20 0.6012    1.5151     0 12.51792  0.00000  5.00000
GRID             1    20 0.0000    0.9675     0 12.29508  0.00108  5.00000
GRID             0    20 0.7804    1.6091     0 12.29508  0.00108  2.50000
GRID             1    20 0.0000    0.6290     0 11.87327  0.06372  2.50000
GRID             0    20 0.8779    4.1604     0 11.87327  0.06372 -2.50000
GRID             1    20 0.0000    0.9542     0 12.92455 -0.04700 -2.50000
GRID             0    20 0.9998    2776.1     0 12.92455 -0.04700 -5.00000
GRID             1    20 0.0000    1.0450     0 12.86129 -0.00060 -5.00000


                                                   N
         Iteration N Obs      R Objective Subit        a        b        c
OLS              0    20 0.6685    0.6290     0 11.87327  0.06372  2.50000
OLS              1    20 0.6649    0.5871     3 11.79268  0.10083  2.11710
OLS              2    20 0.6713    0.5740     2 11.71445  0.14901  1.81658
OLS              3    20 0.6726    0.5621     2 11.63772  0.20595  1.58705
OLS              4    20 0.6678    0.5471     2 11.56098  0.26987  1.40903
OLS              5    20 0.6587    0.5295     2 11.48317  0.33953  1.26760
OLS              6    20 0.6605    0.5235     1 11.32436  0.48846  1.03784
OLS              7    20 0.6434    0.4997     2 11.18704  0.62475  0.90793
OLS              8    20 0.6294    0.4805     1 10.93520  0.87965  0.73319
OLS              9    20 0.6031    0.4530     1 10.55670  1.26879  0.57385
OLS             10    20 0.6052    0.4526     0  9.62442  2.23114  0.36146
OLS             11    20 0.1652    0.2948     0  8.56683  3.31774  0.32417
OLS             12    20 0.0008    0.2868     0  8.38015  3.50974  0.32664


     NOTE: At OLS Iteration 12 CONVERGE=0.001 Criteria Met.
```

Figure 14.24. ITPRINT Listing

Because no initial values for A or B were provided in the PARAMETERS statement or were read in with a PARMSDATA= or ESTDATA= option, A and B were given the default value of 0.0001 for the first iteration. At the second grid point, C=5, the values of A and B obtained from the previous iterations are used for the initial iteration. If initial values are provided for parameters, the parameters start at those initial values at each grid point.

Guessing Starting Values from the Logic of the Model

Example 14.1 of the logistic growth curve model of the U.S. population illustrates the need for reasonable starting values. This model can be written

$$pop = \frac{a}{1 + \exp(b - c(t - 1790))}$$

where *t* is time in years. The model is estimated using decennial census data of the U.S. population in millions. If this simple but highly nonlinear model is estimated using the default starting values, the estimation fails to converge.

To find reasonable starting values, first consider the meaning of *a* and *c*. Taking the limit as time increases, *a* is the limiting or maximum possible population. So, as a starting value for *a*, several times the most recent population known can be used, for example, one billion (1000 million).

Dividing the time derivative by the function to find the growth rate and taking the limit as *t* moves into the past, you can determine that *c* is the initial growth rate. You can examine the data and compute an estimate of the growth rate for the first few decades, or you can pick a number that sounds like a plausible population growth rate figure, such as 2%.

To find a starting value for *b*, let *t* equal the base year used, 1790, which causes *c* to drop out of the formula for that year, and then solve for the value of *b* that is consistent with the known population in 1790 and with the starting value of *a*. This yields $b = \ln(a/3.9 - 1)$ or about 5.5, where *a* is 1000 and 3.9 is roughly the population for 1790 given in the data. The estimates converge using these starting values.

Convergence Problems

When estimating nonlinear models, you may encounter some of the following convergence problems.

Unable to Improve

The optimization algorithm may be unable to find a step that improves the objective function. If this happens in the Gauss-Newton method, the step size is halved to find a change vector for which the objective improves. In the Marquardt method, λ will be increased to find a change vector for which the objective improves. If, after MAXSUBITER= step-size halvings or increases in λ, the change vector still does not produce a better objective value, the iterations are stopped and an error message is printed.

Failure of the algorithm to improve the objective value can be caused by a CONVERGE= value that is too small. Look at the convergence measures reported at the point of failure. If the estimates appear to be approximately converged, you can accept the NOT CONVERGED results reported, or you can try re-running the FIT task with a larger CONVERGE= value.

If the procedure fails to converge because it is unable to find a change vector that improves the objective value, check your model and data to ensure that all parameters are identified and data values are reasonably scaled. Then, re-run the model with different starting values. Also, consider using the Marquardt method if Gauss-Newton fails; the Gauss-Newton method can get into trouble if the Jacobian matrix is nearly singular or ill-conditioned. Keep in mind that a nonlinear model may be well-identified and well-conditioned for parameter values close to the solution values but unidentified or numerically ill-conditioned for other parameter values. The choice of starting values can make a big difference.

Nonconvergence

The estimates may diverge into areas where the program overflows or the estimates may go into areas where function values are illegal or too badly scaled for accurate calculation. The estimation may also take steps that are too small or that make only marginal improvement in the objective function and, thus, fail to converge within the iteration limit.

When the estimates fail to converge, collinearity diagnostics for the Jacobian crossproducts matrix are printed if there are 20 or fewer parameters estimated. See "Linear Dependencies" later in this section for an explanation of these diagnostics.

Inadequate Convergence Criterion

If convergence is obtained, the resulting estimates will only approximate a minimum point of the objective function. The statistical validity of the results is based on the exact minimization of the objective function, and for nonlinear models the quality of the results depends on the accuracy of the approximation of the minimum. This is controlled by the convergence criterion used.

There are many nonlinear functions for which the objective function is quite flat in a large region around the minimum point so that many quite different parameter vectors may satisfy a weak convergence criterion. By using different starting values, different convergence criteria, or different minimization methods, you can produce very different estimates for such models.

You can guard against this by running the estimation with different starting values and different convergence criteria and checking that the estimates produced are essentially the same. If they are not, use a smaller CONVERGE= value.

Local Minimum

You may have converged to a local minimum rather than a global one. This problem is difficult to detect because the procedure will appear to have succeeded. You can guard against this by running the estimation with different starting values or with a different minimization technique. The START= option can be used to automatically perform a grid search to aid in the search for a global minimum.

Discontinuities

The computational methods assume that the model is a continuous and smooth function of the parameters. If this is not the case, the methods may not work.

If the model equations or their derivatives contain discontinuities, the estimation will usually succeed, provided that the final parameter estimates lie in a continuous interval and that the iterations do not produce parameter values at points of discontinuity or parameter values that try to cross asymptotes.

One common case of discontinuities causing estimation failure is that of an asymptotic discontinuity between the final estimates and the initial values. For example, consider the following model, which is basically linear but is written with one parameter in reciprocal form:

```
   y = a + b * x1 + x2 / c;
```

By placing the parameter C in the denominator, a singularity is introduced into the parameter space at C=0. This is not necessarily a problem, but if the correct estimate of C is negative while the starting value is positive (or vice versa), the asymptotic discontinuity at 0 will lie between the estimate and the starting value. This means that the iterations have to pass through the singularity to get to the correct estimates. The situation is shown in Figure 14.25.

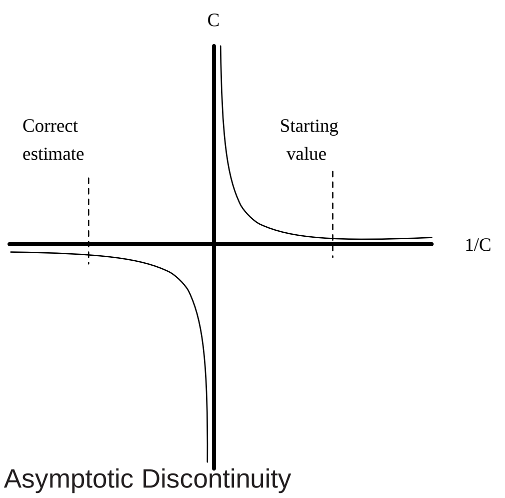


Figure 14.25. Asymptotic Discontinuity

Because of the incorrect sign of the starting value, the C estimate goes off towards positive infinity in a vain effort to get past the asymptote and onto the correct arm of the hyperbola. As the computer is required to work with ever closer approximations to infinity, the numerical calculations break down and an "objective function was not improved" convergence failure message is printed. At this point, the iterations terminate with an extremely large positive value for C. When the sign of the starting value for C is changed, the estimates converge quickly to the correct values.

Linear Dependencies

In some cases, the Jacobian matrix may not be of full rank; parameters may not be fully identified for the current parameter values with the current data. When linear dependencies occur among the derivatives of the model, some parameters appear with a standard error of 0 and with the word BIASED printed in place of the *t* statistic. When this happens, collinearity diagnostics for the Jacobian crossproducts matrix are printed if the DETAILS option is specified and there are twenty or fewer parameters estimated. Collinearity diagnostics are also printed out automatically when a minimization method fails, or when the COLLIN option is specified.

For each parameter, the proportion of the variance of the estimate accounted for by each *principal component* is printed. The principal components are constructed from the eigenvalues and eigenvectors of the correlation matrix (scaled covariance matrix). When collinearity exists, a principal component is associated with proportion of the variance of more than one parameter. The numbers reported are proportions so they will remain between 0 and 1. If two or more parameters have large proportion values associated with the same principle component, then two problems can occur: the computation of the parameter estimates are slow or nonconvergent; and the parameter estimates have inflated variances (Belsley 1980, p. 105-117).

For example, the following cubic model is fit to a quadratic data set:

```
proc model data=test3;
   exogenous x1 ;
   parms b1 a1 c1 ;
   y1 = a1 * x1 + b1 * x1 * x1  + c1 * x1 * x1 *x1;
   fit y1/ collin ;
run;
```

The collinearity diagnostics are shown in Figure 14.26.

```
                                The MODEL Procedure

                              Collinearity Diagnostics

                                Condition    -----Proportion of Variation----
  Number       Eigenvalue          Number           b1           a1           c1

       1         2.942920          1.0000       0.0001       0.0004       0.0002
       2         0.056638          7.2084       0.0001       0.0357       0.0148
       3         0.000442         81.5801       0.9999       0.9639       0.9850
```

Figure 14.26. Collinearity Diagnostics

Notice that the proportions associated with the smallest eigenvalue are almost 1. For this model, removing any of the parameters will decrease the variances of the remaining parameters.

In many models the collinearity might not be clear cut. Collinearity is not necessarily something you remove. A model may need to be reformulated to remove the redundant parameterization or the limitations on the estimatability of the model can be accepted.

Collinearity diagnostics are also useful when an estimation does not converge. The diagnostics provide insight into the numerical problems and can suggest which parameters need better starting values. These diagnostics are based on the approach of Belsley, Kuh, and Welsch (1980).

Iteration History

The options ITPRINT, ITDETAILS, XPX, I, and ITALL specify a detailed listing of each iteration of the minimization process.

ITPRINT Option

The ITPRINT information is selected whenever any iteration information is requested.

The following information is displayed for each iteration:

N	the number of usable observations
Objective	the corrected objective function value
Trace(S)	the trace of the **S** matrix
subit	the number of subiterations required to find a λ or a damping factor that reduces the objective function
R	the R convergence measure

The estimates for the parameters at each iteration are also printed.

ITDETAILS Option

The additional values printed for the ITDETAILS option are:

Theta	is the angle in degrees between Δ, the parameter change vector, and the negative gradient of the objective function.
Phi	is the directional derivative of the objective function in the Δ direction scaled by the objective function.
Stepsize	is the value of the damping factor used to reduce Δ if the Gauss-Newton method is used.
Lambda	is the value of λ if the Marquardt method is used.
Rank(XPX)	If the projected Jacobian crossproducts matrix is singular, the rank of the $\mathbf{X}'\mathbf{X}$ matrix is output.

The definitions of PPC and R are explained in the section "Convergence Criteria." When the values of PPC are large, the parameter associated with the criteria is displayed in parentheses after the value.

XPX and I Options

The XPX and the I options select the printing of the augmented $\mathbf{X}'\mathbf{X}$ matrix and the augmented $\mathbf{X}'\mathbf{X}$ matrix after a *sweep* operation (Goodnight 1979) has been performed on it. An example of the output from the following statements is shown in Figure 14.27.

```
proc model data=test2 ;
   y1 = a1 * x2 * x2 - exp( d1*x1);
   y2 = a2 * x1 * x1 + b2 * exp( d2*x2);
   fit y1 y2 / XPX I ;
run;
```

```
                              The MODEL Procedure
                                OLS Estimation

                 Cross Products for System  At OLS Iteration 0

                  a1          d1          a2          b2          d2    Residual

a1           1839468   -33818.35         0.0        0.00    0.000000     3879959
d1            -33818     1276.45         0.0        0.00    0.000000      -76928
a2                 0        0.00     42925.0     1275.15    0.154739      470686
b2                 0        0.00      1275.2       50.01    0.003867       16055
d2                 0        0.00         0.2        0.00    0.000064           2
Residual     3879959   -76928.14    470686.3    16055.07    2.329718    24576144


                   XPX Inverse for System  At OLS Iteration 0

                  a1          d1          a2          b2          d2    Residual

a1          0.000001    0.000028    0.000000      0.0000        0.00           2
d1          0.000028    0.001527    0.000000      0.0000        0.00          -9
a2          0.000000    0.000000    0.000097     -0.0025       -0.08           6
b2          0.000000    0.000000   -0.002455      0.0825        0.95         172
d2          0.000000    0.000000   -0.084915      0.9476    15746.71       11931
Residual    1.952150   -8.546875    5.823969    171.6234    11930.89    10819902
```

Figure 14.27. XPX and I Options Output

The first matrix, labeled "Cross Products," for OLS estimation is

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{r} \\ \mathbf{r}'\mathbf{X} & \mathbf{r}'\mathbf{r} \end{bmatrix}$$

The column labeled "Residual" in the output is the vector $\mathbf{X}'\mathbf{r}$, which is the gradient of the objective function. The diagonal scalar value $\mathbf{r}'\mathbf{r}$ is the objective function uncorrected for degrees of freedom. The second matrix, labeled "XPX Inverse," is created through a sweep operation on the augmented $\mathbf{X}'\mathbf{X}$ matrix to get:

$$\begin{bmatrix} (\mathbf{X}'\mathbf{X})^{-1} & (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{r} \\ (\mathbf{X}'\mathbf{r})'(\mathbf{X}'\mathbf{X})^{-1} & \mathbf{r}'\mathbf{r} - (\mathbf{X}'\mathbf{r})'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{r} \end{bmatrix}$$

Note that the residual column is the change vector used to update the parameter estimates at each iteration. The corner scalar element is used to compute the R convergence criteria.

ITALL Option

The ITALL option, in addition to causing the output of all of the preceding options, outputs the **S** matrix, the inverse of the **S** matrix, the CROSS matrix, and the swept CROSS matrix. An example of a portion of the CROSS matrix for the preceding example is shown in Figure 14.28.

```
                              The MODEL Procedure
                                 OLS Estimation

                   Crossproducts Matrix  At OLS Iteration 0

                          1     @PRED.y1/@a1     @PRED.y1/@d1     @PRED.y2/@a2

1                     50.00             6409          -239.16           1275.0
@PRED.y1/@a1        6409.08          1839468        -33818.35         187766.1
@PRED.y1/@d1        -239.16           -33818          1276.45          -7253.0
@PRED.y2/@a2        1275.00           187766         -7253.00          42925.0
@PRED.y2/@b2          50.00             6410          -239.19           1275.2
@PRED.y2/@d2           0.00                1            -0.03              0.2
RESID.y1           14699.97          3879959        -76928.14         420582.9
RESID.y2           16052.76          4065028        -85083.68         470686.3

                   Crossproducts Matrix  At OLS Iteration 0

                  @PRED.y2/@b2      @PRED.y2/@d2       RESID.y1       RESID.y2

1                        50.00          0.003803          14700          16053
@PRED.y1/@a1           6409.88          0.813934        3879959        4065028
@PRED.y1/@d1           -239.19         -0.026177         -76928         -85084
@PRED.y2/@a2           1275.15          0.154739         420583         470686
@PRED.y2/@b2             50.01          0.003867          14702          16055
@PRED.y2/@d2              0.00          0.000064              2              2
RESID.y1              14701.77          1.820356       11827102       12234106
RESID.y2              16055.07          2.329718       12234106       12749042
```

Figure 14.28. ITALL Option Cross-Products Matrix Output

Computer Resource Requirements

If you are estimating large systems, you need to be aware of how PROC MODEL uses computer resources such as memory and the CPU so they can be used most efficiently.

Saving Time with Large Data Sets

If your input data set has many observations, the FIT statement does a large number of model program executions. A pass through the data is made at least once for each iteration and the model program is executed once for each observation in each pass. If you refine the starting estimates by using a smaller data set, the final estimation with the full data set may require fewer iterations.

For example, you could use

```
proc model;
   /* Model goes here */
   fit / data=a(obs=25);
   fit / data=a;
```

where OBS=25 selects the first 25 observations in A. The second FIT statement produces the final estimates using the full data set and starting values from the first run.

Fitting the Model in Sections to Save Space and Time

If you have a very large model (with several hundred parameters, for example), the procedure uses considerable space and time. You may be able to save resources by breaking the estimation process into several steps and estimating the parameters in subsets.

You can use the FIT statement to select for estimation only the parameters for selected equations. Do not break the estimation into too many small steps; the total computer time required is minimized by compromising between the number of FIT statements that are executed and the size of the crossproducts matrices that must be processed.

When the parameters are estimated for selected equations, the entire model program must be executed even though only a part of the model program may be needed to compute the residuals for the equations selected for estimation. If the model itself can be broken into sections for estimation (and later combined for simulation and forecasting), then more resources can be saved.

For example, to estimate the following four equation model in two steps, you could use

```
proc model data=a outmodel=part1;
   parms a0-a2 b0-b2 c0-c3 d0-d3;
   y1 = a0 + a1*y2 + a2*x1;
   y2 = b0 + b1*y1 + b2*x2;
   y3 = c0 + c1*y1 + c2*y4 + c3*x3;
   y4 = d0 + d1*y1 + d2*y3 + d3*x4;
   fit y1 y2;
   fit y3 y4;
   fit y1 y2 y3 y4;
run;
```

You should try estimating the model in pieces to save time only if there are more than 14 parameters; the preceding example takes more time, not less, and the difference in memory required is trivial.

Memory Requirements for Parameter Estimation

PROC MODEL is a large program, and it requires much memory. Memory is also required for the SAS System, various data areas, the model program and associated tables and data vectors, and a few crossproducts matrices. For most models, the memory required for PROC MODEL itself is much larger than that required for the model program, and the memory required for the model program is larger than that required for the crossproducts matrices.

The number of bytes needed for two crossproducts matrices, four **S** matrices, and three parameter covariance matrices is

$$8 \times (2 + k + m + g)^2 + 16 \times g^2 + 12 \times (p + 1)^2$$

plus lower-order terms. *m* is the number of unique nonzero derivatives of each residual with respect to each parameter, *g* is the number of equations, *k* is the number of instruments, and *p* is the number of parameters. This formula is for the memory required for 3SLS. If you are using OLS, a reasonable estimate of the memory required for large problems (greater than 100 parameters) is to divide the value obtained from the formula in half.

Consider the following model program.

```
proc model data=test2 details;
   exogenous x1 x2;
   parms b1 100 a1 a2 b2 2.5 c2 55;
   y1 = a1 * y2 + b1 * x1 * x1;
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2;
   fit y1 y2 / n3sls;
   inst b1 b2 c2 x1 ;
run;
```

The DETAILS option prints the storage requirements information shown in Figure 14.29.

```
                              The MODEL Procedure

                      Storage Requirements for this Problem

                      Order of XPX Matrix                6
                      Order of S Matrix                  2
                      Order of Cross Matrix             13
                      Total Nonzero Derivatives          5
                      Distinct Variable Derivatives      5
                      Size of Cross matrix             728
```

Figure 14.29. Storage Requirements Information

The matrix $\mathbf{X}'\mathbf{X}$ augmented by the residual vector is called the XPX matrix in the output, and it has the size $m + 1$. The order of the **S** matrix, 2 for this example, is the value of *g*. The CROSS matrix is made up of the *k* unique instruments, a constant column representing the intercept terms, followed by the *m* unique Jacobian variables plus a constant column representing the parameters with constant derivatives, followed by the *g* residuals.

The size of two CROSS matrices in bytes is

$$8 \times (2 + k + m + g)^2 + 2 + k + m + g$$

Note that the CROSS matrix is symmetric, so only the diagonal and the upper triangular part of the matrix is stored. For examples of the CROSS and XPX matrices see "Iteration History" in this section.

The MEMORYUSE Option

The MEMORYUSE option on the FIT, SOLVE, MODEL, or RESET statement may be used to request a comprehensive memory usage summary.

Figure 14.30 shows an example of the output produced by the MEMORYUSE option.

```
                           The MODEL Procedure

                     Memory Usage Summary (in bytes)

                Symbols                                 5368
                Strings                                 1057
                Lists                                   1472
                Arrays                                    84
                Statements                               704
                Opcodes                                  800
                Parsing                                  640
                Executable                               220
                Block option                               0
                Cross reference                            0
                Flow analysis                           1024
                Derivatives                             9406
                Data vector                              240
                Cross matrix                             728
                X'X matrix                               392
                S matrix                                  96
                GMM memory                                 0
                Jacobian                                   0
                Work vectors                             692
                Overhead                                1906
                -----------------------    --------------
                Total                                  24829
```

Figure 14.30. MEMORYUSE Option Output for SOLVE Task

Definitions of the memory components follows:

symbols	memory used to store information about variables in the model
strings	memory used to store the variable names and labels
lists	space used to hold lists of variables
arrays	memory used by ARRAY statements
statements	memory used for the list of programming statements in the model
opcodes	memory used to store the code compiled to evaluate the expression in the model program
parsing	memory used in parsing the SAS statements
executable	the compiled model program size (not correct yet)
block option	memory used by the BLOCK option
cross ref.	memory used by the XREF option
flow analysis	memory used to compute the interdependencies of the variables
derivatives	memory used to compute and store the analytical derivatives
data vector	memory used for the program data vector
cross matrix	memory used for one or more copies of the Cross matrix
$\mathbf{X}'\mathbf{X}$ matrix	memory used for one or more copies of the $\mathbf{X}'\mathbf{X}$ matrix
S matrix	memory used for the covariance matrix
GMM memory	additional memory used for the GMM and ITGMM methods
Jacobian	memory used for the Jacobian matrix for SOLVE and FIML
work vectors	memory used for miscellaneous work vectors
overhead	other miscellaneous memory

Testing for Normality

The NORMAL option on the FIT statement performs multivariate and univariate tests of normality.

The three multivariate tests provided are Mardia's skewness test and kurtosis test (Mardia 1980) and the Henze-Zirkler $T_{n,\beta}$ test (Henze and Zirkler 1990). The two univariate tests provided are the Shapiro-Wilk W test and the Kolmogorov-Smirnov test. (For details on the univariate tests, refer to "Tests for Normality" in "The UNIVARIATE Procedure" chapter in the *SAS Procedures Guide*.) The null hypothesis for all these tests is that the residuals are normally distributed.

For a random sample $X_1, \ldots, X_n$, $X_i \in \mathrm{R}^{\mathrm{d}}$, where *d* is the dimension of X_i and *n* is the number of observations, a measure of multivariate skewness is

$$b_{1,d} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} [(X_i - \mu)' S^{-1} (X_j - \mu)]^3$$

where **S** is the sample covariance matrix of **X**. For weighted regression, both **S** and $(X_i - \mu)$ are computed using the weights supplied by the WEIGHT statement or the _WEIGHT_ variable.

Mardia showed that under the null hypothesis $\frac{n}{6} b_{1,d}$ is asymptotically distributed as $\chi^2(d(d+1)(d+2)/6)$.

A measure of multivariate kurtosis is given by

$$b_{2,d} = \frac{1}{n} \sum_{i=1}^{n} [(X_i - \mu)' S^{-1} (X_i - \mu)]^2$$

Mardia showed that under the null hypothesis $b_{2,d}$is asymptotically normally distributed with mean $d(d+2)$ and variance $8d(d+2)/n$.

The Henze-Zirkler test is based on a nonnegative functional $D(.,.)$that measures the distance between two distribution functions and has the property that

$$D(\mathrm{N_d}(0, \mathrm{I_d}), \mathrm{Q}) = 0$$

if and only if

$$Q = \mathrm{N_d}(0, \mathrm{I_d})$$

where $\mathrm{N_d}(\mu, \Sigma_{\mathrm{d}})$ is a *d*-dimensional normal distribution.

The distance measure $D(.,.)$ can be written as

$$D_\beta(P, Q) = \int_{\mathrm{R^d}} |\hat{P}(t) - \hat{Q}(t)|^2 \varphi_\beta(t) dt$$

where $\hat{P}(t)$ and $\hat{Q}(t)$ are the Fourier transforms of P and Q, and $\varphi_\beta(t)$ is a weight or a kernel function. The density of the normal distribution $\mathrm{N_d}(0, \beta^2\mathrm{I_d})$ is used as $\varphi_\beta(t)$

$$\varphi_\beta(t) = (2\pi\beta^2)^{\frac{-d}{2}}\exp(\frac{-|\mathrm{t}|^2}{2\beta^2}), \quad \mathrm{t} \in \mathrm{R^d}$$

where $|t| = (t^{'}t)^{0.5}$.

The parameter β depends on n as

$$\beta_d(n) = \frac{1}{\sqrt{2}}(\frac{2d+1}{4})^{1/(d+4)}n^{1/(d+4)}$$

The test statistic computed is called $T_\beta(d)$ and is approximately distributed as a log normal. The log normal distribution is used to compute the null hypothesis probability.

$$\begin{aligned} T_\beta(d) = \quad & \tfrac{1}{n^2} \quad \sum_{j=1}^{n}\sum_{k=1}^{n}\exp(-\frac{\beta^2}{2}|Y_j - Y_k|^2) \\ - \quad & 2(1+\beta^2)^{-d/2}\frac{1}{n}\sum_{j=1}^{n}\exp(-\frac{\beta^2}{2(1+\beta^2)}|Y_j|^2) + (1+2\beta^2)^{-d/2} \end{aligned}$$

where

$$|Y_j - Y_k|^2 = (X_j - X_k)'S^{-1}(X_j - X_k)$$

$$|Y_j|^2 = (X_j - \bar{X})'S^{-1}(X_j - \bar{X})$$

Monte Carlo simulations suggest that $T_\beta(d)$ has good power against distributions with heavy tails.

The Shapiro-Wilk W test is computed only when the number of observations (*n*) is less than 2000.

The following is an example of the output produced by the NORMAL option.

```
                         The MODEL Procedure

                            Normality Test
Equation         Test Statistic       Value      Prob

y1               Shapiro-Wilk W        0.37    <.0001
y2               Shapiro-Wilk W        0.84    <.0001
System           Mardia Skewness      286.4    <.0001
                 Mardia Kurtosis      31.28    <.0001
                 Henze-Zirkler T       7.09    <.0001
```

Figure 14.31. Normality Test Output

Heteroscedasticity

One of the key assumptions of regression is that the variance of the errors is constant across observations. If the errors have constant variance, the errors are called *homoscedastic*. Typically, residuals are plotted to assess this assumption. Standard estimation methods are inefficient when the errors are *heteroscedastic* or have nonconstant variance.

Heteroscedasticity Tests

The MODEL procedure now provides two tests for heteroscedasticity of the errors: White's test and the modified Breusch-Pagan test.

Both White's test and the Breusch-Pagan are based on the residuals of the fitted model. For systems of equations, these tests are computed separately for the residuals of each equation.

The residuals of an estimation are used to investigate the heteroscedasticity of the true disturbances.

The WHITE option tests the null hypothesis

$$H_0 : \sigma_i^2 = \sigma^2 \quad \text{for all i}$$

White's test is general because it makes no assumptions about the form of the heteroscedasticity (White 1980). Because of its generality, White's test may identify specification errors other than heteroscedasticity (Thursby 1982). Thus White's test may be significant when the errors are homoscedastic but the model is misspecified in other ways.

White's test is equivalent to obtaining the error sum of squares for the regression of the squared residuals on a constant and all the unique variables in $\mathbf{J}\otimes\mathbf{J}$, where the matrix **J** is composed of the partial derivatives of the equation residual with respect to the estimated parameters.

Note that White's test in the MODEL procedure is different than White's test in the REG procedure requested by the SPEC option. The SPEC option produces the test from Theorem 2 on page 823 of White (1980). The WHITE option, on the other hand, produces the statistic from Corollary 1 on page 825 of White (1980).

The modified Breusch-Pagan test assumes that the error variance varies with a set of regressors, which are listed in the BREUSCH= option.

Define the matrix Z to be composed of the values of the variables listed in the BREUSCH= option, such that $z_{i,j}$ is the value of the *j*th variable in the BREUSCH= option for the *i*th observation. The null hypothesis of the Breusch-Pagan test is

$$H_0 : \sigma_i^2 = \sigma^2(\alpha_0 + \alpha^{'}\mathbf{z_i})$$

where σ_i^2 is the error variance for the *i*th observation, and α_0 and α are regression coefficients.

The test statistic for the Breusch-Pagan test is

$$bp = \frac{1}{v}(\mathbf{u} - \bar{u}\mathbf{i})^{'} Z(Z^{'}Z)^{-1}Z^{'}(\mathbf{u} - \bar{u}\mathbf{i})$$

where $\mathbf{u} = (e_1^2, e_2^2, \ldots, e_n^2)$, $\mathbf{i}$ is a $n \times 1$ vector of ones, and

$$v = \frac{1}{n}\sum_{i=1}^{n}(e_i^2 - \frac{\mathbf{e}^{'}\mathbf{e}}{n})^2$$

This is a modified version of the Breusch-Pagan test, which is less sensitive to the assumption of normality than the original test (Greene 1993, p. 395).

The statements in the following example produce the output in Figure 14.32:

```
proc model data=schools;
   parms const inc inc2;

   exp = const + inc * income + inc2 * income * income;
   incsq = income * income;

   fit exp / white breusch=(1 income incsq);
run;
```

```
                              The MODEL Procedure

                            Heteroscedasticity Test
Equation      Test              Statistic   DF  Pr > ChiSq  Variables

exp           White's Test          21.16    4      0.0003  Cross of all vars
              Breusch-Pagan         15.83    2      0.0004  1, income, incsq
```

Figure 14.32. Output for Heteroscedasticity Tests

Correcting for Heteroscedasticity

There are two methods for improving the efficiency of the parameter estimation in the presence of heteroscedastic errors. If the error variance relationships are known, weighted regression can be used or an error model can be estimated. For details on error model estimation see section "Error Covariance Stucture Specification". If the error variance relationship is unknown, GMM estimation can be used.

Weighted Regression

The WEIGHT statement can be used to correct for the heteroscedasticity. Consider the following model, which has a heteroscedastic error term:

$$y_t = 250(e^{-0.2t} - e^{-0.8t}) + \sqrt{(9/t)}\epsilon_t$$

The data for this model is generated with the following SAS statements.

```
data test;
   do t=1 to 25;
      y = 250 * (exp( -0.2 * t ) - exp( -0.8 * t )) +
          sqrt( 9 / t ) * rannor(1);
      output;
   end;
run;
```

If this model is estimated with OLS,

```
proc model data=test;
   parms b1 0.1 b2 0.9;
   y = 250 * ( exp( -b1 * t ) - exp( -b2 * t ) );
   fit y;
run;
```

the estimates shown in Figure 14.33 are obtained for the parameters.

```
                              The MODEL Procedure

                       Nonlinear OLS Parameter Estimates

                                     Approx                       Approx
          Parameter       Estimate      Std Err     t Value     Pr > |t|

          b1              0.200977      0.00101      198.60       <.0001
          b2              0.826236      0.00853       96.82       <.0001
```

Figure 14.33. Unweighted OLS Estimates

If both sides of the model equation are multiplied by $\sqrt{t}$, the model will have a homoscedastic error term. This multiplication or weighting is done through the WEIGHT statement. The WEIGHT statement variable operates on the squared residuals as

$$\epsilon_t^{'}\epsilon_t = weight \times \mathbf{q}_t^{'}\mathbf{q}_t$$

so that the WEIGHT statement variable represents the square of the model multiplier. The following PROC MODEL statements corrects the heteroscedasticity with a WEIGHT statement

```
proc model data=test;
   parms b1 0.1 b2 0.9;
   y = 250 * ( exp( -b1 * t ) - exp( -b2 * t ) );
   fit y;
   weight t;
run;
```

Note that the WEIGHT statement follows the FIT statement. The weighted estimates are shown in Figure 14.34.

```
                              The MODEL Procedure

                       Nonlinear OLS Parameter Estimates

                                          Approx                       Approx
          Parameter       Estimate       Std Err     t Value     Pr > |t|

          b1              0.200503      0.000844      237.53       <.0001
          b2              0.816701        0.0139       58.71       <.0001
```

Figure 14.34. Weighted OLS Estimates

The weighted OLS estimates are identical to the output produced by the following PROC MODEL example:

```
   proc model data=test;
      parms b1 0.1 b2 0.9;
      y = 250 * ( exp( -b1 * t ) - exp( -b2 * t ) );
      _weight_ = t;
      fit y;
   run;
```

If the WEIGHT statement is used in conjunction with the _WEIGHT_ variable, the two values are multiplied together to obtain the weight used.

The WEIGHT statement and the _WEIGHT_ variable operate on all the residuals in a system of equations. If a subset of the equations needs to be weighted, the residuals for each equation can be modified through the RESID. variable for each equation. The following example demonstrates the use of the RESID. variable to make a homoscedastic error term:

```
   proc model data=test;
      parms b1 0.1 b2 0.9;
      y = 250 * ( exp( -b1 * t ) - exp( -b2 * t ) );
      resid.y = resid.y * sqrt(t);
      fit y;
   run;
```

These statements produce estimates of the parameters and standard errors that are identical to the weighted OLS estimates. The reassignment of the RESID.Y variable must be done after Y is assigned, otherwise it would have no effect. Also, note that the residual (RESID.Y) is multiplied by $\sqrt{t}$. Here the multiplier is acting on the residual before it is squared.

GMM Estimation

If the form of the heteroscedasticity is unknown, generalized method of moments estimation (GMM) can be used. The following PROC MODEL statements use GMM to estimate the example model used in the preceding section:

```
   proc model data=test;
      parms b1 0.1 b2 0.9;
      y = 250 * ( exp( -b1 * t ) - exp( -b2 * t ) );
      fit y / gmm;
      instruments b1 b2;
   run;
```

GMM is an instrumental method, so instrument variables must be provided.

GMM estimation generates estimates for the parameters shown in Figure 14.35.

```
                              The MODEL Procedure

                       Nonlinear GMM Parameter Estimates

                                        Approx                       Approx
          Parameter       Estimate     Std Err     t Value     Pr > |t|

          b1              0.200487    0.000807      248.38       <.0001
          b2              0.822148      0.0142       57.95       <.0001
```

Figure 14.35. GMM Estimation for Heteroscedasticity

Transformation of Error Terms

In PROC MODEL you can control the form of the error term. By default the error term is assumed to be additive. This section demonstrates how to specify nonadditive error terms and discusses the effects of these transformations.

Models with Nonadditive Errors

The estimation methods used by PROC MODEL assume that the error terms of the equations are independently and identically distributed with zero means and finite variances. Furthermore, the methods assume that the RESID.*name* equation variable for normalized form equations or the EQ.*name* equation variable for general form equations contains an estimate of the error term of the true stochastic model whose parameters are being estimated. Details on RESID.*name* and EQ.*name* equation variables are in the section "Model Translations."

To illustrate these points, consider the common loglinear model

$$y = \alpha x^{\beta} \qquad (1)$$

$$\ln y = a + b\ln(x) \qquad (2)$$

where a=log(α) and b=β. Equation (2) is called the *log form* of the equation in contrast to equation (1), which is called the *level form* of the equation. Using the SYSLIN procedure, you can estimate equation (2) by specifying

```
   proc syslin data=in;
      model logy=logx;
   run;
```

where LOGY and LOGX are the logs of Y and X computed in a preceding DATA step. The resulting values for INTERCEPT and LOGX correspond to *a* and *b* in equation (2).

Using the MODEL procedure, you can try to estimate the parameters in the level form (and avoid the DATA step) by specifying

```
proc model data=in;
   parms alpha beta;
   y = alpha * x ** beta;
   fit y;
run;
```

where ALPHA and BETA are the parameters in equation (1).

Unfortunately, at least one of the preceding is wrong; an ambiguity results because equations (1) and (2) contain no explicit error term. The SYSLIN and MODEL procedures both deal with additive errors; the residual used (the estimate of the error term in the equation) is the difference between the predicted and actual values (of LOGY for PROC SYSLIN and of Y for PROC MODEL in this example). If you perform the regressions discussed previously, PROC SYSLIN estimates equation (3) while PROC MODEL estimates equation (4).

$$\ln y = a + b\ln(x) + \epsilon \qquad (3)$$

$$y = \alpha x^{\beta} + \xi \qquad (4)$$

These are different statistical models. Equation (3) is the log form of equation (5)

$$y = \alpha x^{\beta} \mu \qquad (5)$$

where μ=e$^{\epsilon}$. Equation (4), on the other hand, cannot be linearized because the error term ξ (different from μ) is additive in the level form.

You must decide whether your model is equation (4) or (5). If the model is equation (4), you should use PROC MODEL. If you linearize equation (1) without considering the error term and apply SYSLIN to MODEL LOGY=LOGX, the results will be wrong. On the other hand, if your model is equation (5) (in practice it usually is), and you want to use PROC MODEL to estimate the parameters in the *level* form, you must do something to account for the multiplicative error.

PROC MODEL estimates parameters by minimizing an objective function. The objective function is computed using either the RESID.-prefixed equation variable or the EQ.-prefixed equation variable. You must make sure that these prefixed equation variables are assigned an appropriate error term. If the model has additive errors that satisfy the assumptions, nothing needs to be done. In the case of equation (5), the error is nonadditive and the equation is in normalized form, so you must alter the value of RESID.Y.

The following assigns a valid estimate of μ to RESID.Y:

```
y = alpha * x ** beta;
resid.y = actual.y / pred.y;
```

However, μ=e$^{\epsilon}$ and, therefore, μ cannot have a mean of zero and you cannot consistently estimate α and β by minimizing the sum of squares of an estimate of μ. Instead, you use $\epsilon = \ln\mu$.

```
proc model data=in;
   parms alpha beta;
   y = alpha * x ** beta;
   resid.y = log( actual.y / pred.y );
   fit y;
run;
```

If the model was expressed in general form, this transformation becomes

```
proc model data=in;
   parms alpha beta;
   EQ.trans = log( y / (alpha * x ** beta));
   fit trans;
run;
```

Both examples produce estimates of α and β of the level form that match the estimates of *a* and *b* of the log form. That is, ALPHA=exp(INTERCEPT) and BETA=LOGX, where INTERCEPT and LOGX are the PROC SYSLIN parameter estimates from the MODEL LOGY=LOGX. The standard error reported for ALPHA is different from that for the INTERCEPT in the log form.

The preceding example is not intended to suggest that loglinear models should be estimated in level form but, rather, to make the following points:

- Nonlinear transformations of equations involve the error term of the equation, and this should be taken into account when transforming models.
- The RESID.-prefixed and the EQ.-prefixed equation variables for models estimated by the MODEL procedure must represent additive errors with zero means.
- You can use assignments to RESID.-prefixed and EQ.-prefixed equation variables to transform error terms.
- Some models do not have additive errors or zero means, and many such models can be estimated using the MODEL procedure. The preceding approach applies not only to multiplicative models but to any model that can be manipulated to isolate the error term.

Predicted Values of Transformed Models

Nonadditive or transformed errors affect the distribution of the predicted values, as well as the estimates. For the preceding loglinear example, the MODEL procedure produces consistent parameter estimates. However, the predicted values for Y computed by PROC MODEL are not unbiased estimates of the expected values of Y, although they do estimate the conditional median Y values.

In general, the predicted values produced for a model with nonadditive errors are not unbiased estimates of the conditional means of the endogenous value. If the model can be transformed to a model with additive errors by using a *monotonic* transformation, the predicted values estimate the conditional medians of the endogenous variable.

For transformed models in which the biasing factor is known, you can use programming statements to correct for the bias in the predicted values as estimates of the endogenous means. In the preceding loglinear case, the predicted values will be biased by the factor exp(σ^2/2). You can produce approximately unbiased predicted values in this case by writing the model as

```
proc model data=in;
   parms alpha beta;
   y=alpha * x ** beta;
   resid.y = log( actual.y / pred.y );

   fit y;
run;
```

Refer to Miller (1984) for a discussion of bias factors for predicted values of transformed models.

Note that models with transformed errors are not appropriate for Monte Carlo simulation using the SDATA= option. PROC MODEL computes the OUTS= matrix from the transformed RESID.-prefixed equation variables, while it uses the SDATA= matrix to generate multivariate normal errors, which are added to the predicted values. This method of computing errors is inconsistent when the equation variables have been transformed.

Error Covariance Structure Specification

One of the key assumptions of regression is that the variance of the errors is constant across observations. Correcting for heteroscedasticity improves the efficiency of the estimates.

Consider the following general form for models:

$$\begin{aligned}
\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) &= \varepsilon_{\mathbf{t}} \\
\varepsilon_{\mathbf{t}} &= H_t * \epsilon_{\mathbf{t}} \\
H_t &= \begin{bmatrix} \sqrt{h_{t,1}} & 0 & \ldots & 0 \\ 0 & \sqrt{h_{t,2}} & \ldots & 0 \\ & & \ddots & \\ 0 & 0 & \ldots & \sqrt{h_{t,g}} \end{bmatrix} \\
\mathbf{h_t} &= \mathbf{g}(\mathbf{y}_t, \mathbf{x}_t, \phi)
\end{aligned}$$

where $\epsilon_{\mathbf{t}} \sim N(0, \Sigma)$.

For models that are homoscedastic,

$$h_t = 1$$

If you had a model which was heteroscedastic with known form you can improve the efficiency of the estimates by performing a weighted regression. The weight variable, using this notation, would be $1/\sqrt{h_t}$.

If the errors for a model are heteroscedastic and the functional form of the variance is known, the model for the variance can now be estimated along with the regression function.

To specify a functional form for the variance, assign the function to an H.*var* variable where *var* is the equation variable. For example, if you wanted to estimate the scale parameter for the variance of a simple regression model

$$y = a * x + b$$

you can specify

```
   proc model data=s;
      y = a * x + b;
      h.y = sigma**2;
   fit y;
```

Consider the same model with the following functional form for the variance:

$$h_t = \sigma^2 * x^{2*\alpha}$$

This would be written as

```
   proc model data=s;
      y = a * x + b;
      h.y = sigma**2 * x**(2*alpha);
   fit y;
```

There are three ways to model the variance in the MODEL procedure; Feasable generalized least squares; Generalized method of moments; and Full information maximum likelihood.

Feasable GLS

A simple approach to estimating a variance function is to estimate the mean parameters θ using some auxilary method, such as OLS, and then use the residuals of that estimation to estimate the parameters ϕ of the variance function. This scheme is called *feasable GLS*. It is posible to use the residuals from an auxilary method for the purpose of estimating ϕ because in many cases the residuals consistently estimate the error terms.

This scheme can be done by hand by performing OLS estimation of $\mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)$, the mean function, then by regressing the residuals squared on h_t, and finally by re-estimating the the mean function using a weight of $1/\sqrt{h_t}$.

For all estimation methods except GMM and FIML, using the H.var syntax specifies that feasable GLS will be used in the estimation. For feasable GLS the mean function is estimated by the usual method. The variance function is then estimated using pseudolikelihood (PL) function of the generated residuals. The objective function for the PL estimation is

$$p_n(\sigma, \theta) = \sum_{i=1}^{n} \left(\frac{(y_i - f(x_i, \hat{\beta}))^2}{\sigma^2 h(z_i, \theta)} + \log[\sigma^2 h(z_i, \theta)] \right)$$

Once the variance function has been estimated the mean function is re-estimated using the variance function as weights. If an S-iterated method is selected, this process is repeated until convergence (iterated feasable GLS).

Note, feasable GLS will not yield consistent estimates when one of the following is true:

- The variance is unbounded.
- There is too much serial dependence in the errors (the dependence does not fade with time).
- A combination of serial dependence and lag dependent variables.

The first two cases are unusual but the third is much more common. Whether iterated feasable GLS avoids consistency problems with the last case is an unanswered research question. For more information see (Davidson and MacKinnon 1993) pages 298-301 or (Gallant 1987) pages 124-125 and (Amemiya 1985) pages 202-203.

One limitation is that parameters can not be shared between the mean equation and the variance equation. This implies that certian GARCH models, cross equation restrictions of parameters, or testing of combinations of parameters in the mean and variance component are not allowed.

Generalized Method of Moments

In GMM, normally the first moment of the mean function is used in the objective function.

$$\begin{aligned} \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta) &= \epsilon_t \\ \mathbf{E}(\epsilon_t) &= 0 \end{aligned}$$

To add the second moment conditions to the estimation, add the equation

$$\mathbf{E}(\varepsilon_t * \varepsilon_t - h_t) = 0$$

to the model. For example if you wanted to estimate σ for linear example above, you can write

```
proc model data=s;
   y = a * x + b;
   eq.two = resid.y**2 - sigma**2;
fit y two/ gmm;
instruments x;
run;
```

This is a popular way to estimate a continuous-time interest rate processes (see (Chan, et al 1992)). The H.var syntax will automatically generate this system of equations.

To further take advantage of the information obtained about the variance, the moment equations can be modified to

$$\begin{aligned} \mathbf{E}(\varepsilon_t/\sqrt{h_t}) &= 0 \\ \mathbf{E}(\varepsilon_t * \varepsilon_t - h_t) &= 0 \end{aligned}$$

For the above example, this can be written as

```
proc model data=s;
   y = a * x + b;
   eq.two = resid.y**2 - sigma**2;
   resid.y = resid.y / sigma;
fit y two/ gmm;
instruments x;
run;
```

Note that, if the error model is misspecified in this form of the GMM model, the parameter estimates may be inconsistent.

Full Information Maximum Likelihood

For FIML estimation of variance functions, the concentrated likelihood below is used as the objective function. That is, the mean function will be coupled with the variance function and the system will be solved simultaneously.

$$\begin{aligned} l_n(\phi) &= \frac{ng}{2}(1+\ln(2\pi)) - \sum_{t=1}^{n} \ln\left(\left|\frac{\partial \mathbf{q}(\mathbf{y}_t, \mathbf{x}_t, \theta)}{\partial \mathbf{y}_t}\right|\right) \\ &\quad + \frac{1}{2}\sum_{t=1}^{n}\sum_{i=1}^{g}\left(\ln(h_{t,i}) + \mathbf{q}_i(\mathbf{y}_t, \mathbf{x}_t, \theta)^2/h_{t,i}\right) \end{aligned}$$

where g is the number of equations in the system.

The HESSIAN=GLS option is not available for FIML estimation involving variance functions. The matrix used when HESSIAN=CROSS is specified is a cross products matrix which has been enhanced by the dual quasi-newton approximation.

Examples

You can specify a GARCH(1,1) model as follows:

```
proc model data=modloc.usd_jpy;

             /* Mean model --------*/
   jpyret = intercept ;

             /* Variance model ----------------*/
   h.jpyret = arch0 + arch1 * zlag( resid.jpyret * resid.jpyret )
              + garch1 * zlag(h.jpyret) ;
```

```
      bounds arch0 arch1 garch1 >= 0;

   fit jpyret/method=marquardt fiml;
   run;
```

Note that the BOUNDS statement was used to ensure that the parameters were positive, a requirement for GARCH models.

EGARCH models are used because there is no restrictions on the parameters. You can specify a EGARCH(1,1) model as follows:

```
   proc model data=sasuser.usd_dem ;

             /* Mean model ----------*/
     demret = intercept ;

                /* Variance model ----------------*/
     if ( _OBS_ =1 )  then
       h.demret = exp( earch0/ (1. - egarch1)  );
     else
       h.demret = exp( earch0 + earch1 * zlag( g)
                             + egarch1 * log(zlag(h.demret)));
     g = theta * nresid.demret + abs( nresid.demret ) - sqrt(2/3.1415);

                       /* Fit and save the model */
   fit demret/method=marquardt fiml  maxiter=100
   run;
```

Ordinary Differential Equations

Ordinary differential equations (ODEs) are also called *initial value problems* because a time zero value for each first-order differential equation is needed. The following is an example of a first-order system of ODEs:

$$\begin{aligned} y' &= -0.1y + 2.5z^2 \\ z' &= -z \\ y_0 &= 0 \\ z_0 &= 1 \end{aligned}$$

Note that you must provide an initial value for each ODE.

As a reminder, any *n*-order differential equation can be modeled as a system of first-order differential equations. For example, consider the differential equation

$$\begin{aligned} y^{''} &= by^{'} + cy \\ y_0 &= 0 \\ y_0^{'} &= 1 \end{aligned}$$

which can be written as the system of differential equations

$$y^{'} = z$$

$$\begin{aligned} z' &= by' + cy \\ y_0 &= 0 \\ z_0 &= 1 \end{aligned}$$

This differential system can be simulated as follows:

```
data t;
   time=0; output;
   time=1; output;
   time=2; output;
run;

proc model data=t ;
   dependent y 0 z 1;
   parm b -2 c -4;
      /* Solve  y''=b y' + c y --------------*/

   dert.y = z;
   dert.z = b * dert.y + c * y;

   solve y z / dynamic solveprint;
run;
```

The preceding statements produce the following output. These statements produce additional output, which is not shown.

```
                              The MODEL Procedure
                            Simultaneous Simulation

            Observation    1    Missing       2    CC    -1.000000
                                Iterations    0


                                Solution Values

                                   y              z

                            0.000000       1.000000


Observation    2    Iterations    0    CC     0.000000    ERROR.y     0.000000


                                Solution Values

                                   y              z

                           0.2096398      -.2687053


Observation    3    Iterations    0    CC     9.464802    ERROR.y    -0.234405


                                Solution Values

                                   y              z

                           -.0247649      -.1035929
```

The differential variables are distinguished by the derivative with respect to time (DERT.) prefix. Once you define the DERT. variable, you can use it on the right-hand side of another equation. The differential equations must be expressed in normal form; implicit differential equations are not allowed, and other terms on the left-hand side are not allowed.

The TIME variable is the *implied with respect to* variable for all DERT. variables. The TIME variable is also the only variable that must be in the input data set.

You can provide initial values for the differential equations in the data set, in the declaration statement (as in the previous example), or in statements in the code. Using the previous example, you can specify the initial values as

```
proc model data=t ;
   dependent y z ;
   parm b -2 c -4;
      /* Solve  y’’=b y’ + c y --------------*/
   if ( time=0 ) then
      do;
         y=0;
         z=1;
      end;
   else
      do;
         dert.y = z;
         dert.z = b * dert.y + c * y;
      end;
   end;
   solve y z / dynamic solveprint;
run;
```

If you do not provide an initial value, 0 is used.

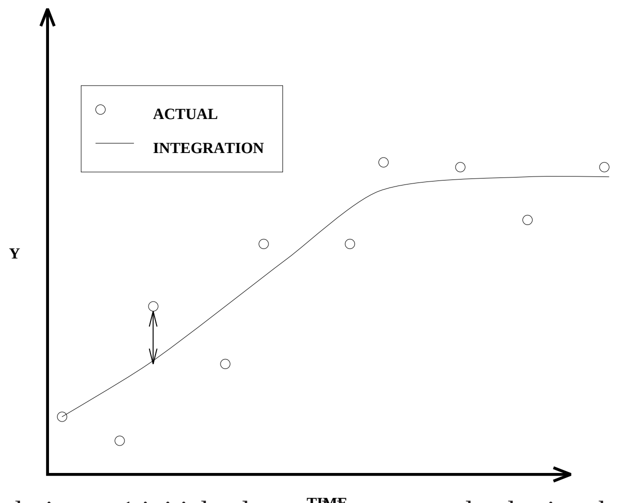
DYNAMIC and STATIC Simulation

Note that, in the previous example, the DYNAMIC option was specified in the SOLVE statement. The DYNAMIC and STATIC options work the same for differential equations as they do for dynamic systems. In the differential equation case, the DYNAMIC option makes the initial value needed at each observation the computed value from the previous iteration. For a static simulation, the data set must contain values for the integrated variables. For example, if DERT.Y and DERT.Z are the differential variables, you must include Y and Z in the input data set in order to do a static simulation of the model.

If the simulation is dynamic, the initial values for the differential equations are obtained from the data set, if they are available. If the variable is not in the data set, you can specify the initial value in a declaration statement. If you do not specify an initial value, the value of 0.0 is used.

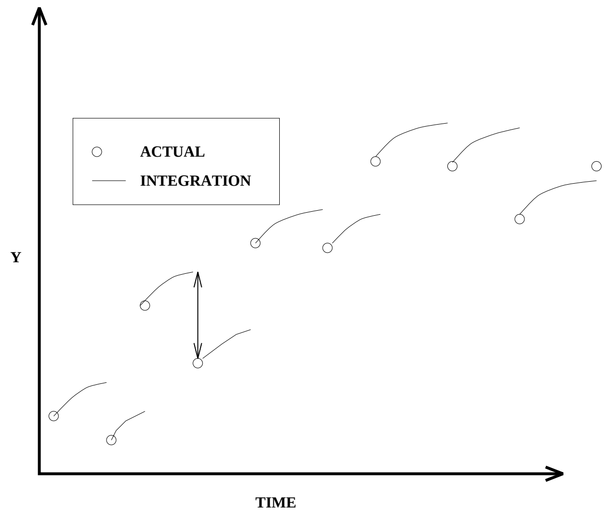
A dynamic solution is obtained by solving one initial value problem for all the data. A graph of a simple dynamic simulation is shown in Figure 14.36. If the time variable for the current observation is less than the time variable for the previous observation, the integration is restarted from this point. This allows for multiple samples in one data file.

Figure 14.36. Dynamic Solution



In a static solution, n-1 initial value problems are solved using the first n-1 data values as initial values. The equations are integrated using the ith data value as an initial value to the i+1 data value. Figure 14.37 displays a static simulation of noisy data from a simple differential equation. The static solution does not propagate errors in initial values as the dynamic solution does.

Figure 14.37. Static Solution



For estimation, the DYNAMIC and STATIC options in the FIT statement perform the same functions as they do in the SOLVE statement. Components of differential systems that have missing values or are not in the data set are simulated dynamically. For example, often in multiple compartment kinetic models, only one compartment is monitored. The differential equations describing the unmonitored compartments are simulated dynamically.

For estimation, it is important to have accurate initial values for ODEs that are not in the data set. If an accurate initial value is not known, the initial value can be made an unknown parameter and estimated. This allows for errors in the initial values but increases the number of parameters to estimate by the number of equations.

Estimation of Differential Equations

Consider the kinetic model for the accumulation of mercury (Hg) in mosquito fish (Matis, Miller, and Allen 1991, p. 177). The model for this process is the one-compartment constant infusion model shown in Figure 14.38.

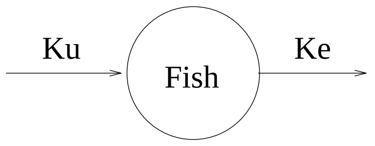


Figure 14.38. One-Compartment Constant Infusion Model

The differential equation that models this process is

$$\begin{aligned} \frac{dconc}{dt} &= k_u - k_e conc \\ conc_0 &= 0 \end{aligned}$$

The analytical solution to the model is

$$conc = (k_u/k_e)(1 - \exp(-\mathrm{k_e t}))$$

The data for the model are

```
data fish;
   input day conc;
   datalines;
0.0    0.0
1.0    0.15
2.0    0.2
3.0    0.26
4.0    0.32
6.0    0.33
;
run;
```

To fit this model in differential form, use the following statements:

```
proc model data=fish;
   parm ku ke;

   dert.conc = ku - ke * conc;

   fit conc / time=day;
run;
```

The results from this estimation are shown in Figure 14.39.

```
                          The MODEL Procedure

                  Nonlinear OLS Parameter Estimates

                                    Approx                     Approx
     Parameter       Estimate      Std Err    t Value     Pr > |t|

     ku              0.180159       0.0312       5.78       0.0044
     ke              0.524661       0.1181       4.44       0.0113
```

Figure 14.39. Static Estimation Results for Fish Model

To perform a dynamic estimation of the differential equation, add the DYNAMIC option to the FIT statement.

```
proc model data=fish;
   parm ku .3 ke .3;

   dert.conc = ku - ke * conc;

   fit conc / time = day dynamic;
run;
```

The equation DERT.CONC is integrated from $conc(0) = 0$. The results from this estimation are shown in Figure 14.40.

```
                          The MODEL Procedure

                  Nonlinear OLS Parameter Estimates

                                    Approx                     Approx
     Parameter       Estimate      Std Err    t Value     Pr > |t|

     ku              0.167109       0.0170       9.84       0.0006
     ke              0.469033       0.0731       6.42       0.0030
```

Figure 14.40. Dynamic Estimation Results for Fish Model

To perform a dynamic estimation of the differential equation and estimate the initial value, use the following statements:

```
proc model data=fish;
   parm ku .3 ke .3 conc0 0;
```

```
      dert.conc = ku - ke * conc;

      fit conc initial=(conc = conc0) / time = day dynamic;
   run;
```

The INITIAL= option in the FIT statement is used to associate the initial value of a differential equation with a parameter. The results from this estimation are shown in Figure 14.41.

```
                              The MODEL Procedure

                       Nonlinear OLS Parameter Estimates

                                    Approx                    Approx
   Parameter       Estimate       Std Err    t Value     Pr > |t|

   ku              0.164408        0.0230       7.14       0.0057
   ke               0.45949        0.0943       4.87       0.0165
   conc0           0.003798        0.0174       0.22       0.8414
```

Figure 14.41. Dynamic Estimation with Initial Value for Fish Model

Finally, to estimate the fish model using the analytical solution, use the following statements:

```
   proc model data=fish;
      parm ku .3  ke .3;

      conc = (ku/ ke)*( 1 -exp(-ke * day));

      fit conc;
   run;
```

The results from this estimation are shown in Figure 14.42.

```
                              The MODEL Procedure

                       Nonlinear OLS Parameter Estimates

                                    Approx                    Approx
   Parameter       Estimate       Std Err    t Value     Pr > |t|

   ku              0.167109        0.0170       9.84       0.0006
   ke              0.469033        0.0731       6.42       0.0030
```

Figure 14.42. Analytical Estimation Results for Fish Model

A comparison of the results among the four estimations reveals that the two dynamic estimations and the analytical estimation give nearly identical results (identical to the default precision). The two dynamic estimations are identical because the estimated initial value (0.00013071) is very close to the initial value used in the first dynamic estimation (0). Note also that the static model did not require an initial guess for the parameter values. Static estimation, in general, is more forgiving of bad initial values.

The form of the estimation that is preferred depends mostly on the model and data.

If a very accurate initial value is known, then a dynamic estimation makes sense. If, additionally, the model can be written analytically, then the analytical estimation is computationally simpler. If only an approximate initial value is known and not modeled as an unknown parameter, the static estimation is less sensitive to errors in the initial value.

The form of the error in the model is also an important factor in choosing the form of the estimation. If the error term is additive and independent of previous error, then the dynamic mode is appropriate. If, on the other hand, the errors are cumulative, a static estimation is more appropriate. See the section "Monte Carlo Simulation" for an example.

Auxiliary Equations

Auxiliary equations can be used with differential equations. These are equations that need to be satisfied with the differential equations at each point between each data value. They are automatically added to the system, so you do not need to specify them in the SOLVE or FIT statement.

Consider the following example.

The Michaelis-Menten Equations describe the kinetics of an enzyme-catalyzed reaction. The enzyme is E, and S is called the *substrate*. The enzyme first reacts with the substrate to form the enzyme-substrate complex ES, which then breaks down in a second step to form enzyme and products P.

The reaction rates are described by the following system of differential equations:

$$\begin{aligned}\frac{d[ES]}{dt} &= k_1([E]-[ES])[S] - k_2[ES] - k_3[ES] \\ \frac{d[S]}{dt} &= -k_1([E]-[ES])[S] + k_2[ES] \\ [E] &= [E]_{tot} - [ES]\end{aligned}$$

The first equation describes the rate of formation of ES from E + S. The rate of formation of ES from E + P is very small and can be ignored. The enzyme is in either the complexed or the uncomplexed form. So if the total ($[E]_{tot}$) concentration of enzyme and the amount bound to the substrate is known, $[E]$ can be obtained by conservation.

In this example, the conservation equation is an auxiliary equation and is coupled with the differential equations for integration.

Time Variable

You must provide a time variable in the data set. The name of the time variable defaults to TIME. You can use other variables as the time variable by specifying the TIME= option in the FIT or SOLVE statement. The time intervals need not be evenly spaced. If the time variable for the current observation is less than the time variable for the previous observation, the integration is restarted.

Differential Equations and Goal Seeking

Consider the following differential equation

$$y^{'} = a{*}x$$

and the data set

```
data t2;
   y=0; time=0; output;
   y=2; time=1; output;
   y=3; time=2; output;
run;
```

The problem is to find values for X that satisfy the differential equation and the data in the data set. Problems of this kind are sometimes referred to as *goal seeking problems* because they require you to search for values of X that will satisfy the goal of Y.

This problem is solved with the following statements:

```
proc model data=t2 ;
   dependent x 0;
   independent y;
   parm a 5;
   dert.y = a * x;
   solve x / out=foo;
run;

proc print data=foo; run;
```

The output from the PROC PRINT statement is shown in Figure 14.43.

```
Obs    _TYPE_      _MODE_     _ERRORS_        x          y       time

 1     PREDICT    SIMULATE        0        0.00000    0.00000      0
 2     PREDICT    SIMULATE        0        0.80000    2.00000      1
 3     PREDICT    SIMULATE        0       -0.40000    3.00000      2
```

Figure 14.43. Dynamic Solution

Note that an initial value of 0 is provided for the X variable because it is undetermined at TIME = 0.

In the preceding goal seeking example, X is treated as a linear function between each set of data points (see Figure 14.44).

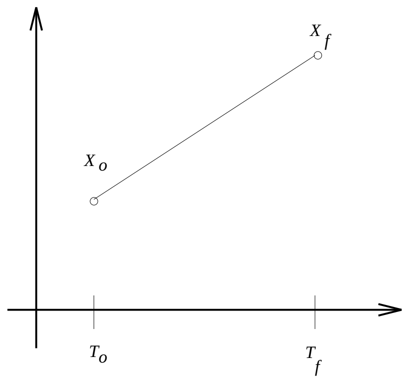


Figure 14.44. Form of X Used for Integration in Goal Seeking

If you integrate $y^{'} = ax$ manually, you have

$$\begin{aligned} x(t) &= \frac{t_f - t}{t_f - t_0} x_0 + \frac{-T_0}{t_f - t_0} x_f \\ y &= \int_{t_o}^{t_f} ax(t)dt \\ &= a\frac{1}{t_f - t_0}(t(t_f x_0 - t_0 x_f) + \frac{1}{2}t^2(x_f - x_0))|_{t_0}^{t_f} \end{aligned}$$

For observation 2, this reduces to

$$\begin{aligned} y &= \frac{1}{2}a{*}x_f \\ 2 &= 2.5{*}x_f \end{aligned}$$

So $x = 0.8$ for this observation.

Goal seeking for the TIME variable is not allowed.

Restrictions and Bounds on Parameters

Using the BOUNDS and RESTRICT statements, PROC MODEL can compute optimal estimates subject to equality or inequality constraints on the parameter estimates.

Equality restrictions can be written as a vector function

$$\mathbf{h}(\theta) = 0$$

Inequality restrictions are either active or inactive. When an inequality restriction is active, it is treated as an equality restriction. All inactive inequality restrictions can be written as a vector function

$$F(\theta) \geq 0$$

Strict inequalities, such as $(f(\theta) > 0)$, are transformed into inequalities as $f(\theta) \times (1 - \epsilon) - \epsilon \geq 0$, where the tolerance ϵ is controlled by the EPSILON= option on the FIT statement and defaults to 10^{-8}. The *i*th inequality restriction becomes active if $F_i < 0$ and remains active until its lagrange multiplier becomes negative. Lagrange multipliers are computed for all the nonredundant equality restrictions and all the active inequality restrictions.

For the following, assume the vector $\mathbf{h}(\theta)$ contains all the current active restrictions. The constraint matrix A is

$$A(\hat{\theta}) = \frac{\partial \mathbf{h}(\hat{\theta})}{\partial \hat{\theta}}$$

The covariance matrix for the restricted parameter estimates is computed as

$$Z(Z'HZ)^{-1}Z'$$

where H is Hessian or approximation to the Hessian of the objective function $((X'(\mathrm{diag(S)})^{-1} \otimes \mathrm{I})\mathrm{X})$ for OLS), and Z is the last $(np - nc)$ columns of Q. Q is from an LQ factorization of the constraint matrix, *nc* is the number of active constraints, and *np* is the number of parameters. Refer to Gill, Murray, and Wright (1981) for more details on LQ factorization. The covariance column in Table 14.1 summarizes the Hessian approximation used for each estimation method.

The covariance matrix for the Lagrange multipliers is computed as

$$(AH^{-1}A')^{-1}$$

The *p*-value reported for a restriction is computed from a beta distribution rather than a *t*-distribution because the numerator and the denominator of the *t*-ratio for an estimated Lagrange multiplier are not independent.

The Lagrange multipliers for the active restrictions are printed with the parameter estimates. The Lagrange multiplier estimates are computed using the relationship

$$A^{'}\lambda = \mathrm{g}$$

where the dimension of the constraint matrix *A* is the number of constraints by the number of parameters, λ is the vector of Lagrange multipliers, and *g* is the gradient of the objective function at the final estimates.

The final gradient includes the effects of the estimated S matrix. For example, for OLS the final gradient would be:

$$\mathrm{g} = X'(\mathrm{diag(S)}^{-1}\otimes\mathrm{I})\mathrm{r}$$

where *r* is the residual vector. Note that when nonlinear restrictions are imposed, the convergence measure R may have values greater than one for some iterations.

Tests on Parameters

In general, the hypothesis tested can be written as

$$H_0 : \mathbf{h}(\theta) = 0$$

where $\mathbf{h}(\theta)$ is a vector valued function of the parameters θ given by the *r* expressions specified on the TEST statement.

Let $\hat{V}$ be the estimate of the covariance matrix of $\hat{\theta}$. Let $\hat{\theta}$ be the unconstrained estimate of θ and $\tilde{\theta}$ be the constrained estimate of θ such that $h(\tilde{\theta}) = 0$. Let

$$A(\theta) = \partial h(\theta)/\partial\theta \mid_{\hat{\theta}}$$

Let *r* be the dimension of $h(\theta)$ and *n* be the number of observations. Using this notation, the test statistics for the three kinds of tests are computed as follows.

The Wald test statistic is defined as

$$W = h^{'}(\hat{\theta})\left(A(\hat{\theta})\hat{V}A^{'}(\hat{\theta})\right)^{-1}h(\hat{\theta})$$

The Wald test is not invariant to reparameterization of the model (Gregory 1985, Gallant 1987, p. 219). For more information on the theoretical properties of the Wald test see Phillips and Park 1988.

The Lagrange multiplier test statistic is

$$R = \lambda^{'}A(\tilde{\theta})\tilde{V}^{-1}A^{'}(\tilde{\theta})\lambda$$

where λ is the vector of Lagrange multipliers from the computation of the restricted estimate $\tilde{\theta}$.

The Lagrange multiplier test statistic is equivalent to Rao's efficient score test statistic:

$$R = (\partial L(\tilde{\theta})/\partial\theta)' \tilde{V}^{-1} (\partial L(\tilde{\theta})/\partial\theta)$$

where L is the log likelihood function for the estimation method used. For OLS and SUR the Lagrange multiplier test statistic is computed as:

$$R = [(\partial \hat{S}(\tilde{\theta})/\partial\theta)' \tilde{V}^{-1} (\partial \hat{S}(\tilde{\theta})/\partial\theta)]/\hat{S}(\tilde{\theta})$$

where $\hat{S}(\tilde{\theta})$ is the corresponding objective function value at the constrained estimate.

The likelihood ratio test statistic is

$$T = 2\left(L(\tilde{\theta}) - L(\hat{\theta})\right)$$

where $\tilde{\theta}$ represents the constrained estimate of θ and L is the concentrated log likelihood value.

For OLS and SUR, the likelihood ratio test statistic is computed as:

$$T = 0.5 \times df \times (n - nparms) \times (\hat{S}(\tilde{\theta}) - \hat{S}(\hat{\theta}))/\hat{S}(\hat{\theta})$$

where df is the difference in degrees of freedom for the full and restricted models, and $nparms$ is the number of parameters in the full system.

The Likelihood ratio test is not appropriate for models with nonstationary serially correlated errors (Gallant 1987, p. 139). The likelihood ratio test should not be used for dynamic systems, for systems with lagged dependent variables, or with the FIML estimation method unless certain conditions are met (see Gallant 1987, p. 479).

For each kind of test, under the null hypothesis the test statistic is asymptotically distributed as a χ^2 random variable with *r* degrees of freedom, where *r* is the number of expressions on the TEST statement. The *p*-values reported for the tests are computed from the $\chi^2(r)$ distribution and are only asymptotically valid.

Monte Carlo simulations suggest that the asymptotic distribution of the Wald test is a poorer approximation to its small sample distribution than the other two tests. However, the Wald test has the least computational cost, since it does not require computation of the constrained estimate $\tilde{\theta}$.

The following is an example of using the TEST statement to perform a likelihood ratio test for a compound hypothesis.

```
   test a*exp(-k) = 1-k, d = 0 ,/ lr;
```

It is important to keep in mind that although individual *t* tests for each parameter are printed by default into the parameter estimates table, they are only asymptotically valid for nonlinear models. You should be cautious in drawing any inferences from these *t* tests for small samples.

Hausman Specification Test

Hausman's specification test, or *m*-statistic, can be used to test hypotheses in terms of bias or inconsistency of an estimator. This test was also proposed by Wu (1973). Hausman's *m*-statistic is as follows.

Given two estimators, $\hat{\beta}_0$ and $\hat{\beta}_1$, where under the null hypothesis both estimators are consistent but only $\hat{\beta}_0$ is asymptotically efficient and under the alternative hypothesis only $\hat{\beta}_1$ is consistent, the *m*-statistic is

$$m = \hat{q}'(\hat{V}_1 - \hat{V}_0)^{-}\hat{q}$$

where $\hat{V}_1$ and $\hat{V}_0$ represent consistent estimates of the asymptotic covariance matrices of $\hat{\beta}_1$ and $\hat{\beta}_0$, and

$$q = \hat{\beta}_1 - \hat{\beta}_0$$

The *m*-statistic is then distributed χ^2 with k degrees of freedom, where k is the rank of the matrix $(\hat{V}_1 - \hat{V}_0)$. A generalized inverse is used, as recommended by Hausman (1982).

In the MODEL procedure, Hausman's *m*-statistic can be used to determine if it is necessary to use an instrumental variables method rather than a more efficient OLS estimation. Hausman's *m*-statistic can also be used to compare 2SLS with 3SLS for a class of estimators for which 3SLS is asymptotically efficient (similarly for OLS and SUR).

Hausman's *m*-statistic can also be used, in principle, to test the null hypothesis of normality when comparing 3SLS to FIML. Because of the poor performance of this form of the test, it is not offered in the MODEL procedure. Refer to R.C. Fair (1984, pp. 246-247) for a discussion of why Hausman's test fails for common econometric models.

To perform a Hausman's specification test, specify the HAUSMAN option in the FIT statement. The selected estimation methods are compared using Hausman's *m*-statistic.

In the following example, OLS, SUR, 2SLS, 3SLS, and FIML are used to estimate a model, and Hausman's test is requested.

```
proc model data=one out=fiml2;
   endogenous y1 y2;

   y1 = py2 * y2 + px1 * x1 + interc;
```

```
      y2 = py1* y1 + pz1 * z1 + d2;

      fit y1 y2 / ols sur 2sls 3sls fiml hausman;
      instruments x1 z1;
   run;
```

The output specified by the HAUSMAN option produces the following results.

```
                              The MODEL Procedure

                     Hausman's Specification Test Results
         Comparing    To            DF    Statistic    Pr > ChiSq

         OLS          SUR            6        32.47        <.0001
         OLS          2SLS           6        13.86        0.0313
         OLS          3SLS           6        -0.07          .
         2SLS         3SLS           6         0.00        1.0000
```

Figure 14.45. Hausman's Specification Test Results

Figure 14.45 indicates that 2SLS, a system estimation method, is preferred over OLS. The model needs an IV estimator but not a full error covariance matrix. Note that the FIML estimation results are not compared.

Chow Tests

The Chow test is used to test for break points or structural changes in a model. The problem is posed as a partitioning of the data into two parts of size n_1 and n_2. The null hypothesis to be tested is

$$H_o: \quad \beta_1 = \beta_2 = \beta$$

where β_1 is estimated using the first part of the data and β_2 is estimated using the second part.

The test is performed as follows (refer to Davidson and MacKinnon 1993, p. 380).

1. The p parameters of the model are estimated.
2. A second linear regression is performed on the residuals, $\hat{u}$, from the nonlinear estimation in step one.

   $$\hat{u} = \hat{X}b + \text{residuals}$$

   where $\hat{X}$ is Jacobian columns that are evaluated at the parameter estimates. If the estimation is an instrumental variables estimation with matrix of instruments W, then the following regression is performed:

   $$\hat{u} = P_{W*}\hat{X}b + \text{residuals}$$

   where P_{W*} is the projection matrix.

3. The restricted SSE (RSSE) from this regression is obtained. An SSE for each subsample is then obtained using the same linear regression.

4. The *F* statistic is then

$$f = \frac{(RSSE - SSE_1 - SSE_2)/p}{(SSE_1 + SSE_2)/(n-2p)}$$

   This test has p and $n - 2p$ degrees of freedom.

Chow's test is not applicable if $\min(n_1, n_2) < p$, since one of the two subsamples does not contain enough data to estimate β. In this instance, the *predictive Chow test* can be used. The predictive Chow test is defined as

$$f = \frac{(RSSE - SSE_1){\times}(n_1 - p)}{SSE_1{*}n_2}$$

where $n_1 > p$. This test can be derived from the Chow test by noting that the $SSE_2 = 0$ when $n_2 <= p$ and by adjusting the degrees of freedom appropriately.

You can select the Chow test and the predictive Chow test by specifying the CHOW=*arg* and the PCHOW=*arg* options in the FIT statement, where *arg* is either the number of observations in the first sample or a parenthesized list of first sample sizes. If the sizes for the second or the first group are less than the number of parameters, then a PCHOW test is automatically used. These tests statistics are not produced for GMM and FIML estimations.

The following is an example of the use of the Chow test.

```
data exp;
   x=0;
   do time=1 to 100;
      if time=50 then x=1;
      y = 35 * exp( 0.01 * time ) + rannor( 123 ) + x * 5;
      output;
   end;
run;

proc model data=exp;
   parm zo 35 b;
      dert.z = b * z;
      y=z;
   fit y init=(z=zo) / chow =(40 50 60) pchow=90;
run;
```

The data set introduced an artificial structural change into the model (the structural change effects the intercept parameter). The output from the requested Chow tests are shown in Figure 14.46.

```
                              The MODEL Procedure

                             Structural Change Test

                           Break
       Test                Point    Num DF    Den DF    F Value    Pr > F

       Chow                   40         2        96      12.95    <.0001
       Chow                   50         2        96     101.37    <.0001
       Chow                   60         2        96      26.43    <.0001
       Predictive Chow        90        11        87       1.86    0.0566
```

Figure 14.46. Chow's Test Results

Profile Likelihood Confidence Intervals

Wald-based and likelihood ratio-based confidence intervals are available in the MODEL procedure for computing a confidence interval on an estimated parameter. A confidence interval on a parameter θ can be constructed by inverting a Wald-based or a likelihood ratio-based test.

The approximate $100(1-\alpha)$ % Wald confidence interval for a parameter θ is

$$\hat{\theta} \pm z_{1-\alpha/2}\hat{\sigma}$$

where z_p is the $100p$th percentile of the standard normal distribution, $\hat{\theta}$ is the maximum likelihood estimate of θ, and $\hat{\sigma}$ is the standard error estimate of $\hat{\theta}$.

A likelihood ratio-based confidence interval is derived from the χ^2 distribution of the generalized likelihood ratio test. The approximate $1-\alpha$ confidence interval for a parameter θ is

$$\theta : 2[l(\hat{\theta}) - l(\theta)] \leq q_{1,1-\alpha} = 2l^*$$

where $q_{1,1-\alpha}$ is the $(1-\alpha)$ quantile of the χ^2 with one degree of freedom, and $l(\theta)$ is the log likelihood as a function of one parameter. The endpoints of a confidence interval are the zeros of the function $l(\theta) - l^*$. Computing a likelihood ratio-based confidence interval is an iterative process. This process must be performed twice for each parameter, so the computational cost is considerable. Using a modified form of the algorithm recommended by Venzon and Moolgavkar (1988), you can determine that the cost of each endpoint computation is approximately the cost of estimating the original system.

To request confidence intervals on estimated parameters, specify the following option in the FIT statement:

PRL= WALD | LR | BOTH

By default the PRL option produces 95% likelihood ratio confidence limits. The coverage of the confidence interval is controlled by the ALPHA= option in the FIT statement.

The following is an example of the use of the confidence interval options.

```
data exp;
   do time = 1 to 20;
      y = 35 * exp( 0.01 * time ) + 5*rannor( 123 );
   output;
   end;
run;

proc model data=exp;
   parm zo 35 b;
      dert.z = b * z;
      y=z;
   fit y init=(z=zo) / prl=both;
   test zo = 40.475437 ,/lr;
run;
```

The output from the requested confidence intervals and the TEST statement are shown in Figure 14.47

```
                              The MODEL Procedure

                       Nonlinear OLS Parameter Estimates

                                        Approx                     Approx
          Parameter       Estimate     Std Err    t Value     Pr > |t|

          zo              36.58933      1.9471      18.79       <.0001
          b               0.006497     0.00464       1.40       0.1780


                                 Test Results

Test            Type            Statistic    Pr > ChiSq    Label

Test0           L.R.                 3.81        0.0509    zo = 40.475437


                                Parameter Wald
                             95% Confidence Intervals
                Parameter           Value       Lower       Upper

                zo                36.5893     32.7730     40.4056
                b                 0.00650    -0.00259      0.0156


                          Parameter Likelihood Ratio
                             95% Confidence Intervals
                Parameter           Value       Lower       Upper

                zo                36.5893     32.8381     40.4921
                b                 0.00650    -0.00264      0.0157
```

Figure 14.47. Confidence Interval Estimation

Note that the likelihood ratio test reported the probability that $zo = 40.47543$ is 5% but $zo = 40.47543$ is the upper bound of a 95% confidence interval. To understand this conundrum, note that the TEST statement is using the likelihood ratio statistic to test the null hypothesis $H_0 : zo = 40.47543$ with the alternate that $H_a : zo \neq 40.47543$. The upper confidence interval can be viewed as a test with the null hypothesis $H_0 : zo <= 40.47543$.

Choice of Instruments

Several of the estimation methods supported by PROC MODEL are instrumental variables methods. There is no standard method for choosing instruments for nonlinear regression. Few econometric textbooks discuss the selection of instruments for nonlinear models. Refer to Bowden, R.J. and Turkington, D.A. (1984, p. 180-182) for more information.

The purpose of the instrumental projection is to purge the regressors of their correlation with the residual. For nonlinear systems, the regressors are the partials of the residuals with respect to the parameters.

Possible instrumental variables include

- any variable in the model that is independent of the errors
- lags of variables in the system
- derivatives with respect to the parameters, if the derivatives are independent of the errors
- low degree polynomials in the exogenous variables
- variables from the data set or functions of variables from the data set.

Selected instruments must not

- depend on any variable endogenous with respect to the equations estimated
- depend on any of the parameters estimated
- be lags of endogenous variables if there is serial correlation of the errors.

If the preceding rules are satisfied and there are enough observations to support the number of instruments used, the results should be consistent and the efficiency loss held to a minimum.

You need at least as many instruments as the maximum number of parameters in any equation, or some of the parameters cannot be estimated. Note that *number of instruments* means linearly independent instruments. If you add an instrument that is a linear combination of other instruments, it has no effect and does not increase the effective number of instruments.

You can, however, use too many instruments. In order to get the benefit of instrumental variables, you must have more observations than instruments. Thus, there is a trade-off; the instrumental variables technique completely eliminates the simultaneous equation bias only in large samples. In finite samples, the larger the excess of observations over instruments, the more the bias is reduced. Adding more instruments may improve the efficiency, but after some point efficiency declines as the excess of observations over instruments becomes smaller and the bias grows.

The instruments used in an estimation are printed out at the beginning of the estimation. For example, the following statements produce the instruments list shown in Figure 14.48.

```
proc model data=test2;
   exogenous x1 x2;
   parms b1 a1 a2 b2 2.5 c2 55;
   y1 = a1 * y2 + b1 * exp(x1);
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2;
   fit y1 y2 / n2sls;
   inst b1 b2 c2 x1 ;
run;
```

```
                                 The MODEL Procedure

                             The 2 Equations to Estimate

                             y1 =  F(b1, a1(y2))
                             y2 =  F(a2(y1), b2, c2)
                      Instruments  1 x1 @y1/@b1 @y2/@b2 @y2/@c2
```

Figure 14.48. Instruments Used Message

This states that an intercept term, the exogenous variable X1, and the partial derivatives of the equations with respect to B1, B2, and C2, were used as instruments for the estimation.

Examples

Suppose that Y1 and Y2 are endogenous variables, that X1 and X2 are exogenous variables, and that A, B, C, D, E, F, and G are parameters. Consider the following model:

```
y1 = a + b * x1 + c * y2 + d * lag(y1);
y2 = e + f * x2 + g * y1;
fit y1 y2;
instruments exclude=(c g);
```

The INSTRUMENTS statement produces X1, X2, LAG(Y1), and an intercept as instruments.

In order to estimate the Y1 equation by itself, it is necessary to include X2 explicitly in the instruments since F, in this case, is not included in the estimation

```
y1 = a + b * x1 + c * y2 + d * lag(y1);
y2 = e + f * x2 + g * y1;
fit y1;
instruments x2 exclude=(c);
```

This produces the same instruments as before. You can list the parameter associated with the lagged variable as an instrument instead of using the EXCLUDE= option. Thus, the following is equivalent to the previous example:

```
y1 = a + b * x1 + c * y2 + d * lag(y1);
y2 = e + f * x2 + g * y1;
fit y1;
instruments x1 x2 d;
```

For an example of declaring instruments when estimating a model involving identities, consider Klein's Model I

```
proc model data=klien;
   endogenous c p w i x wsum k y;
   exogenous  wp g t year;
   parms c0-c3 i0-i3 w0-w3;
   a: c = c0 + c1 * p + c2 * lag(p) + c3 * wsum;
   b: i = i0 + i1 * p + i2 * lag(p) + i3 * lag(k);
   c: w = w0 + w1 * x + w2 * lag(x) + w3 * year;
   x = c + i + g;
   y = c + i + g-t;
   p = x-w-t;
   k = lag(k) + i;
   wsum = w + wp;
```

The three equations to estimate are identified by the labels A, B, and C. The parameters associated with the predetermined terms are C2, I2, I3, W2, and W3 (and the intercepts, which are automatically added to the instruments). In addition, the system includes five identities that contain the predetermined variables G, T, LAG(K), and WP. Thus, the INSTRUMENTS statement can be written as

```
lagk = lag(k);
instruments c2 i2 i3 w2 w3 g t wp lagk;
```

where LAGK is a program variable used to hold LAG(K). However, this is more complicated than it needs to be. Except for LAG(K), all the predetermined terms in the identities are exogenous variables, and LAG(K) is already included as the coefficient of I3. There are also more parameters for predetermined terms than for endogenous terms, so you might prefer to use the EXCLUDE= option. Thus, you can specify the same instruments list with the simpler statement

```
instruments _exog_ exclude=(c1 c3 i1 w1);
```

To illustrate the use of polynomial terms as instrumental variables, consider the following model:

```
y1 = a + b * exp( c * x1 ) + d * log( x2 ) + e * exp( f * y2 );
```

The parameters are A, B, C, D, E, and F, and the right-hand-side variables are X1, X2, and Y2. Assume that X1 and X2 are exogenous (independent of the error), while Y2 is endogenous. The equation for Y2 is not specified, but assume that it includes the variables X1, X3, and Y1, with X3 exogenous, so the exogenous variables of the full system are X1, X2, and X3. Using as instruments quadratic terms in the exogenous variables, the model is specified to PROC MODEL as follows.

```
proc model;
   parms a b c d e f;
   y1 = a + b * exp( c * x1 ) + d * log( x2 ) + e * exp( f * y2 );
   instruments inst1-inst9;
   inst1 = x1; inst2 = x2; inst3 = x3;
   inst4 = x1 * x1; inst5 = x1 * x2; inst6 = x1 * x3;
   inst7 = x2 * x2; inst8 = x2 * x3; inst9 = x3 * x3;
   fit y1 / 2sls;
run;
```

It is not clear what degree polynomial should be used. There is no way to know how good the approximation is for any degree chosen, although the first-stage R^2s may help the assessment.

First-Stage R^2s

When the FSRSQ option is used on the FIT statement, the MODEL procedure prints a column of first-stage R^2 (FSRSQ) statistics along with the parameter estimates. The FSRSQ measures the fraction of the variation of the derivative column associated with the parameter that remains after projection through the instruments.

Ideally, the FSRSQ should be very close to 1.00 for exogenous derivatives. If the FSRSQ is small for an endogenous derivative, it is unclear whether this reflects a poor choice of instruments or a large influence of the errors in the endogenous right-hand-side variables. When the FSRSQ for one or more parameters is small, the standard errors of the parameter estimates are likely to be large.

Note that you can make all the FSRSQs larger (or 1.00) by including more instruments, because of the disadvantage discussed previously. The FSRSQ statistics reported are unadjusted R^2s and do not include a degrees-of-freedom correction.

Autoregressive Moving Average Error Processes

Autoregressive moving average error processes (ARMA errors) and other models involving lags of error terms can be estimated using FIT statements and simulated or forecast using SOLVE statements. ARMA models for the error process are often used for models with autocorrelated residuals. The %AR macro can be used to specify models with autoregressive error processes. The %MA macro can be used to specify models with moving average error processes.

Autoregressive Errors

A model with first-order autoregressive errors, AR(1), has the form

$$y_t = f(x_t, \theta) + \mu_t$$

$$\mu_t = \phi\mu_{t-1} + \epsilon_t$$

while an AR(2) error process has the form

$$\mu_t = \phi_1\mu_{t-1} + \phi_2\mu_{t-2} + \epsilon_t$$

and so forth for higher-order processes. Note that the ϵ_t's are independent and identically distributed and have an expected value of 0.

An example of a model with an AR(2) component is

$$y = \alpha + \beta x_1 + \mu_t$$

$$\mu_t = \phi_1 \mu_{t-1} + \phi_2 \mu_{t-2} + \epsilon_t$$

You would write this model as follows:

```
proc model data=in;
   parms a b p1 p2;
   y = a + b * x1 + p1 * zlag1(y - (a + b * x1)) +
         p2 * zlag2(y - (a + b * x1));
   fit y;
run;
```

or equivalently using the %AR macro as

```
proc model data=in;
   parms a b;
   y = a + b * x1;
   %ar( y, 2 );
   fit y;
run;
```

Moving Average Models

A model with first-order moving average errors, MA(1), has the form

$$y_t = f(x_t) + \mu_t$$

$$\mu_t = \epsilon_t - \theta_1 \epsilon_{t-1}$$

where ϵ_t is identically and independently distributed with mean zero. An MA(2) error process has the form

$$\mu_t = \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2}$$

and so forth for higher-order processes.

For example, you can write a simple linear regression model with MA(2) moving average errors as

```
proc model data=inma2;
   parms a b ma1 ma2;
   y = a + b * x + ma1 * zlag1( resid.y ) +
       ma2 * zlag2( resid.y );
   fit;
run;
```

where MA1 and MA2 are the moving average parameters.

Note that RESID.Y is automatically defined by PROC MODEL as

```
pred.y = a + b * x + ma1 * zlag1( resid.y ) +
       ma2 * zlag2( resid.y );
resid.y = actual.y - pred.y;
```

Note that RESID.Y is ϵ_t.

The ZLAG function must be used for MA models to truncate the recursion of the lags. This ensures that the lagged errors start at zero in the lag-priming phase and do not propagate missing values when lag-priming period variables are missing, and ensures that the future errors are zero rather than missing during simulation or forecasting. For details on the lag functions, see the section "Lag Logic."

This model written using the %MA macro is

```
proc model data=inma2;
   parms a b;
   y =  a + b * x;
   %ma(y, 2);
   fit;
run;
```

General Form for ARMA Models

The general ARMA(*p*,*q*) process has the following form

$$\mu_t = \phi_1 \mu_{t-1} + \ldots + \phi_p \mu_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \ldots - \theta_q \epsilon_{t-q}$$

An ARMA(*p*,*q*) model can be specified as follows

```
yhat =  ... compute structural predicted value here ... ;
yarma = ar1 * zlag1( y - yhat ) + ...  /* ar part */
                                + ar(p) * zlag(p)( y - yhat )
     + ma1 * zlag1( resid.y )   + ...  /* ma part */
                                + ma(q) * zlag(q)( resid.y );
y = yhat + yarma;
```

where AR*i* and MA*j* represent the autoregressive and moving average parameters for the various lags. You can use any names you want for these variables, and there are many equivalent ways that the specification could be written.

Vector ARMA processes can also be estimated with PROC MODEL. For example, a two-variable AR(1) process for the errors of the two endogenous variables Y1 and Y2 can be specified as follows

```
y1hat =  ... compute structural predicted value here ... ;

y1    = y1hat + ar1_1 * zlag1( y1 - y1hat )   /* ar part y1,y1 */
              + ar1_2 * zlag1( y2 - y2hat );  /* ar part y1,y2 */

y21hat =  ... compute structural predicted value here ... ;
```

```
y2     = y2hat + ar2_2 * zlag1( y2 - y2hat )   /* ar part y2,y2 */
               + ar2_1 * zlag1( y1 - y1hat );  /* ar part y2,y1 */
```

Convergence Problems with ARMA Models

ARMA models can be difficult to estimate. If the parameter estimates are not within the appropriate range, a moving average model's residual terms will grow exponentially. The calculated residuals for later observations can be very large or can overflow. This can happen either because improper starting values were used or because the iterations moved away from reasonable values.

Care should be used in choosing starting values for ARMA parameters. Starting values of .001 for ARMA parameters usually work if the model fits the data well and the problem is well-conditioned. Note that an MA model can often be approximated by a high order AR model, and vice versa. This may result in high collinearity in mixed ARMA models, which in turn can cause serious ill-conditioning in the calculations and instability of the parameter estimates.

If you have convergence problems while estimating a model with ARMA error processes, try to estimate in steps. First, use a FIT statement to estimate only the structural parameters with the ARMA parameters held to zero (or to reasonable prior estimates if available). Next, use another FIT statement to estimate the ARMA parameters only, using the structural parameter values from the first run. Since the values of the structural parameters are likely to be close to their final estimates, the ARMA parameter estimates may now converge. Finally, use another FIT statement to produce simultaneous estimates of all the parameters. Since the initial values of the parameters are now likely to be quite close to their final joint estimates, the estimates should converge quickly if the model is appropriate for the data.

AR Initial Conditions

The initial lags of the error terms of AR(*p*) models can be modeled in different ways. The autoregressive error startup methods supported by SAS/ETS procedures are the following:

CLS	conditional least squares (ARIMA and MODEL procedures)
ULS	unconditional least squares (AUTOREG, ARIMA, and MODEL procedures)
ML	maximum likelihood (AUTOREG, ARIMA, and MODEL procedures)
YW	Yule-Walker (AUTOREG procedure only)
HL	Hildreth-Lu, which deletes the first *p* observations (MODEL procedure only)

See Chapter 8, for an explanation and discussion of the merits of various AR(p) startup methods.

The CLS, ULS, ML, and HL initializations can be performed by PROC MODEL. For AR(1) errors, these initializations can be produced as shown in Table 14.2. These methods are equivalent in large samples.

Table 14.2. Initializations Performed by PROC MODEL: AR(1) ERRORS

Method	**Formula**
conditional least squares	Y=YHAT+AR1*ZLAG1(Y-YHAT);

unconditional least squares	Y=YHAT+AR1*ZLAG1(Y-YHAT);
	IF _OBS_=1 THEN
	RESID.Y=SQRT(1-AR1**2)*RESID.Y;

maximum likelihood	Y=YHAT+AR1*ZLAG1(Y-YHAT);
	W=(1-AR1**2)**(-1/(2*_NUSED_));
	IF _OBS_=1 THEN W=W*SQRT(1-AR1**2);
	RESID.Y=W*RESID.Y;

Hildreth-Lu	Y=YHAT+AR1*LAG1(Y-YHAT);

MA Initial Conditions

The initial lags of the error terms of MA(q) models can also be modeled in different ways. The following moving average error startup paradigms are supported by the ARIMA and MODEL procedures:

ULS	unconditional least squares
CLS	conditional least squares
ML	maximum likelihood

The conditional least-squares method of estimating moving average error terms is not optimal because it ignores the startup problem. This reduces the efficiency of the estimates, although they remain unbiased. The initial lagged residuals, extending before the start of the data, are assumed to be 0, their unconditional expected value. This introduces a difference between these residuals and the generalized least-squares residuals for the moving average covariance, which, unlike the autoregressive model, persists through the data set. Usually this difference converges quickly to 0, but for nearly noninvertible moving average processes the convergence is quite slow. To minimize this problem, you should have plenty of data, and the moving average parameter estimates should be well within the invertible range.

This problem can be corrected at the expense of writing a more complex program. Unconditional least-squares estimates for the MA(1) process can be produced by specifying the model as follows:

```
yhat =  ... compute structural predicted value here ... ;
if _obs_ = 1 then do;
   h = sqrt( 1 + ma1 ** 2 );
   y = yhat;
   resid.y = ( y - yhat ) / h;
   end;
else do;
   g = ma1 / zlag1( h );
   h = sqrt( 1 + ma1 ** 2 - g ** 2 );
   y = yhat + g * zlag1( resid.y );
   resid.y = ( ( y - yhat) - g * zlag1( resid.y ) ) / h;
   end;
```

Moving-average errors can be difficult to estimate. You should consider using an AR(*p*) approximation to the moving average process. A moving average process can usually be well-approximated by an autoregressive process if the data have not been smoothed or differenced.

The %AR Macro

The SAS macro %AR generates programming statements for PROC MODEL for autoregressive models. The %AR macro is part of SAS/ETS software and no special options need to be set to use the macro. The autoregressive process can be applied to the structural equation errors or to the endogenous series themselves.

The %AR macro can be used for

- univariate autoregression
- unrestricted vector autoregression
- restricted vector autoregression.

Univariate Autoregression

To model the error term of an equation as an autoregressive process, use the following statement after the equation:

```
%ar( varname, nlags )
```

For example, suppose that Y is a linear function of X1 and X2, and an AR(2) error. You would write this model as follows:

```
proc model data=in;
   parms a b c;
   y = a + b * x1 + c * x2;
   %ar( y, 2 )
   fit y / list;
run;
```

The calls to %AR must come *after* all of the equations that the process applies to.

The proceding macro invocation, %AR(y,2), produces the statements shown in the LIST output in Figure 14.49.

```
                              The MODEL Procedure

                        Listing of Compiled Program Code
          Stmt    Line:Col      Statement as Parsed

             1    5738:50       PRED.y = a + b * x1 + c * x2;
             1    5738:50       RESID.y = PRED.y - ACTUAL.y;
             1    5738:50       ERROR.y = PRED.y - y;
             2    7987:23       _PRED__y = PRED.y;
             3    8003:15       #OLD_PRED.y = PRED.y + y_l1
                                * ZLAG1( y - _PRED__y ) + y_l2
                                * ZLAG2( y - _PRED__y );
             3    8003:15       PRED.y = #OLD_PRED.y;
             3    8003:15       RESID.y = PRED.y - ACTUAL.y;
             3    8003:15       ERROR.y = PRED.y - y;
```

Figure 14.49. LIST Option Output for an AR(2) Model

The _PRED__ prefixed variables are temporary program variables used so that the lags of the residuals are the correct residuals and not the ones redefined by this equation. Note that this is equivalent to the statements explicitly written in the "General Form for ARMA Models" earlier in this section.

You can also restrict the autoregressive parameters to zero at selected lags. For example, if you wanted autoregressive parameters at lags 1, 12, and 13, you can use the following statements:

```
proc model data=in;
   parms a b c;
   y = a + b * x1 + c * x2;
   %ar( y, 13, , 1 12 13 )
   fit y / list;
run;
```

These statements generate the output shown in Figure 14.50.

```
                              The MODEL Procedure

                        Listing of Compiled Program Code
          Stmt    Line:Col      Statement as Parsed

             1    8182:50       PRED.y = a + b * x1 + c * x2;
             1    8182:50       RESID.y = PRED.y - ACTUAL.y;
             1    8182:50       ERROR.y = PRED.y - y;
             2    8631:23       _PRED__y = PRED.y;
             3    8647:15       #OLD_PRED.y = PRED.y + y_l1 * ZLAG1( y -
                                _PRED__y ) + y_l12 * ZLAG12( y -
                                _PRED__y ) + y_l13 * ZLAG13(
                                y - _PRED__y );
             3    8647:15       PRED.y = #OLD_PRED.y;
             3    8647:15       RESID.y = PRED.y - ACTUAL.y;
             3    8647:15       ERROR.y = PRED.y - y;
```

Figure 14.50. LIST Option Output for an AR Model with Lags at 1, 12, and 13

There are variations on the conditional least-squares method, depending on whether observations at the start of the series are used to "warm up" the AR process. By default, the %AR conditional least-squares method uses all the observations and assumes zeros for the initial lags of autoregressive terms. By using the M= option, you can request that %AR use the unconditional least-squares (ULS) or maximum-likelihood (ML) method instead. For example,

```
proc model data=in;
   y = a + b * x1 + c * x2;
   %ar( y, 2, m=uls )
   fit y;
run;
```

Discussions of these methods is provided in the "AR Initial Conditions" earlier in this section.

By using the M=CLS*n* option, you can request that the first *n* observations be used to compute estimates of the initial autoregressive lags. In this case, the analysis starts with observation *n*+1. For example:

```
proc model data=in;
   y = a + b * x1 + c * x2;
   %ar( y, 2, m=cls2 )
   fit y;
run;
```

You can use the %AR macro to apply an autoregressive model to the endogenous variable, instead of to the error term, by using the TYPE=V option. For example, if you want to add the five past lags of Y to the equation in the previous example, you could use %AR to generate the parameters and lags using the following statements:

```
proc model data=in;
   parms a b c;
   y = a + b * x1 + c * x2;
   %ar( y, 5, type=v )
   fit y / list;
run;
```

The preceding statements generate the output shown in Figure 14.51.

```
                              The MODEL Procedure

                        Listing of Compiled Program Code
          Stmt    Line:Col      Statement as Parsed

             1    8892:50       PRED.y = a + b * x1 + c * x2;
             1    8892:50       RESID.y = PRED.y - ACTUAL.y;
             1    8892:50       ERROR.y = PRED.y - y;
             2    9301:15       #OLD_PRED.y = PRED.y + y_l1 * ZLAG1( y )
                                + y_l2 * ZLAG2( y ) + y_l3 * ZLAG3( y )
                                + y_l4 * ZLAG4( y ) + y_l5 * ZLAG5( y );
             2    9301:15       PRED.y = #OLD_PRED.y;
             2    9301:15       RESID.y = PRED.y - ACTUAL.y;
             2    9301:15       ERROR.y = PRED.y - y;
```

Figure 14.51. LIST Option Output for an AR model of Y

This model predicts Y as a linear combination of X1, X2, an intercept, and the values of Y in the most recent five periods.

Unrestricted Vector Autoregression

To model the error terms of a set of equations as a vector autoregressive process, use the following form of the %AR macro after the equations:

```
%ar( process_name, nlags, variable_list )
```

The *process_name* value is any name that you supply for %AR to use in making names for the autoregressive parameters. You can use the %AR macro to model several different AR processes for different sets of equations by using different process names for each set. The process name ensures that the variable names used are unique. Use a short *process_name* value for the process if parameter estimates are to be written to an output data set. The %AR macro tries to construct parameter names less than or equal to eight characters, but this is limited by the length of *name*, which is used as a prefix for the AR parameter names.

The *variable_list* value is the list of endogenous variables for the equations.

For example, suppose that errors for equations Y1, Y2, and Y3 are generated by a second-order vector autoregressive process. You can use the following statements:

```
proc model data=in;
   y1 = ... equation for y1 ...;
   y2 = ... equation for y2 ...;
   y3 = ... equation for y3 ...;
   %ar( name, 2, y1 y2 y3 )
   fit y1 y2 y3;
run;
```

which generates the following for Y1 and similar code for Y2 and Y3:

```
y1 = pred.y1 + name1_1_1*zlag1(y1-name_y1) +
     name1_1_2*zlag1(y2-name_y2) +
     name1_1_3*zlag1(y3-name_y3) +
     name2_1_1*zlag2(y1-name_y1) +
     name2_1_2*zlag2(y2-name_y2) +
     name2_1_3*zlag2(y3-name_y3) ;
```

Only the conditional least-squares (M=CLS or M=CLS*n*) method can be used for vector processes.

You can also use the same form with restrictions that the coefficient matrix be 0 at selected lags. For example, the statements

```
proc model data=in;
   y1 = ... equation for y1 ...;
   y2 = ... equation for y2 ...;
   y3 = ... equation for y3 ...;
   %ar( name, 3, y1 y2 y3, 1 3 )
   fit y1 y2 y3;
```

apply a third-order vector process to the equation errors with all the coefficients at lag 2 restricted to 0 and with the coefficients at lags 1 and 3 unrestricted.

You can model the three series Y1-Y3 as a vector autoregressive process in the variables instead of in the errors by using the TYPE=V option. If you want to model Y1-Y3 as a function of past values of Y1-Y3 and some exogenous variables or constants, you can use %AR to generate the statements for the lag terms. Write an equation for each variable for the nonautoregressive part of the model, and then call %AR with the TYPE=V option. For example,

```
proc model data=in;
   parms a1-a3 b1-b3;
   y1 = a1 + b1 * x;
   y2 = a2 + b2 * x;
   y3 = a3 + b3 * x;
   %ar( name, 2, y1 y2 y3, type=v )
   fit y1 y2 y3;
run;
```

The nonautoregressive part of the model can be a function of exogenous variables, or it may be intercept parameters. If there are no exogenous components to the vector autoregression model, including no intercepts, then assign zero to each of the variables. There must be an assignment to each of the variables before %AR is called.

```
proc model data=in;
   y1=0;
   y2=0;
   y3=0;
   %ar( name, 2, y1 y2 y3, type=v )
   fit y1 y2 y3;
```

This example models the vector Y=(Y1 Y2 Y3)$'$ as a linear function only of its value in the previous two periods and a white noise error vector. The model has 18=(3 $\times$ 3 + 3 $\times$ 3) parameters.

Syntax of the %AR Macro

There are two cases of the syntax of the %AR macro. The first has the general form

%AR *(name, nlag [,endolist [,laglist]] [***,M=***method] [***,TYPE=***V])*

where

name	specifies a prefix for %AR to use in constructing names of variables needed to define the AR process. If the *endolist* is not specified, the endogenous list defaults to *name*, which must be the name of the equation to which the AR error process is to be applied. The *name* value cannot exceed eight characters.
nlag	is the order of the AR process.
endolist	specifies the list of equations to which the AR process is to be applied. If more than one name is given, an unrestricted vector process is created with the structural residuals of all the equations included as regressors in each of the equations. If not specified, *endolist* defaults to *name*.
laglist	specifies the list of lags at which the AR terms are to be added. The coefficients of the terms at lags not listed are set to 0. All of the listed lags must be less than or equal to *nlag*, and there must be no duplicates. If not specified, the *laglist* defaults to all lags 1 through *nlag*.

M=*method*	specifies the estimation method to implement. Valid values of M= are CLS (conditional least-squares estimates), ULS (unconditional least-squares estimates), and ML (maximum-likelihood estimates). M=CLS is the default. Only M=CLS is allowed when more than one equation is specified. The ULS and ML methods are not supported for vector AR models by %AR.
TYPE=V	specifies that the AR process is to be applied to the endogenous variables themselves instead of to the structural residuals of the equations.

Restricted Vector Autoregression

You can control which parameters are included in the process, restricting those parameters that you do not include to 0. First, use %AR with the DEFER option to declare the variable list and define the dimension of the process. Then, use additional %AR calls to generate terms for selected equations with selected variables at selected lags. For example,

```
proc model data=d;
   y1 = ... equation for y1 ...;
   y2 = ... equation for y2 ...;
   y3 = ... equation for y3 ...;
   %ar( name, 2, y1 y2 y3, defer )
   %ar( name, y1, y1 y2 )
   %ar( name, y2 y3, , 1 )
   fit y1 y2 y3;
run;
```

The error equations produced are

```
y1 = pred.y1 + name1_1_1*zlag1(y1-name_y1) +
     name1_1_2*zlag1(y2-name_y2) + name2_1_1*zlag2(y1-name_y1) +
     name2_1_2*zlag2(y2-name_y2) ;
y2 = pred.y2 + name1_2_1*zlag1(y1-name_y1) +
     name1_2_2*zlag1(y2-name_y2) + name1_2_3*zlag1(y3-name_y3) ;
y3 = pred.y3 + name1_3_1*zlag1(y1-name_y1) +
     name1_3_2*zlag1(y2-name_y2) + name1_3_3*zlag1(y3-name_y3) ;
```

This model states that the errors for Y1 depend on the errors of both Y1 and Y2 (but not Y3) at both lags 1 and 2, and that the errors for Y2 and Y3 depend on the previous errors for all three variables, but only at lag 1.

%AR Macro Syntax for Restricted Vector AR

An alternative use of %AR is allowed to impose restrictions on a vector AR process by calling %AR several times to specify different AR terms and lags for different equations.

The first call has the general form

%AR(*name, nlag, endolist, DEFER)* where

name	specifies a prefix for %AR to use in constructing names of variables needed to define the vector AR process.
nlag	specifies the order of the AR process.
endolist	specifies the list of equations to which the AR process is to be applied.
DEFER	specifies that %AR is not to generate the AR process but is to wait for further information specified in later %AR calls for the same *name* value.

The subsequent calls have the general form

```
%AR( name, eqlist, varlist, laglist,TYPE= )
```

where

name	is the same as in the first call.
eqlist	specifies the list of equations to which the specifications in this %AR call are to be applied. Only names specified in the *endolist* value of the first call for the *name* value can appear in the list of equations in *eqlist*.
varlist	specifies the list of equations whose lagged structural residuals are to be included as regressors in the equations in *eqlist*. Only names in the *endolist* of the first call for the *name* value can appear in *varlist*. If not specified, *varlist* defaults to *endolist*.
laglist	specifies the list of lags at which the AR terms are to be added. The coefficients of the terms at lags not listed are set to 0. All of the listed lags must be less than or equal to the value of *nlag*, and there must be no duplicates. If not specified, *laglist* defaults to all lags 1 through *nlag*.

The %MA Macro

The SAS macro %MA generates programming statements for PROC MODEL for moving average models. The %MA macro is part of SAS/ETS software and no special options are needed to use the macro. The moving average error process can be applied to the structural equation errors. The syntax of the %MA macro is the same as the %AR macro except there is no TYPE= argument.

When you are using the %MA and %AR macros combined, the %MA macro must follow the %AR macro. The following SAS/IML statements produce an ARMA(1, (1 3)) error process and save it in the data set MADAT2.

```
 /* use IML module to simulate a MA process  */
proc iml;
   phi={1 .2};
```

```
      theta={ 1 .3 0 .5};
      y=armasim(phi, theta, 0,.1, 200,32565);
      create madat2 from y[colname='y'];
      append;
   quit;
```

The following PROC MODEL statements are used to estimate the parameters of this model using maximum likelihood error structure:

```
   title1 'Maximum Likelihood ARMA(1, (1 3))';
   proc model data=madat2;
      y=0;
      %ar(y,1,,  M=ml)
      %ma(y,3,,1 3,  M=ml)  /* %MA always after %AR */
      fit y;
   run;
```

The estimates of the parameters produced by this run are shown in Figure 14.52.

```
                       Maximum Likelihood ARMA(1, (1 3))

                              The MODEL Procedure

                   Nonlinear OLS Summary of Residual Errors

                    DF     DF                                               Adj
  Equation       Model  Error        SSE        MSE  Root MSE  R-Square     R-Sq

  y                  3    197     2.6383     0.0134    0.1157   -0.0067  -0.0169
  RESID.y                 197     1.9957     0.0101    0.1007


                       Nonlinear OLS Parameter Estimates

                                Approx                  Approx
Parameter      Estimate    Std Err    t Value    Pr > |t|   Label

y_l1           -0.10067     0.1187     -0.85      0.3973   AR(y) y lag1
                                                           parameter
y_m1            -0.1934     0.0939     -2.06      0.0408   MA(y) y lag1
                                                           parameter
y_m3           -0.59384     0.0601     -9.88      <.0001   MA(y) y lag3
                                                           parameter
```

Figure 14.52. Estimates from an ARMA(1, (1 3)) Process

Syntax of the %MA Macro

There are two cases of the syntax for the %MA macro. The first has the general form

%MA *(name, nlag [*,*endolist [*,*laglist]] [*,**M=***method])*

where

name	specifies a prefix for %MA to use in constructing names of variables needed to define the MA process and is the default *endolist*.
nlag	is the order of the MA process.

endolist	specifies the equations to which the MA process is to be applied. If more than one name is given, CLS estimation is used for the vector process.
laglist	specifies the lags at which the MA terms are to be added. All of the listed lags must be less than or equal to *nlag*, and there must be no duplicates. If not specified, the *laglist* defaults to all lags 1 through *nlag*.
M=*method*	specifies the estimation method to implement. Valid values of M= are CLS (conditional least-squares estimates), ULS (unconditional least-squares estimates), and ML (maximum-likelihood estimates). M=CLS is the default. Only M=CLS is allowed when more than one equation is specified on the *endolist*.

%MA Macro Syntax for Restricted Vector Moving Average

An alternative use of %MA is allowed to impose restrictions on a vector MA process by calling %MA several times to specify different MA terms and lags for different equations.

The first call has the general form

%MA(*name, nlag, endolist,* **DEFER** *)*

where

name	specifies a prefix for %MA to use in constructing names of variables needed to define the vector MA process.
nlag	specifies the order of the MA process.
endolist	specifies the list of equations to which the MA process is to be applied.
DEFER	specifies that %MA is not to generate the MA process but is to wait for further information specified in later %MA calls for the same *name* value.

The subsequent calls have the general form

```
%MA( name, eqlist, varlist, laglist )
```

where

name	is the same as in the first call.
eqlist	specifies the list of equations to which the specifications in this %MA call are to be applied.
varlist	specifies the list of equations whose lagged structural residuals are to be included as regressors in the equations in *eqlist*.
laglist	specifies the list of lags at which the MA terms are to be added.

Distributed Lag Models and the %PDL Macro

In the following example, the variable *y* is modeled as a linear function of *x*, the first lag of *x*, the second lag of *x*, and so forth:

$$y_t = a + b_0 x_t + b_1 x_{t-1} + b_2 x_{t-2} + b_3 x_{t-3} + \ldots + b_n x_{t-l}$$

Models of this sort can introduce a great many parameters for the lags, and there may not be enough data to compute accurate independent estimates for them all. Often, the number of parameters is reduced by assuming that the lag coefficients follow some pattern. One common assumption is that the lag coefficients follow a polynomial in the lag length

$$b_i = \sum_{j=0}^{d} \alpha_j (i)^j$$

where *d* is the degree of the polynomial used. Models of this kind are called *Almon lag models*, *polynomial distributed lag models*, or *PDLs* for short. For example, Figure 14.53 shows the lag distribution that can be modeled with a low order polynomial. Endpoint restrictions can be imposed on a PDL to require that the lag coefficients be 0 at the 0th lag, or at the final lag, or at both.

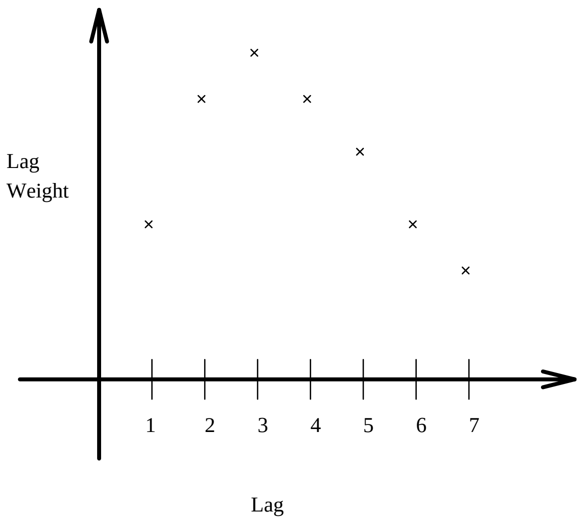


Figure 14.53. Polynomial Distributed Lags

For linear single-equation models, SAS/ETS software includes the PDLREG procedure for estimating PDL models. See Chapter 15, “The PDLREG Procedure,” for a more detailed discussion of polynomial distributed lags and an explanation of endpoint restrictions.

Polynomial and other distributed lag models can be estimated and simulated or forecast with PROC MODEL. For polynomial distributed lags, the %PDL macro can generate the needed programming statements automatically.

The %PDL Macro

The SAS macro %PDL generates the programming statements to compute the lag coefficients of polynomial distributed lag models and to apply them to the lags of variables or expressions.

To use the %PDL macro in a model program, you first call it to declare the lag distribution; later, you call it again to apply the PDL to a variable or expression. The first call generates a PARMS statement for the polynomial parameters and assignment statements to compute the lag coefficients. The second call generates an expression that applies the lag coefficients to the lags of the specified variable or expression. A PDL can be declared only once, but it can be used any number of times (that is, the second call can be repeated).

The initial declaratory call has the general form

%PDL*(pdlname, nlags, degree*, R=*code*, OUTEST=*dataset)*

where *pdlname* is a name (up to eight characters) that you give to identify the PDL, *nlags* is the lag length, and *degree* is the degree of the polynomial for the distribution. The R=*code* is optional for endpoint restrictions. The value of *code* can be FIRST (for upper), LAST (for lower), or BOTH (for both upper and lower endpoints). See chapter pdlreg, "The PDLREG Procedure," for a discussion of endpoint restrictions. The option OUTEST=*dataset* creates a data set containing the estimates of the parameters and their covariance matrix.

The later calls to apply the PDL have the general form

```
%PDL( pdlname, expression )
```

where *pdlname* is the name of the PDL and *expression* is the variable or expression to which the PDL is to be applied. The *pdlname* given must be the same as the name used to declare the PDL.

The following statements produce the output in Figure 14.54:

```
proc model data=in list;
   parms int pz;
   %pdl(xpdl,5,2);
   y = int + pz * z + %pdl(xpdl,x);
   %ar(y,2,M=ULS);
   id i;
fit y / out=model1 outresid converge=1e-6;
run;
```

```
                              The MODEL Procedure

                           Nonlinear OLS  Estimates

                            Approx                 Approx
Term           Estimate    Std Err   t Value    Pr > |t|   Label

XPDL_L0        1.568788     0.0992     15.81      <.0001   PDL(XPDL,5,2)
                                                           coefficient for lag0
XPDL_L1        0.564917     0.0348     16.24      <.0001   PDL(XPDL,5,2)
                                                           coefficient for lag1
XPDL_L2        -0.05063     0.0629     -0.80      0.4442   PDL(XPDL,5,2)
                                                           coefficient for lag2
XPDL_L3        -0.27785     0.0549     -5.06      0.0010   PDL(XPDL,5,2)
                                                           coefficient for lag3
XPDL_L4        -0.11675     0.0390     -2.99      0.0173   PDL(XPDL,5,2)
                                                           coefficient for lag4
XPDL_L5         0.43267     0.1445      2.99      0.0172   PDL(XPDL,5,2)
                                                           coefficient for lag5
```

Figure 14.54. %PDL Macro ESTIMATE Statement Output

This second example models two variables, Y1 and Y2, and uses two PDLs:

```
proc model data=in;
   parms int1 int2;
   %pdl( logxpdl, 5, 3 )
   %pdl( zpdl, 6, 4 )
   y1 = int1 + %pdl( logxpdl, log(x) ) + %pdl( zpdl, z );
   y2 = int2 + %pdl( zpdl, z );
   fit y1 y2;
run;
```

A (5,3) PDL of the log of X is used in the equation for Y1. A (6,4) PDL of Z is used in the equations for both Y1 and Y2. Since the same ZPDL is used in both equations, the lag coefficients for Z are the same for the Y1 and Y2 equations, and the polynomial parameters for ZPDL are shared by the two equations. See Example 14.5 for a complete example and comparison with PDLREG.

Input Data Sets

DATA= Input Data Set

For FIT tasks, the DATA= option specifies which input data set to use in estimating parameters. Variables in the model program are looked up in the DATA= data set and, if found, their attributes (type, length, label, and format) are set to be the same as those in the DATA= data set (if not defined otherwise within PROC MODEL), and values for the variables in the program are read from the data set.

ESTDATA= Input Data Set

The ESTDATA= option specifies an input data set that contains an observation giving values for some or all of the model parameters. The data set can also contain observations giving the rows of a covariance matrix for the parameters.

Parameter values read from the ESTDATA= data set provide initial starting values for parameters estimated. Observations providing covariance values, if any are present in the ESTDATA= data set, are ignored.

The ESTDATA= data set is usually created by the OUTEST= option in a previous FIT statement. You can also create an ESTDATA= data set with a SAS DATA step program. The data set must contain a numeric variable for each parameter to be given a value or covariance column. The name of the variable in the ESTDATA= data set must match the name of the parameter in the model. Parameters with names longer than eight characters cannot be set from an ESTDATA= data set. The data set must also contain a character variable _NAME_ of length 8. _NAME_ has a blank value for the observation that gives values to the parameters. _NAME_ contains the name of a parameter for observations defining rows of the covariance matrix.

More than one set of parameter estimates and covariances can be stored in the ESTDATA= data set if the observations for the different estimates are identified by the variable _TYPE_. _TYPE_ must be a character variable of length 8. The TYPE= option is used to select for input the part of the ESTDATA= data set for which the _TYPE_ value matches the value of the TYPE= option.

The following SAS statements generate the ESTDATA= data set shown in Figure 14.55. The second FIT statement uses the TYPE= option to select the estimates from the GMM estimation as starting values for the FIML estimation.

```
         /* Generate test data */
data gmm2;
    do t=1 to 50;
       x1 = sqrt(t) ;
       x2 = rannor(10) * 10;
       y1 = -.002 * x2 * x2 - .05 / x2 - 0.001 * x1 * x1;
       y2 = 0.002* y1 + 2 * x2 * x2 + 50 / x2 + 5 * rannor(1);
       y1 = y1 + 5 * rannor(1);
       z1 = 1; z2 = x1 * x1; z3 = x2 * x2; z4 = 1.0/x2;
       output;
    end;
run;

proc model data=gmm2 ;
   exogenous x1 x2;
   parms a1 a2 b1 2.5 b2 c2 55 d1;
   inst b1 b2 c2 x1 x2;
   y1 = a1 * y2 + b1 * x1 * x1 + d1;
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2 + d1;

   fit y1 y2 / 3sls gmm kernel=(qs,1,0.2) outest=gmmest;

   fit y1 y2 / fiml type=gmm estdata=gmmest;
run;

proc print data=gmmest;
run;
```

```
                 _
                 S       _
  _  _           T       N
  N  T           A       U
  A  Y           T       S
O M  P           U       E
b E  E           S       D     a          a        b       b       c        d
s _  _           _       _     1          2        1       2       2        1

1   3SLS 0 Converged 50 -.002229607 -1.25002 0.025827 1.99609 49.8119 -0.44533
2   GMM  0 Converged 50 -.002013073 -1.53882 0.014908 1.99419 49.8035 -0.64933
```

Figure 14.55. ESTDATA= Data Set

MISSING= PAIRWISE | DELETE

When missing values are encountered for any one of the equations in a system of equations, the default action is to drop that observation for all of the equations. The new MISSING=PAIRWISE option on the FIT statement provides a different method of handling missing values that avoids losing data for nonmissing equations for the observation. This is especially useful for SUR estimation on equations with unequal numbers of observations.

The option MISSING=PAIRWISE specifies that missing values are tracked on an equation-by-equation basis. The MISSING=DELETE option specifies that the entire observation is omitted from the analysis when any equation has a missing predicted or actual value for the equation. The default is MISSING=DELETE.

When you specify the MISSING=PAIRWISE option, the S matrix is computed as

$$S = D(R'R)D$$

where D is a diagonal matrix that depends on the VARDEF= option, the matrix R is $(\mathbf{r}_1, \ldots, \mathbf{r}_g)$, and $\mathbf{r}_i$ is the vector of residuals for the *i*th equation with r_{ij} replaced with zero when r_{ij} is missing.

For MISSING=PAIRWISE, the calculation of the diagonal element $d_{i,i}$ of **D** is based on n_i, the number of nonmissing observations for the *i*th equation, instead of on *n* or, for VARDEF=WGT or WDF, on the sum of the weights for the nonmissing observations for the *i*th equation instead of on the sum of the weights for all observations. Refer to the description of the VARDEF= option for the definition of **D**.

The degrees of freedom correction for a shared parameter is computed using the average number of observations used in its estimation.

The MISSING=PAIRWISE option is not valid for the GMM and FIML estimation methods.

For the instrumental variables estimation methods (2SLS, 3SLS), when an instrument is missing for an observation, that observation is dropped for all equations, regardless of the MISSING= option.

PARMSDATA= Input Data Set

The option PARMSDATA= reads values for all parameters whose names match the names of variables in the PARMSDATA= data set. Values for any or all of the parameters in the model can be reset using the PARMSDATA= option. The PARMSDATA= option goes on the PROC MODEL statement, and the data set is read before any FIT or SOLVE statements are executed.

Together, the OUTPARMS= and PARMSDATA= options allow you to change part of a model and recompile the new model program without the need to reestimate equations that were not changed.

Suppose you have a large model with parameters estimated and you now want to replace one equation, Y, with a new specification. Although the model program must be recompiled with the new equation, you don't need to reestimate all the equations, just the one that changed.

Using the OUTPARMS= and PARMSDATA= options, you could do the following:

```
proc model model=oldmod outparms=temp; run;
proc model outmodel=newmod parmsdata=temp data=in;
   ...  include new model definition with changed y eq. here ...
   fit y;
run;
```

The model file NEWMOD will then contain the new model and its estimated parameters plus the old models with their original parameter values.

SDATA= Input Data Set

The SDATA= option allows a cross-equation covariance matrix to be input from a data set. The **S** matrix read from the SDATA= data set, specified in the FIT statement, is used to define the objective function for the OLS, N2SLS, SUR, and N3SLS estimation methods and is used as the initial **S** for the methods that iterate the **S** matrix.

Most often, the SDATA= data set has been created by the OUTS= or OUTSUSED= option on a previous FIT statement. The OUTS= and OUTSUSED= data sets from a FIT statement can be read back in by a FIT statement in the same PROC MODEL step.

You can create an input SDATA= data set using the DATA step. PROC MODEL expects to find a character variable _NAME_ in the SDATA= data set as well as variables for the equations in the estimation or solution. For each observation with a _NAME_ value matching the name of an equation, PROC MODEL fills the corresponding row of the **S** matrix with the values of the names of equations found in the data set. If a row or column is omitted from the data set, a 1 is placed on the diagonal for the row or column. Missing values are ignored, and since the **S** matrix is symmetric, you can include only a triangular part of the **S** matrix in the SDATA= data set with the omitted part indicated by missing values. If the SDATA= data set contains multiple observations with the same _NAME_, the last values supplied for the _NAME_ are used. The structure of the expected data set is further described in the "OUTS=Data Set" section.

Use the TYPE= option on the PROC MODEL or FIT statement to specify the type of estimation method used to produce the **S** matrix you want to input.

The following SAS statements are used to generate an **S** matrix from a GMM and a 3SLS estimation and to store that estimate in the data set GMMS:

```
proc model data=gmm2 ;
   exogenous x1 x2;
   parms a1 a2 b1 2.5 b2 c2 55 d1;
   inst b1 b2 c2 x1 x2;
   y1 = a1 * y2 + b1 * x1 * x1 + d1;
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2 + d1;

   fit y1 y2 / 3sls gmm kernel=(qs,1,0.2) outest=gmmest outs=gmms;
run;
```

The data set GMMS is shown in Figure 14.56.

```
           Obs    _NAME_    _TYPE_    _NUSED_       y1         y2

            1       y1       3SLS        50      27.1032    38.1599
            2       y2       3SLS        50      38.1599    74.6253
            3       y1       GMM         50      27.4205    46.4028
            4       y2       GMM         50      46.4028    99.4656
```

Figure 14.56. SDATA= Data Set

VDATA= Input data set

The VDATA= option allows a variance matrix for GMM estimation to be input from a data set. When the VDATA= option is used on the PROC MODEL or FIT statement, the matrix that is input is used to define the objective function and is used as the initial V for the methods that iterate the V matrix.

Normally the VDATA= matrix is created from the OUTV= option on a previous FIT statement. Alternately an input VDATA= data set can be created using the DATA step. Each row and column of the V matrix is associated with an equation and an instrument. The position of each element in the V matrix can then be indicated by an equation name and an instrument name for the row of the element and an equation name and an instrument name for the column. Each observation in the VDATA= data set is an element in the V matrix. The row and column of the element are indicated by four variables EQ_ROW, INST_ROW, EQ_COL, and INST_COL which contain the equation name or instrument name. The variable name for an element is VALUE. Missing values are set to 0. Because the variance matrix is symmetric, only a triangular part of the matrix needs to be input.

The following SAS statements are used to generate a **V** matrix estimation from GMM and to store that estimate in the data set GMMV:

```
proc model data=gmm2 ;
   exogenous x1 x2;
   parms a1 a2 b2 b1 2.5 c2 55 d1;
   inst b1 b2 c2 x1 x2;
   y1 = a1 * y2 + b1 * x1 * x1 + d1;
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2 + d1;
   fit y1 y2 / gmm outv=gmmv;
run;
```

The data set GMM2 was generated by the example in the preceding ESTDATA= section. The **V** matrix stored in GMMV is selected for use in an additional GMM estimation by the following FIT statement:

```
fit y1 y2 / gmm vdata=gmmv;
run;

proc print data=gmmv(obs=15);
run;
```

A partial listing of the GMMV data set is shown in Figure 14.57. There are a total of 78 observations in this data set. The **V** matrix is 12 by 12 for this example.

```
Obs    _TYPE_    EQ_ROW    EQ_COL    INST_ROW        INST_COL             VALUE

  1     GMM        Y1        Y1      1               1                  1509.59
  2     GMM        Y1        Y1      X1              1                  8257.41
  3     GMM        Y1        Y1      X1              X1                47956.08
  4     GMM        Y1        Y1      X2              1                  7136.27
  5     GMM        Y1        Y1      X2              X1                44494.70
  6     GMM        Y1        Y1      X2              X2               153135.59
  7     GMM        Y1        Y1      @PRED.Y1/@B1    1                 47957.10
  8     GMM        Y1        Y1      @PRED.Y1/@B1    X1               289178.68
  9     GMM        Y1        Y1      @PRED.Y1/@B1    X2               275074.36
 10     GMM        Y1        Y1      @PRED.Y1/@B1    @PRED.Y1/@B1    1789176.56
 11     GMM        Y1        Y1      @PRED.Y2/@B2    1                152885.91
 12     GMM        Y1        Y1      @PRED.Y2/@B2    X1               816886.49
 13     GMM        Y1        Y1      @PRED.Y2/@B2    X2              1121114.96
 14     GMM        Y1        Y1      @PRED.Y2/@B2    @PRED.Y1/@B1    4576643.57
 15     GMM        Y1        Y1      @PRED.Y2/@B2    @PRED.Y2/@B2   28818318.24
```

Figure 14.57. The First 15 Observations in the VDATA= Data Set

Output Data Sets

OUT= Data Set

For normalized form equations, the OUT= data set specified on the FIT statement contains residuals, actuals, and predicted values of the dependent variables computed from the parameter estimates. For general form equations, actual values of the endogenous variables are copied for the residual and predicted values.

The variables in the data set are as follows:

- BY variables
- RANGE variable
- ID variables
- _ESTYPE_, a character variable of length 8 identifying the estimation method: OLS, SUR, N2SLS, N3SLS, ITOLS, ITSUR, IT2SLS, IT3SLS, GMM, ITGMM, or FIML

- _TYPE_, a character variable of length 8 identifying the type of observation: RESIDUAL, PREDICT, or ACTUAL
- _WEIGHT_, the weight of the observation in the estimation. The _WEIGHT_ value is 0 if the observation was not used. It is equal to the product of the _WEIGHT_ model program variable and the variable named in the WEIGHT statement, if any, or 1 if weights were not used.
- the WEIGHT statement variable if used
- the model variables. The dependent variables for the normalized-form equations in the estimation contain residuals, actuals, or predicted values, depending on the _TYPE_ variable, whereas the model variables that are not associated with estimated equations always contain actual values from the input data set.
- any other variables named in the OUTVARS statement. These can be program variables computed by the model program, CONTROL variables, parameters, or special variables in the model program.

The following SAS statements are used to generate and print an OUT= data set:

```
proc model data=gmm2;
   exogenous x1 x2;
   parms a1 a2 b2 b1 2.5 c2 55 d1;
   inst b1 b2 c2 x1 x2;
   y1 = a1 * y2 + b1 * x1 * x1 + d1;
   y2 = a2 * y1 + b2 * x2 * x2 + c2 / x2 + d1;

   fit y1 y2 / 3sls gmm out=resid outall ;
run;

proc print data=resid(obs=20);
run;
```

The data set GMM2 was generated by the example in the preceding ESTDATA= section above. A partial listing of the RESID data set is shown in Figure 14.58.

```
Obs   _ESTYPE_    _TYPE_    _WEIGHT_      x1          x2       y1          y2

  1     3SLS     ACTUAL         1      1.00000    -1.7339   -3.05812    -23.071
  2     3SLS     PREDICT        1      1.00000    -1.7339   -0.36806    -19.351
  3     3SLS     RESIDUAL       1      1.00000    -1.7339   -2.69006     -3.720
  4     3SLS     ACTUAL         1      1.41421    -5.3046    0.59405     43.866
  5     3SLS     PREDICT        1      1.41421    -5.3046   -0.49148     45.588
  6     3SLS     RESIDUAL       1      1.41421    -5.3046    1.08553     -1.722
  7     3SLS     ACTUAL         1      1.73205    -5.2826    3.17651     51.563
  8     3SLS     PREDICT        1      1.73205    -5.2826   -0.48281     41.857
  9     3SLS     RESIDUAL       1      1.73205    -5.2826    3.65933      9.707
 10     3SLS     ACTUAL         1      2.00000    -0.6878    3.66208    -70.011
 11     3SLS     PREDICT        1      2.00000    -0.6878   -0.18592    -76.502
 12     3SLS     RESIDUAL       1      2.00000    -0.6878    3.84800      6.491
 13     3SLS     ACTUAL         1      2.23607    -7.0797    0.29210     99.177
 14     3SLS     PREDICT        1      2.23607    -7.0797   -0.53732     92.201
 15     3SLS     RESIDUAL       1      2.23607    -7.0797    0.82942      6.976
 16     3SLS     ACTUAL         1      2.44949    14.5284    1.86898    423.634
 17     3SLS     PREDICT        1      2.44949    14.5284   -1.23490    421.969
 18     3SLS     RESIDUAL       1      2.44949    14.5284    3.10388      1.665
 19     3SLS     ACTUAL         1      2.64575    -0.6968   -1.03003    -72.214
 20     3SLS     PREDICT        1      2.64575    -0.6968   -0.10353    -69.680
```

Figure 14.58. The OUT= Data Set

OUTEST= Data Set

The OUTEST= data set contains parameter estimates and, if requested, estimates of the covariance of the parameter estimates.

The variables in the data set are as follows:

- BY variables
- _NAME_, a character variable of length 8, blank for observations containing parameter estimates or a parameter name for observations containing covariances
- _TYPE_, a character variable of length 8 identifying the estimation method: OLS, SUR, N2SLS, N3SLS, ITOLS, ITSUR, IT2SLS, IT3SLS, GMM, ITGMM, or FIML
- the parameters estimated.

If the COVOUT option is specified, an additional observation is written for each row of the estimate of the covariance matrix of parameter estimates, with the _NAME_ values containing the parameter names for the rows. Parameter names longer than eight characters are truncated.

OUTPARMS= Data Set

The option OUTPARMS= writes all the parameter estimates to an output data set. This output data set contains one observation and is similar to the OUTEST= data set, but it contains all the parameters, is not associated with any FIT task, and contains no covariances. The OUTPARMS= option is used on the PROC MODEL statement, and the data set is written at the end, after any FIT or SOLVE steps have been performed.

OUTS= Data Set

The OUTS= SAS data set contains the estimate of the covariance matrix of the residuals across equations. This matrix is formed from the residuals that are computed using the parameter estimates.

The variables in the OUTS= data set are as follows:

- BY variables
- _NAME_, a character variable containing the name of the equation
- _TYPE_, a character variable of length 8 identifying the estimation method: OLS, SUR, N2SLS, N3SLS, ITOLS, ITSUR, IT2SLS, IT3SLS, GMM, ITGMM, or FIML
- variables with the names of the equations in the estimation.

Each observation contains a row of the covariance matrix. The data set is suitable for use with the SDATA= option on a subsequent FIT or SOLVE statement. (See "Tests on Parameters" in this chapter for an example of the SDATA= option.)

OUTSUSED= Data Set

The OUTSUSED= SAS data set contains the covariance matrix of the residuals across equations that is used to define the objective function. The form of the OUTSUSED= data set is the same as that for the OUTS= data set.

Note that OUTSUSED= is the same as OUTS= for the estimation methods that iterate the **S** matrix (ITOLS, IT2SLS, ITSUR, and IT3SLS). If the SDATA= option is specified in the FIT statement, OUTSUSED= is the same as the SDATA= matrix read in for the methods that do not iterate the **S** matrix (OLS, SUR, N2SLS, and N3SLS).

OUTV= Data Set

The OUTV= data set contains the estimate of the variance matrix, V. This matrix is formed from the instruments and the residuals that are computed using the parameter estimates obtained from the initial 2SLS estimation when GMM estimation is selected. If an estimation method other than GMM or ITGMM is requested and OUTV= is specified, a V matrix is created using computed estimates. In the case that a VDATA= data set is used, this becomes the OUTV= data set. For ITGMM, the OUTV= data set is the matrix formed from the instruments and the residuals computed using the final parameter estimates.

ODS Table Names

PROC MODEL assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in the following table. For more information on ODS, see Chapter 6, "Using the Output Delivery System."

Table 14.3. ODS Tables Produced in PROC MODEL

ODS Table Name	Description	Option
	ODS Tables Created by the FIT Statement	
AugGMMCovariance	Cross products matrix	GMM
ChowTest	Structural change test	CHOW=
CollinDiagnostics	Collinearity Diagnostics	
ConfInterval	Profile likelihood Confidence Intervals	PRL=
ConvCrit	Convergence criteria for estimation	default
ConvergenceStatus	Convergence status	default
CorrB	Correlations of parameters	COVB/CORRB
CorrResiduals	Correlations of residuals	CORRS/COVS
CovB	Covariance of parameters	COVB/CORRB
CovResiduals	Covariance of residuals	CORRS/COVS
Crossproducts	Cross products matrix	ITALL/ITPRINT
DatasetOptions	Data sets used	default
DetResidCov	Determinant of the Residuals	DETAILS
DWTest	Durbin Watson Test	DW=
Equations	Listing of equations to estimate	default
EstSummaryMiss	Model Summary Statistics for PAIRWISE	MISSING=
EstSummaryStats	Objective, Objective * N	default
GMMCovariance	Cross products matrix	GMM
Godfrey	Godfrey's Serial Correlation Test	GF=
HausmanTest	Hausman's test table	HAUSMAN
HeteroTest	Heteroscedasticity test tables	BREUSCH/PAGEN
InvXPXMat	X'X inverse for System	I
IterInfo	Iteration printing	ITALL/ITPRINT
LagLength	Model lag length	default
MinSummary	Number of parameters, estimation kind	default
MissingValues	Missing values generated by the program	default
ModSummary	Listing of all categorized variables	default
ModVars	Listing of Model variables and parameters	default
NormalityTest	Normality test table	NORMAL
ObsSummary	Identifies observations with errors	default
ObsUsed	Observations read, used, and missing.	default
ParameterEstimates	Parameter Estimates	default
ParmChange	Parameter Change Vector	
ResidSummary	Summary of the SSE, MSE for the equations	default
SizeInfo	Storage Requirement for estimation	DETAILS

Table 14.3. (continued)

ODS Table Name	Description	Option
TermEstimates	Nonlinear OLS and ITOLS Estimates	OLS/ITOLS
TestResults	Test statement table	
WgtVar	The name of the weight variable	
XPXMat	X'X for System	XPX
ODS Tables Created by the SOLVE Statement		
DatasetOptions	Data sets used	default
DescriptiveStatistics	Descriptive Statistics	STATS
FitStatistics	Fit statistics for simulation	STATS
LagLength	Model lag length	default
ModSummary	Listing of all categorized variables	default
ObsSummary	Simulation trace output	SOLVEPRINT
ObsUsed	Observations read, used, and missing.	default
SimulationSummary	Number of variables solved for	default
SolutionVarList	Solution Variable Lists	default
TheilRelStats	Theil Relative Change Error Statistics	THEIL
TheilStats	Theil Forecast Error Statistics	THEIL
ODS Tables Created by the FIT and SOLVE Statements		
AdjacencyMatrix	Adjacency Graph	GRAPH
BlockAnalysis	Block analysis	BLOCK
BlockStructure	Block structure	BLOCK
CodeDependency	Variable cross reference	LISTDEP
CodeList	Listing of programs statements	LISTCODE
CrossReference	Cross Reference Listing For Program	
DepStructure	Dependency Structure of the System	BLOCK
DerList	Derivative variables	LISTDER
FirstDerivatives	First derivative table	LISTDER
InterIntg	Integration Iteration Output	INTGPRINT
MemUsage	Memory usage statistics	MEMORYUSE
ParmReadIn	Parameter estimates read in	ESTDATA=
ProgList	Listing of Compiled Program Code	
RangeInfo	RANGE statement specification	
SortAdjacencyMatrix	Sorted adjacency Graph	GRAPH
TransitiveClosure	Transitive closure Graph	GRAPH

Simulation Details

The *solution* given the vector $\mathbf{k}$, of the following nonlinear system of equations is the vector $\mathbf{u}$ which satisfies this equation:

$$\mathbf{q}(\mathbf{u}, \mathbf{k}, \theta) = 0$$

A *simulation* is a set of solutions $\mathbf{u}_t$ for a specific sequence of vectors $\mathbf{k}_t$.

Model simulation can be performed to

- check how well the model predicts the actual values over the historical period
- investigate the sensitivity of the solution to changes in the input values or parameters
- examine the dynamic characteristics of the model
- check the stability of the simultaneous solution
- estimate the statistical distribution of the predicted values of the nonlinear model using Monte Carlo methods

By combining the various solution modes with different input data sets, model simulation can answer many different questions about the model. This section presents details of model simulation and solution.

Solution Modes

The following solution modes are commonly used:

- *Dynamic simultaneous forecast* mode is used for forecasting with the model. Collect the historical data on the model variables, the future assumptions of the exogenous variables, and any prior information on the future endogenous values, and combine them in a SAS data set. Use the FORECAST option on the SOLVE statement.
- *Dynamic simultaneous simulation* mode is often called *ex-post simulation, historical simulation*, or *ex-post forecasting*. Use the DYNAMIC option. This mode is the default.
- *Static simultaneous simulation* mode can be used to examine the within-period performance of the model without the complications of previous period errors. Use the STATIC option.
- *NAHEAD=n dynamic simultaneous simulation* mode can be used to see how well *n*-period-ahead forecasting would have performed over the historical period. Use the NAHEAD=*n* option.

The different solution modes are explained in detail in the following sections.

Dynamic and Static Simulations

In model simulation, either solved values or actual values from the data set can be used to supply lagged values of an endogenous variable. A *dynamic* solution refers to a solution obtained by using only solved values for the lagged values. Dynamic mode is used both for forecasting and for simulating the dynamic properties of the model.

A *static* solution refers to a solution obtained by using the actual values when available for the lagged endogenous values. Static mode is used to simulate the behavior of the model without the complication of previous period errors. Dynamic simulation is the default.

If you wish to use static values for lags only for the first *n* observations, and dynamic values thereafter, specify the START=*n* option. For example, if you want a dynamic simulation to start after observation twenty-four, specify START=24 on the SOLVE statement. If the model being simulated had a value lagged for four time periods, then this value would start using dynamic values when the simulation reached observation number 28.

n-Period-Ahead Forecasting

Suppose you want to regularly forecast 12 months ahead and produce a new forecast each month as more data becomes available. *n*-period-ahead forecasting allows you to test how well you would have done over time had you been using your model to forecast 1 year ahead.

To see how well a model predicts *n* time periods in the future, perform an *n*-period-ahead forecast on real data and compare the forecast values with the actual values.

n-period-ahead forecasting refers to using dynamic values for the lagged endogenous variables only for lags *1* through *n-1*. For example, 1-period-ahead forecasting, specified by the NAHEAD=1 option on the SOLVE statement, is the same as if a static solution had been requested. Specifying NAHEAD=2 produces a solution that uses dynamic values for lag one and static, actual, values for longer lags.

The following example is a 2-year-ahead dynamic simulation. The output is shown in Figure 14.59.

```
data yearly;
   input year x1 x2 x3 y1 y2 y3;
   datalines;
84 4 9  0  7  4  5
85 5 6  1  1  27  4
86 3 8  2  5  8  2
87 2 10 3  0  10 10
88 4 7  6  20 60 40
89 5 4  8  40 40 40
90 3 2  10 50 60 60
91 2 5  11 40 50 60
;
run;

proc model data=yearly outmodel=foo;
   endogenous y1 y2 y3;
   exogenous  x1 x2 x3;

   y1 = 2 + 3*x1 - 2*x2 + 4*x3;
   y2 = 4 + lag2( y3 ) + 2*y1 + x1;
   y3 = lag3( y1 ) + y2 - x2;

   solve y1 y2 y3 / nahead=2 out=c;
run;

proc print data=c;run;
```

```
                          The MODEL Procedure
          Dynamic Simultaneous 2-Periods-Ahead Forecasting Simulation

                            Data Set Options

                            DATA=    YEARLY
                            OUT=     C


                           Solution Summary

                   Variables Solved                3
                   Simulation Lag Length           3
                   Solution Method            NEWTON
                   CONVERGE=                    1E-8
                   Maximum CC                      0
                   Maximum Iterations              1
                   Total Iterations                8
                   Average Iterations              1


                         Observations Processed

                              Read      20
                              Lagged    12
                              Solved     8
                              First      5
                              Last       8


                   Variables Solved For    y1 y2 y3
```

Figure 14.59. NAHEAD Summary Report

```
Obs   _TYPE_     _MODE_    _LAG_   _ERRORS_   y1    y2    y3   x1   x2   x3

 1    PREDICT   SIMULATE     0         0       0    10     7    2   10    3
 2    PREDICT   SIMULATE     1         0      24    58    52    4    7    6
 3    PREDICT   SIMULATE     1         0      41   101   102    5    4    8
 4    PREDICT   SIMULATE     1         0      47   141   139    3    2   10
 5    PREDICT   SIMULATE     1         0      42   130   145    2    5   11
```

Figure 14.60. C Data Set

The proceding 2-year-ahead simulation can be emulated without using the NAHEAD= option by the following PROC MODEL statements:

```
proc model data=test model=foo;
  range year = 87 to 88;
  solve y1 y2 y3 / dynamic solveprint;
run;
```

```
     range year = 88 to 89;
     solve y1 y2 y3 / dynamic solveprint;
   run;

     range year = 89 to 90;
     solve y1 y2 y3 / dynamic solveprint;
   run;

     range year = 90 to 91;
     solve y1 y2 y3 / dynamic solveprint;
```

The totals shown under "Observations Processed" in Figure 14.59 are equal to the sum of the four individual runs.

Simulation and Forecasting

You can perform a simulation of your model or use the model to produce forecasts. *Simulation* refers to the determination of the endogenous or dependent variables as a function of the input values of the other variables, even when actual data for some of the solution variables are available in the input data set. The simulation mode is useful for verifying the fit of the model parameters. Simulation is selected by the SIMULATE option on the SOLVE statement. Simulation mode is the default.

In forecast mode, PROC MODEL solves only for those endogenous variables that are missing in the data set. The actual value of an endogenous variable is used as the solution value whenever nonmissing data for it are available in the input data set. Forecasting is selected by the FORECAST option on the SOLVE statement.

For example, an econometric forecasting model can contain an equation to predict future tax rates, but tax rates are usually set in advance by law. Thus, for the first year or so of the forecast, the predicted tax rate should really be exogenous. Or, you may want to use a prior forecast of a certain variable from a short-run forecasting model to provide the predicted values for the earlier periods of a longer-range forecast of a long-run model. A common situation in forecasting is when historical data needed to fill the initial lags of a dynamic model are available for some of the variables but have not yet been obtained for others. In this case, the forecast must start in the past to supply the missing initial lags. Clearly, you should use the actual data that are available for the lags. In all the preceding cases, the forecast should be produced by running the model in the FORECAST mode; simulating the model over the future periods would not be appropriate.

Monte Carlo Simulation

The accuracy of the forecasts produced by PROC MODEL depends on four sources of error (Pindyck 1981, 405-406):

- The system of equations contains an implicit random error term ϵ

  $$\mathbf{g}(\mathbf{y}, \mathbf{x}, \hat{\theta}) = \epsilon$$

  where **y**, **x**, **g**, $\hat{\theta}$, and ϵ are vector valued.
- The estimated values of the parameters, $\hat{\theta}$, are themselves random variables.

- The exogenous variables may have been forecast themselves and therefore may contain errors.
- The system of equations may be incorrectly specified; the model only approximates the process modeled.

The RANDOM= option is used to request Monte Carlo (or stochastic) simulations to generate confidence intervals for errors arising from the first two sources. The Monte Carlo simulations can be performed with ϵ, θ, or both vectors represented as random variables. The SEED= option is used to control the random number generator for the simulations. SEED=0 forces the random number generator to use the system clock as its seed value.

In Monte Carlo simulations, repeated simulations are performed on the model for random perturbations of the parameters and the additive error term. The random perturbations follow a multivariate normal distribution with expected value of 0 and covariance described by a covariance matrix of the parameter estimates in the case of θ, or a covariance matrix of the equation residuals for the case of ϵ. PROC MODEL can generate both covariance matrices or you can provide them.

The ESTDATA= option specifies a data set containing an estimate of the covariance matrix of the parameter estimates to use for computing perturbations of the parameters. The ESTDATA= data set is usually created by the FIT statement with the OUTEST= and OUTCOV options. When the ESTDATA= option is specified, the matrix read from the ESTDATA= data set is used to compute vectors of random shocks or perturbations for the parameters. These random perturbations are computed at the start of each repetition of the solution and added to the parameter values. The perturbed parameters are fixed throughout the solution range. If the covariance matrix of the parameter estimates is not provided, the parameters are not perturbed.

The SDATA= option specifies a data set containing the covariance matrix of the residuals to use for computing perturbations of the equations. The SDATA= data set is usually created by the FIT statement with the OUTS= option. When SDATA= is specified, the matrix read from the SDATA= data set is used to compute vectors of random shocks or perturbations for the equations. These random perturbations are computed at each observation. The simultaneous solution satisfies the model equations plus the random shocks. That is, the solution is not a perturbation of a simultaneous solution of the structural equations; rather, it is a simultaneous solution of the stochastic equations using the simulated errors. If the SDATA= option is not specified, the random shocks are not used.

The different random solutions are identified by the _REP_ variable in the OUT= data set. An unperturbed solution with _REP_=0 is also computed when the RANDOM= option is used. RANDOM=*n* produces *n*+1 solution observations for each input observation in the solution range. If the RANDOM= option is not specified, the SDATA= and ESTDATA= options are ignored, and no Monte Carlo simulation is performed.

PROC MODEL does not have an automatic way of modeling the exogenous variables as random variables for Monte Carlo simulation. If the exogenous variables have been forecast, the error bounds for these variables should be included in the error bounds generated for the endogenous variables. If the models for the exogenous variables are included in PROC MODEL, then the error bounds created from a Monte Carlo simulation will contain the uncertainty due to the exogenous variables.

Alternatively, if the distribution of the exogenous variables is known, the built-in random number generator functions can be used to perturb these variables appropriately for the Monte Carlo simulation. For example, if you knew the forecast of an exogenous variable, X, had a standard error of 5.2 and the error was normally distributed, then the following statements could be used to generate random values for X:

```
   x_new = x + 5.2 * rannor(456);
```

During a Monte Carlo simulation the random number generator functions produce one value at each observation. It is important to use a different seed value for all the random number generator functions in the model program; otherwise, the perturbations will be correlated. For the unperturbed solution, _REP_=0, the random number generator functions return 0.

PROC UNIVARIATE can be used to create confidence intervals for the simulation (see the Monte Carlo simulation example in the "Getting Started" section).

Quasi-Random Number Generators

Traditionally high discrepancy psuedo-random number generators are used to generate innovations in Monte Carlo simulations. Loosely translated, a high discrepancy psuedo-random number generator is one in which there is very little correlation between the current number generated and the past numbers generated. This property is ideal if indeed independance of the innovations is required. If, on the other hand, the efficient spanning of a multi-dimensional space is desired, a low discrepancy, quasi-random number generator can be used. A quasi-random number generator produces numbers which have no random component.

A simple one-dimensional quasi-random sequence is the van der Corput sequence. Given a prime number r ($r \geq 2$) any integer has a unique representation in terms of base r. A number in the interval [0,1) can be created by inverting the represention base power by base power. For example, consider r=3 and n=1. 1 in base 3 is

$$1_{10} = 1 \cdot 3^0 = 1_3$$

When the powers of 3 are inverted,

$$\phi(1) = \frac{1}{3}$$

Also 11 in base 3 is

$$11_{10} = 1 \cdot 3^2 + 2 \cdot 3^0 = 102_3$$

When the powers of 3 are inverted,

$$\phi(11) = \frac{1}{9} + 2 \cdot \frac{1}{3} = \frac{7}{9}$$

The first 10 numbers in this squence $\phi(1)\ldots\phi(10)$ are provided below

$$0, \frac{1}{3}, \frac{2}{3}, \frac{1}{9}, \frac{4}{9}, \frac{7}{9}, \frac{2}{9}, \frac{5}{9}, \frac{8}{9}, \frac{1}{27}$$

As the sequence proceeds it fills in the gaps in a uniform fashion.

Several authors have expanded this idea to many dimensions. Two versions supported by the MODEL procedure are the Sobol sequence (QUASI=SOBOL) and the Faure sequence (QUASI=FAURE). The Sobol sequence is based on binary numbers an is generally computationaly faster than the Faure sequence. The Faure sequence uses the dimensionality of the problem to determine the number base to use to generate the sequence. The Faure sequence has better distributional properties than the Sobol sequence for dimensions greater than 8.

As an example of the difference between a pseudo random number and a quasi random number consider simulating a bivariate normal with 100 draws.

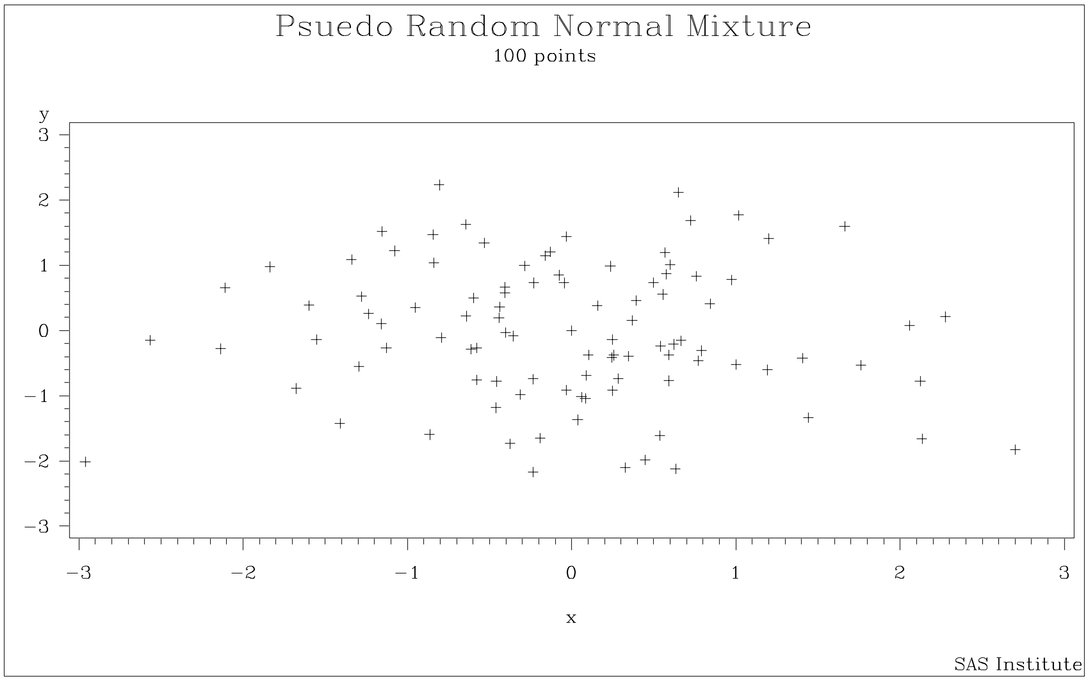


Figure 14.61. A Bivariate Normal using 100 pseudo random draws

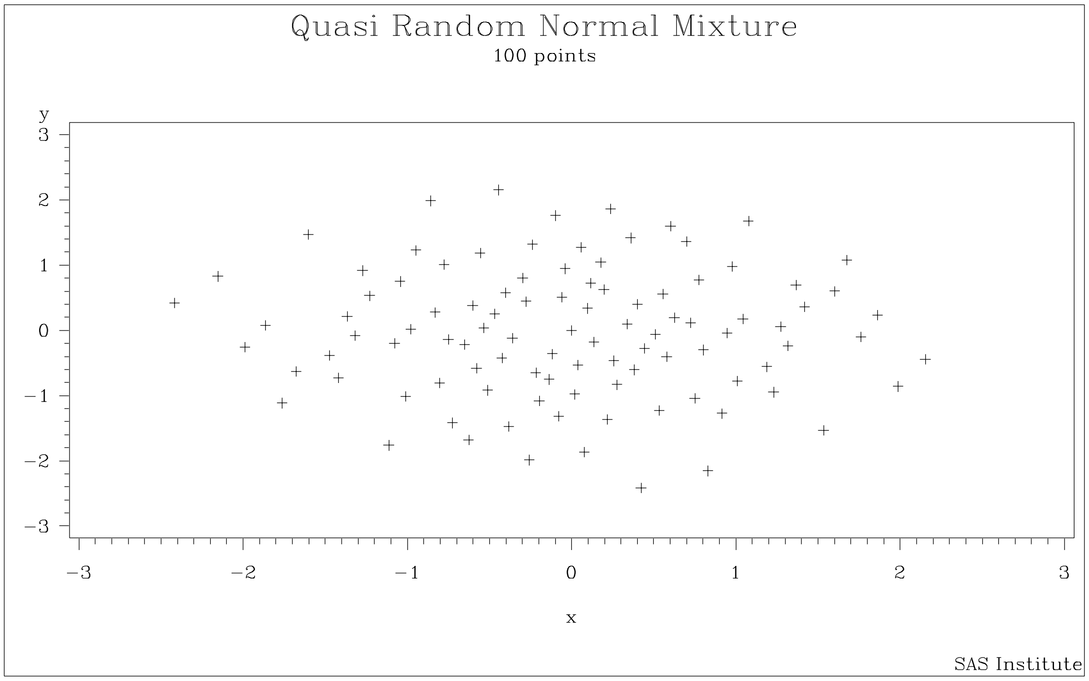


Figure 14.62. A Bivariate Normal using 100 Faure random draws

Solution Mode Output

The following SAS statements dynamically forecast the solution to a nonlinear equation:

```
proc model data=sashelp.citimon;
   parameters a 0.010708  b  -0.478849 c 0.929304;
   lhur = 1/(a * ip) + b + c * lag(lhur);
   solve lhur / out=sim forecast dynamic;
run;
```

The first page of output produced by the SOLVE step is shown in Figure 14.63. This is the summary description of the model. The error message states that the simulation was aborted at observation 144 because of missing input values.

```
                              The MODEL Procedure

                                 Model Summary

                           Model Variables         1
                           Parameters              3
                           Equations               1
                           Number of Statements    1
                           Program Lag Length      1


             Model Variables  LHUR
                  Parameters  a(0.010708) b(-0.478849) c(0.929304)
                   Equations  LHUR



                              The MODEL Procedure
                        Dynamic Single-Equation Forecast

ERROR: Solution values are missing because of missing input values for
       observation 144 at NEWTON iteration 0.
NOTE: Additional information on the values of the variables at this
      observation, which may be helpful in determining the cause of the failure
      of the solution process, is printed below.
Iteration Errors - Missing.
NOTE: Simulation aborted.
```

Figure 14.63. Solve Step Summary Output

The second page of output, shown in Figure 14.64, gives more information on the failed observation.

```
                              The MODEL Procedure
                        Dynamic Single-Equation Forecast

ERROR: Solution values are missing because of missing input values for
       observation 144 at NEWTON iteration 0.
NOTE: Additional information on the values of the variables at this
      observation, which may be helpful in determining the cause of the failure
      of the solution process, is printed below.


            Observation    144    Iteration    0    CC    -1.000000
                                  Missing      1
Iteration Errors - Missing.


                     --- Listing of Program Data Vector ---
_N_:                144     ACTUAL.LHUR:          .     ERROR.LHUR:           .
IP:                   .     LHUR:           7.10000     PRED.LHUR:            .
RESID.LHUR:           .     a:              0.01071     b:             -0.47885
c:              0.92930

NOTE: Simulation aborted.
```

Figure 14.64. Solve Step Error Message

From the program data vector you can see the variable IP is missing for observation 144. LHUR could not be computed so the simulation aborted.

The solution summary table is shown in Figure 14.65.

```
                            The MODEL Procedure
                      Dynamic Single-Equation Forecast

                              Data Set Options

                          DATA=    SASHELP.CITIMON
                          OUT=     SIM


                             Solution Summary

                      Variables Solved              1
                      Forecast Lag Length           1
                      Solution Method          NEWTON
                      CONVERGE=                  1E-8
                      Maximum CC                    0
                      Maximum Iterations            1
                      Total Iterations            143
                      Average Iterations            1


                           Observations Processed

                               Read      145
                               Lagged      1
                               Solved    143
                               First       2
                               Last      145
                               Failed      1


                       Variables Solved For    LHUR
```

Figure 14.65. Solution Summary Report

This solution summary table includes the names of the input data set and the output data set followed by a description of the model. The table also indicates the solution method defaulted to Newton's method. The remaining output is defined as follows.

Maximum CC	is the maximum convergence value accepted by the Newton procedure. This number is always less than the value for "CONVERGE=."
Maximum Iterations	is the maximum number of Newton iterations performed at each observation and each replication of Monte Carlo simulations.
Total Iterations	is the sum of the number of iterations required for each observation and each Monte Carlo simulation.
Average Iterations	is the average number of Newton iterations required to solve the system at each step.
Solved	is the number of observations used times the number of random replications selected plus one, for Monte Carlo simulations. The one additional simulation is the original unperturbed solution. For simulations not involving Monte Carlo, this number is the number of observations used.

Summary Statistics

The STATS and THEIL options are used to select goodness of fit statistics. Actual values must be provided in the input data set for these statistics to be printed. When the RANDOM= option is specified, the statistics do not include the unperturbed (_REP_=0) solution.

STATS Option Output

If the STATS and THEIL options are added to the model in the previous section

```
proc model data=sashelp.citimon;
   parameters a 0.010708  b  -0.478849 c 0.929304;
   lhur= 1/(a * ip) + b + c * lag(lhur) ;
   solve lhur / out=sim dynamic stats theil;
   range date to ’01nov91’d;
run;
```

the STATS output in Figure 14.66 and the THEIL output in Figure 14.67 are generated.

```
                              The MODEL Procedure
                       Dynamic Single-Equation Simulation

                    Solution Range DATE = FEB1980 To NOV1991

                            Descriptive Statistics

                                          Actual              Predicted
  Variable       N Obs       N        Mean     Std Dev       Mean    Std Dev

  LHUR             142     142      7.0887      1.4509     7.2473     1.1465


                               Statistics of fit

                         Mean    Mean %  Mean Abs  Mean Abs       RMS     RMS %
Variable          N     Error     Error     Error   % Error     Error     Error

LHUR            142    0.1585    3.5289    0.6937   10.0001    0.7854   11.2452

                               Statistics of fit

                 Variable      R-Square    Label

                 LHUR            0.7049    UNEMPLOYMENT RATE:
                                           ALL WORKERS,
                                           16 YEARS
```

Figure 14.66. STATS Output

The number of observations (Nobs), the number of observations with both predicted and actual values nonmissing (N), and the mean and standard deviation of the actual and predicted values of the determined variables are printed first. The next set of columns in the output are defined as follows.

Mean Error	$\frac{1}{N}\sum_{j=1}^{N}(\hat{y}_j - y_j)$
Mean % Error	$\frac{100}{N}\sum_{j=1}^{N}(\hat{y}_j - y_j)/y_j$
Mean Abs Error	$\frac{1}{N}\sum_{j=1}^{N}\lvert\hat{y}_j - y_j\rvert$
Mean Abs % Error	$\frac{100}{N}\sum_{j=1}^{N}\lvert(\hat{y}_j - y_j)/y_j\rvert$
RMS Error	$\sqrt{\frac{1}{N}\sum_{j=1}^{N}(\hat{y}_j - y_j)^2}$
RMS % Error	$100\sqrt{\frac{1}{N}\sum_{j=1}^{N}((\hat{y}_j - y_j)/y_j)^2}$
R-square	$1 - SSE/CSSA$
SSE	$\sum_{j=1}^{N}(\hat{y}_j - y_j)^2$
SSA	$\sum_{j=1}^{N}(y_j)^2$
CSSA	$SSA - \left(\sum_{j=1}^{N} y_j\right)^2$
$\hat{y}$	predicted value
y	actual value

When the RANDOM= option is specified, the statistics do not include the unperturbed (_REP_=0) solution.

THEIL Option Output

The THEIL option specifies that Theil forecast error statistics be computed for the actual and predicted values and for the relative changes from lagged values. Mathematically, the quantities are

$$\hat{y}c = (\hat{y} - lag(y))/lag(y)$$

$$yc = (y - lag(y))/lag(y)$$

where $\hat{y}c$ is the relative change for the predicted value and *yc* is the relative change for the actual value.

```
                              The MODEL Procedure
                       Dynamic Single-Equation Simulation

                    Solution Range DATE = FEB1980 To NOV1991

                        Theil Forecast Error Statistics

                                             MSE Decomposition Proportions
                                  Corr    Bias     Reg    Dist     Var   Covar
Variable          N       MSE      (R)    (UM)    (UR)    (UD)    (US)    (UC)

LHUR          142.0    0.6168     0.85    0.04    0.01    0.95    0.15    0.81

                        Theil Forecast Error Statistics

                            Inequality Coef
          Variable             U1           U    Label

          LHUR             0.1086      0.0539    UNEMPLOYMENT RATE:
                                                 ALL WORKERS,
                                                 16 YEARS


                Theil Relative Change Forecast Error Statistics

                   Relative Change           MSE Decomposition Proportions
                                  Corr    Bias     Reg    Dist     Var   Covar
Variable          N       MSE      (R)    (UM)    (UR)    (UD)    (US)    (UC)

LHUR          142.0    0.0126    -0.08    0.09    0.85    0.06    0.43    0.47

                Theil Relative Change Forecast Error Statistics

                            Inequality Coef
          Variable             U1           U    Label

          LHUR             4.1226      0.8348    UNEMPLOYMENT RATE:
                                                 ALL WORKERS,
                                                 16 YEARS
```

Figure 14.67. THEIL Output

The columns have the following meaning:

Corr (R) is the correlation coefficient, ρ, between the actual and predicted values.

$$\rho = \frac{\mathrm{cov}(\mathrm{y}, \hat{\mathrm{y}})}{\sigma_{\mathrm{a}}\sigma_{\mathrm{p}}}$$

where σ_p and σ_a are the standard deviations of the predicted and actual values.

Bias (UM) is an indication of systematic error and measures the extent to which the average values of the actual and predicted deviate from each other.

$$\frac{(\mathrm{E}(\mathrm{y}) - \mathrm{E}(\hat{\mathrm{y}}))^2}{\frac{1}{\mathrm{N}}\sum_{\mathrm{t}=1}^{\mathrm{N}} (\mathrm{y_t} - \hat{\mathrm{y}}_\mathrm{t})^2}$$

Reg (UR)	is defined as $(\sigma_p - \rho * \sigma_a)^2/MSE$. Consider the regression $$y = \alpha + \beta\hat{y}$$ If $\hat{\beta} = 1$, UR will equal zero.
Dist (UD)	is defined as $(1-\rho^2)\sigma_a\sigma_a/MSE$ and represents the variance of the residuals obtained by regressing *yc* on $\hat{y}c$.
Var (US)	is the variance proportion. US indicates the ability of the model to replicate the degree of variability in the endogenous variable. $$US = \frac{(\sigma_p - \sigma_a)^2}{MSE}$$
Covar (UC)	represents the remaining error after deviations from average values and average variabilities have been accounted for. $$UC = \frac{2(1-\rho)\sigma_p\sigma_a}{MSE}$$
U1	is a statistic measuring the accuracy of a forecast. $$U1 = \frac{MSE}{\sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t)^2}}$$
U	is the Theil's inequality coefficient defined as follows: $$U = \frac{MSE}{\sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t)^2} + \sqrt{\frac{1}{N}\sum_{t=1}^{N}(\hat{y}_t)^2}}$$
MSE	is the mean square error $$MSE = \frac{1}{N}\sum_{t=1}^{N}(\hat{y}c - yc)^2$$

More information on these statistics can be found in the references Maddala (1977, 344–347) and Pindyck and Rubinfeld (1981, 364–365).

Goal Seeking: Solving for Right-Hand-Side Variables

The process of computing input values needed to produce target results is often called *goal seeking*. To compute a goal-seeking solution, use a SOLVE statement that lists the variables you want to solve for and provide a data set containing values for the remaining variables.

Consider the following demand model for packaged rice

$$quantity\ demanded = \alpha_1 + \alpha_2 price^{2/3} + \alpha_3 income$$

where *price* is the price of the package and *income* is disposable personal income. The only variable the company has control over is the price it charges for rice. This model is estimated using the following simulated data and PROC MODEL statements:

```
data demand;
   do t=1 to 40;
       price = (rannor(10) +5) * 10;
       income = 8000 * t ** (1/8);
       demand = 7200 - 1054 * price ** (2/3) +
                7 * income + 100 * rannor(1);
       output;
   end;
run;

data goal;
    demand = 85000;
    income = 12686;
run;
```

The goal is to find the price the company would have to charge to meet a sales target of 85,000 units. To do this, a data set is created with a DEMAND variable set to 85000 and with an INCOME variable set to 12686, the last income value.

```
proc model data=demand ;
   demand = a1 - a2 * price ** (2/3) + a3 * income;
   fit demand / outest=demest;
run;
```

The desired price is then determined using the following PROC MODEL statement:

```
    solve price / estdata=demest data=goal solveprint;
run;
```

The SOLVEPRINT option prints the solution values, number of iterations, and final residuals at each observation. The SOLVEPRINT output from this solve is shown in Figure 14.68.

```
                              The MODEL Procedure
                           Single-Equation Simulation

 Observation   1   Iterations   6   CC    0.000000   ERROR.demand    0.000000


                                Solution Values

                                     price

                                  33.59016
```

Figure 14.68. Goal Seeking, SOLVEPRINT Output

The output indicates that it took 6 Newton iterations to determine the PRICE of 33.5902, which makes the DEMAND value within 16E-11 of the goal of 85,000 units.

Consider a more ambitious goal of 100,000 units. The output shown in Figure 14.69 indicates that the sales target of 100,000 units is not attainable according to this model.

```
                                The MODEL Procedure
                             Single-Equation Simulation

NOTE: 3 parameter estimates were read from the ESTDATA=DEMEST data set.



                                The MODEL Procedure
                             Single-Equation Simulation

ERROR: Could not reduce norm of residuals in 10 subiterations.

ERROR: The solution failed because 1 equations are missing or have extreme
       values for observation 1 at NEWTON iteration 1.
NOTE: Additional information on the values of the variables at this
      observation, which may be helpful in determining the cause of the failure
      of the solution process, is printed below.


             Observation    1    Iteration    1    CC    -1.000000
                                 Missing      1
Iteration Errors - Missing.


             Observation    1    Iteration    1    CC    -1.000000
                                 Missing      1
ERROR: 2 execution errors for this observation
NOTE: Check for missing input data or uninitialized lags.
      (Note that the LAG and DIF functions return missing values for the
initial lag starting observations. This is a change from the 1982 and earlier
versions of SAS/ETS which returned zero for uninitialized lags.)
NOTE: Simulation aborted.
```

Figure 14.69. Goal Seeking, Convergence Failure

The program data vector indicates that even with PRICE nearly 0 (4.462312E-22) the demand is still 4,164 less than the goal. You may need to reformulate your model or collect more data to more accurately reflect the market response.

Numerical Solution Methods

If the SINGLE option is not used, PROC MODEL computes values that simultaneously satisfy the model equations for the variables named in the SOLVE statement. PROC MODEL provides three iterative methods, Newton, Jacobi, and Seidel, for computing a simultaneous solution of the system of nonlinear equations.

Single-Equation Solution

For normalized-form equation systems, the solution can either simultaneously satisfy all the equations or can be computed for each equation separately, using the actual values of the solution variables in the current period to compute each predicted value. By default, PROC MODEL computes a simultaneous solution. The SINGLE option on the SOLVE statement selects single-equation solutions.

Single-equation simulations are often made to produce residuals (which estimate the random terms of the stochastic equations) rather than the predicted values themselves.

If the input data and range are the same as that used for parameter estimation, a static single-equation simulation will reproduce the residuals of the estimation.

Newton's Method

The NEWTON option on the SOLVE statement requests Newton's method to simultaneously solve the equations for each observation. Newton's method is the default solution method. Newton's method is an iterative scheme that uses the derivatives of the equations with respect to the solution variables, *J*, to compute a change vector as

$$\Delta \mathbf{y}^i = J^{-1}\mathbf{q}(\mathbf{y}^i, \mathbf{x}, \theta)$$

PROC MODEL builds and solves *J* using efficient sparse matrix techniques. The solution variables $\mathbf{y}^i$ at the *i*th iteration are then updated as

$$\mathbf{y}^{i+1} = \mathbf{y}^i + d \times \Delta \mathbf{y}^i$$

d is a damping factor between 0 and 1 chosen iteratively so that

$$\|\mathbf{q}(\mathbf{y}^{i+1}, \mathbf{x}, \theta)\| < \|\mathbf{q}(\mathbf{y}^i, \mathbf{x}, \theta)\|$$

The number of subiterations allowed for finding a suitable *d* is controlled by the MAXSUBITER= option. The number of iterations of Newton's method allowed for each observation is controlled by MAXITER= option. Refer to Ortega and Rheinbolt (1970) for more details.

Jacobi Method

The JACOBI option on the SOLVE statement selects a matrix-free alternative to Newton's method. This method is the traditional nonlinear Jacobi method found in the literature. The Jacobi method as implemented in PROC MODEL substitutes predicted values for the endogenous variables and iterates until a fixed point is reached. Then necessary derivatives are computed only for the diagonal elements of the jacobian, **J**.

If the normalized-form equation is

$$\mathbf{y} = \mathbf{f}(\mathbf{y}, \mathbf{x}, \theta)$$

the Jacobi iteration has the form

$$\mathbf{y}^{i+1} = \mathbf{f}(\mathbf{y}^i, \mathbf{x}, \theta)$$

Seidel Method

The Seidel method is an order-dependent alternative to the Jacobi method. The Seidel method is selected by the SEIDEL option on the SOLVE statement and is applicable only to normalized-form equations. The Seidel method is like the Jacobi method except that in the Seidel method the model is further edited to substitute the predicted values into the solution variables immediately after they are computed. Seidel thus differs from the other methods in that the values of the solution variables are not fixed within an iteration. With the other methods, the order of the equations in the model program makes no difference, but the Seidel method may work much differently when the equations are specified in a different sequence. Note that this fixed point method is the traditional nonlinear Seidel method found in the literature.

The iteration has the form

$$\mathbf{y}_j^{i+1} = \mathbf{f}(\hat{\mathbf{y}}^i, \mathbf{x}, \theta)$$

where $\mathbf{y}_j^{i+1}$ is the *j*th equation variable at the *i*th iteration and

$$\hat{\mathbf{y}}^i = (y_1^{i+1}, y_2^{i+1}, y_3^{i+1}, \ldots, y_{j-1}^{i+1}, y_j^i, y_{j+1}^i, \ldots, y_g^i)'$$

If the model is recursive, and if the equations are in recursive order, the Seidel method will converge at once. If the model is block-recursive, the Seidel method may converge faster if the equations are grouped by block and the blocks are placed in block-recursive order. The BLOCK option can be used to determine the block-recursive form.

Comparison of Methods

Newton's method is the default and should work better than the others for most small- to medium-sized models. The Seidel method is always faster than the Jacobi for recursive models with equations in recursive order. For very large models and some highly nonlinear smaller models, the Jacobi or Seidel methods can sometimes be faster. Newton's method uses more memory than the Jacobi or Seidel methods.

Both the Newton's method and the Jacobi method are order-invariant in the sense that the order in which equations are specified in the model program has no effect on the operation of the iterative solution process. In order-invariant methods, the values of the solution variables are fixed for the entire execution of the model program. Assignments to model variables are automatically changed to assignments to corresponding equation variables. Only after the model program has completed execution are the results used to compute the new solution values for the next iteration.

Troubleshooting Problems

In solving a simultaneous nonlinear dynamic model you may encounter some of the following problems.

Missing Values

For SOLVE tasks, there can be no missing parameter values. If there are missing right-hand-side variables, this will result in a missing left-hand-side variable for that observation.

Unstable Solutions

A solution may exist but be unstable. An unstable system can cause the Jacobi and Seidel methods to diverge.

Explosive Dynamic Systems

A model may have well-behaved solutions at each observation but be dynamically unstable. The solution may oscillate wildly or grow rapidly with time.

Propagation of Errors

During the solution process, solution variables can take on values that cause computational errors. For example, a solution variable that appears in a LOG function may be positive at the solution but may be given a negative value during one of the iterations. When computational errors occur, missing values are generated and propagated, and the solution process may collapse.

Convergence Problems

The following items can cause convergence problems:

- illegal function values (that is $\sqrt{-1}$)
- local minima in the model equation
- no solution exists
- multiple solutions exist
- initial values too far from the solution
- the CONVERGE= value too small.

When PROC MODEL fails to find a solution to the system, the current iteration information and the program data vector are printed. The simulation halts if actual values are not available for the simulation to proceed. Consider the following program:

```
data test1;
   do t=1 to 50;
      x1 = sqrt(t) ;
      y = .;
      output;
   end;

proc model data=test1;
   exogenous x1 ;
   control a1 -1 b1 -29 c1 -4 ;
   y = a1 * sqrt(y) + b1 * x1 * x1 + c1 * lag(x1);
   solve y / out=sim forecast dynamic ;
run;
```

which produces the output shown in Figure 14.70.

```
                              The MODEL Procedure
                        Dynamic Single-Equation Forecast

ERROR: Could not reduce norm of residuals in 10 subiterations.

ERROR: The solution failed because 1 equations are missing or have extreme
       values for observation 1 at NEWTON iteration 1.
NOTE: Additional information on the values of the variables at this
      observation, which may be helpful in determining the cause of the failure
      of the solution process, is printed below.


             Observation    1    Iteration    1    CC    -1.000000
                                 Missing      1
Iteration Errors - Missing.


                     --- Listing of Program Data Vector ---
   _N_:               12     ACTUAL.x1:    1.41421     ACTUAL.y:           .
   ERROR.y:            .     PRED.y:             .     RESID.y:            .
   a1:                -1     b1:               -29     c1:                -4
   x1:           1.41421     y:           -0.00109
   @PRED.y/@y:           .   @ERROR.y/@y:          .



             Observation    1    Iteration    1    CC    -1.000000
                                 Missing      1
ERROR: 1 execution errors for this observation
NOTE: Check for missing input data or uninitialized lags.
      (Note that the LAG and DIF functions return missing values for the
initial lag starting observations. This is a change from the 1982 and earlier
versions of SAS/ETS which returned zero for uninitialized lags.)
NOTE: Simulation aborted.
```

Figure 14.70. SOLVE Convergence Problems

At the first observation the following equation is attempted to be solved:

$$y = -\sqrt{y} - 62$$

There is no solution to this problem. The iterative solution process got as close as it could to making Y negative while still being able to evaluate the model. This problem can be avoided in this case by altering the equation.

In other models, the problem of missing values can be avoided by either altering the data set to provide better starting values for the solution variables or by altering the equations.

You should be aware that, in general, a nonlinear system can have any number of solutions, and the solution found may not be the one that you want. When multiple solutions exist, the solution that is found is usually determined by the starting values for the iterations. If the value from the input data set for a solution variable is missing, the starting value for it is taken from the solution of the last period (if nonmissing) or else the solution estimate is started at 0.

Iteration Output

The iteration output, produced by the ITPRINT option, is useful in determining the cause of a convergence problem. The ITPRINT option forces the printing of the solution approximation and equation errors at each iteration for each observation. A portion of the ITPRINT output from the following statement is shown in Figure 14.71.

```
proc model data=test1;
   exogenous x1 ;
   control a1 -1 b1 -29 c1 -4 ;
   y = a1 * sqrt(abs(y)) + b1 * x1 * x1 + c1 * lag(x1);
   solve y / out=sim forecast dynamic itprint;
run;
```

For each iteration, the equation with the largest error is listed in parentheses after the Newton convergence criteria measure. From this output you can determine which equation or equations in the system are not converging well.

```
                              The MODEL Procedure
                        Dynamic Single-Equation Forecast

Observation    1    Iteration    0    CC    613961.39    ERROR.y    -62.01010


                               Predicted Values

                                          y

                                  0.0001000


                               Iteration Errors

                                          y

                                  -62.01010

Observation    1    Iteration    1    CC    50.902771    ERROR.y    -61.88684


                               Predicted Values

                                          y

                                  -1.215784


                               Iteration Errors

                                          y

                                  -61.88684

Observation    1    Iteration    2    CC     0.364806    ERROR.y    41.752112


                               Predicted Values

                                          y

                                  -114.4503


                               Iteration Errors

                                          y

                                   41.75211
```

Figure 14.71. SOLVE, ITPRINT Output

Numerical Integration

The differential equation system is numerically integrated to obtain a solution for the derivative variables at each data point. The integration is performed by evaluating the provided model at multiple points between each data point. The integration method used is a variable order, variable step-size backward difference scheme; for more detailed information, refer to Aiken (1985) and Byrne (1975). The step size or time step is chosen to satisfy a *local truncation error* requirement. The term *truncation error* comes from the fact that the integration scheme uses a truncated series expansion of the integrated function to do the integration. Because the series is truncated, the integration scheme is within the truncation error of the true value.

To further improve the accuracy of the integration, the total integration time is broken up into small intervals (time steps or step sizes), and the integration scheme is applied to those intervals. The integration at each time step uses the values computed at the previous time step so that the truncation error tends to accumulate. It is usually not possible to estimate the global error with much precision. The best that can be done is to monitor and to control the local truncation error, which is the truncation error committed at each time step relative to

$$d = \max_{0 \leq t \leq T} (\|y(t)\|_{\infty}, 1)$$

where $y(t)$ is the integrated variable. Furthermore, the $y(t)$s are dynamically scaled to within two orders of magnitude one to keep the error monitoring well behaved.

The local truncation error requirement defaults to $1.0E-9$. You can specify the LTEBOUND= option to modify that requirement. The LTEBOUND= option is a relative measure of accuracy, so a value smaller than $1.0E-10$ is usually not practical. A larger bound increases the speed of the simulation and estimation but decreases the accuracy of the results. If the LTEBOUND= option is set too small, the integrator is not able to take time steps small enough to satisfy the local truncation error requirement and still have enough machine precision to compute the results. Since the integrations are scaled to within $1.0E-2$ of one, the simulated values should be correct to at least seven decimal places.

There is a default minimum time step of $1.0E-14$. This minimum time step is controlled by the MINTIMESTEP= option and the machine epsilon. If the minimum time step is smaller than the machine epsilon times the final time value, the minimum time step is increased automatically.

For the points between each observation in the data set, the values for nonintegrated variables in the data set are obtained from a linear interpolation from the two closest points. Lagged variables can be used with integrations, but their values are discrete and are not interpolated between points. Lagging, therefore, can then be used to input step functions into the integration.

The derivatives necessary for estimation (the gradient with respect to the parameters) and goal seeking (the Jacobian) are computed by numerically integrating analytical derivatives. The accuracy of the derivatives is controlled by the same integration techniques mentioned previously.

Limitations

There are limitations to the types of differential equations that can be solved or estimated. One type is an explosive differential equation (finite escape velocity) for which the following differential equation is an example:

$$y^{'} = a{\times}y, \quad a > 0$$

If this differential equation is integrated too far in time, y exceeds the maximum value allowed on the computer, and the integration terminates.

Likewise, differential systems that are singular cannot be solved or estimated in general. For example, consider the following differential system:

$$\begin{aligned} x^{'} &= -y^{'} + 2x + 4y + \exp(\mathrm{t}) \\ y^{'} &= -x^{'} + y + \exp(4{*}\mathrm{t}) \end{aligned}$$

This system has an analytical solution, but an accurate numerical solution is very difficult to obtain. The reason is that $y^{'}$ and $x^{'}$ cannot be isolated on the left-hand side of the equation. If the equation is modified slightly to

$$\begin{aligned} x^{'} &= -y^{'} + 2x + 4y + \exp(\mathrm{t}) \\ y^{'} &= x^{'} + y + \exp(4\mathrm{t}) \end{aligned}$$

the system is nonsingular, but the integration process could still fail or be extremely slow. If the MODEL procedure encounters either system, a warning message is issued.

This system can be rewritten as the following recursive system,

$$\begin{aligned} x^{'} &= 0.5y + 0.5\exp(4t) + x + 1.5y - 0.5\exp(t) \\ y^{'} &= x^{'} + y + \exp(4\mathrm{t}) \end{aligned}$$

which can be estimated and simulated successfully with the MODEL procedure.

Petzold (1982) mentions a class of differential algebraic equations that, when integrated numerically, could produce incorrect or misleading results. An example of such a system mentioned in Petzold (1982) is

$$\begin{aligned} y_2^{'}(t) &= y_1(t) + g_1(t) \\ 0 &= y_2(t) + g_2(t) \end{aligned}$$

The analytical solution to this system depends on g and its derivatives at the current time only and not on its initial value or past history. You should avoid systems of this and other similar forms mentioned in Petzold (1982).

SOLVE Data Sets

SDATA= Input Data Set

The SDATA= option reads a cross-equation covariance matrix from a data set. The covariance matrix read from the SDATA= data set specified on the SOLVE statement is used to generate random equation errors when the RANDOM= option specifies Monte Carlo simulation.

Typically, the SDATA= data set is created by the OUTS= on a previous FIT statement. (The OUTS= data set from a FIT statement can be read back in by a SOLVE statement in the same PROC MODEL step.)

You can create an input SDATA= data set using the DATA step. PROC MODEL expects to find a character variable _NAME_ in the SDATA= data set as well as variables for the equations in the estimation or solution. For each observation with a _NAME_ value matching the name of an equation, PROC MODEL fills the corresponding row of the S matrix with the values of the names of equations found in the data set. If a row or column is omitted from the data set, an identity matrix row or column is assumed. Missing values are ignored. Since the S matrix is symmetric, you can include only a triangular part of the S matrix in the SDATA= data set with the omitted part indicated by missing values. If the SDATA= data set contains multiple observations with the same _NAME_, the last values supplied for the _NAME_ variable are used. The "OUTS= Data Set" section contains more details on the format of this data set.

Use the TYPE= option to specify the type of estimation method used to produce the S matrix you want to input.

ESTDATA= Input Data Set

The ESTDATA= option specifies an input data set that contains an observation with values for some or all of the model parameters. It can also contain observations with the rows of a covariance matrix for the parameters.

When the ESTDATA= option is used, parameter values are set from the first observation. If the RANDOM= option is used and the ESTDATA= data set contains a covariance matrix, the covariance matrix of the parameter estimates is read and used to generate pseudo-random shocks to the model parameters for Monte Carlo simulation. These random perturbations have a multivariate normal distribution with the covariance matrix read from the ESTDATA= data set.

The ESTDATA= data set is usually created by the OUTEST= option in a FIT statement. The OUTEST= data set contains the parameter estimates produced by the FIT statement and also contains the estimated covariance of the parameter estimates if the OUTCOV option is used. This OUTEST= data set can be read in by the ESTDATA= option in a SOLVE statement.

You can also create an ESTDATA= data set with a SAS DATA step program. The data set must contain a numeric variable for each parameter to be given a value or covariance column. The name of the variable in the ESTDATA= data set must match the name of the parameter in the model. Parameters with names longer than eight characters cannot be set from an ESTDATA= data set. The data set must also contain a character variable _NAME_ of length eight. _NAME_ has a blank value for the observation that gives values to the parameters. _NAME_ contains the name of a parameter for observations defining rows of the covariance matrix.

More than one set of parameter estimates and covariances can be stored in the ESTDATA= data set if the observations for the different estimates are identified by the variable _TYPE_. _TYPE_ must be a character variable of length eight. The TYPE= option is used to select for input the part of the ESTDATA= data set for which the value of the _TYPE_ variable matches the value of the TYPE= option.

OUT= Data Set

The OUT= data set contains solution values, residual values, and actual values of the solution variables.

The OUT= data set contains the following variables:

- BY variables
- RANGE variable
- ID variables
- _TYPE_, a character variable of length eight identifying the type of observation. The _TYPE_ variable can be PREDICT, RESIDUAL, ACTUAL, or ERROR.
- _MODE_, a character variable of length eight identifying the solution mode. _MODE_ takes the value FORECAST or SIMULATE.
- if lags are used, a numeric variable, _LAG_, containing the number of dynamic lags that contribute to the solution. The value of _LAG_ is always zero for STATIC mode solutions. _LAG_ is set to a missing value for lag-starting observations.
- _REP_, a numeric variable containing the replication number, if the RANDOM= option is used. For example, if RANDOM=10, each input observation results in eleven output observations with _REP_ values 0 through 10. The observations with _REP_=0 are from the unperturbed solution. (The random-number generator functions are suppressed, and the parameter and endogenous perturbations are zero when _REP_=0.)
- _ERRORS_, a numeric variable containing the number of errors that occurred during the execution of the program for the last iteration for the observation. If the solution failed to converge, this is counted as one error, and the _ERRORS_ variable is made negative.

- solution and other variables. The solution variables contain solution or predicted values for _TYPE_=PREDICT observations, residuals for _TYPE_=RESIDUAL observations, or actual values for _TYPE_=ACTUAL observations. The other model variables, and any other variables read from the input data set, are always actual values from the input data set.
- any other variables named in the OUTVARS statement. These can be program variables computed by the model program, CONTROL variables, parameters, or special variables in the model program. Compound variable names longer than eight characters are truncated in the OUT= data set.

By default only the predicted values are written to the OUT= data set. The OUTRESID, OUTACTUAL, and OUTERROR options are used to add the residual, actual, and ERROR. values to the data set.

For examples of the OUT= data set, see Example 14.6 at the end of this chapter.

DATA= Input Data Set

The input data set should contain all of the exogenous variables and should supply nonmissing values for them for each period to be solved.

Solution variables can be supplied in the input data set and are used as follows:

- to supply initial lags. For example, if the lag length of the model is three, three observations are read in to feed the lags before any solutions are computed.
- to evaluate the goodness of fit. Goodness-of-fit measures are computed based on the difference between the solved values and the actual values supplied from the data set.
- to supply starting values for the iterative solution. If the value from the input data set for a solution variable is missing, the starting value for it is taken from the solution of the last period (if nonmissing) or else the solution estimate is started at zero.
- For STATIC mode solutions, actual values from the data set are used by the lagging functions for the solution variables.
- for FORECAST mode solutions, actual values from the data set are used as the solution values when nonmissing.

Programming Language Overview

Variables in the Model Program

Variable names are alphanumeric but must start with a letter. The length of a variable name is limited to thirty-two characters for non-SAS data set variables

PROC MODEL uses several classes of variables, and different variable classes are treated differently. Variable class is controlled by *declaration statements*. These are the VAR, ENDOGENOUS, and EXOGENOUS statements for model variables, the PARAMETERS statement for parameters, and the CONTROL statement for control class variables. These declaration statements have several valid abbreviations. Various *internal variables* are also made available to the model program to allow communication between the model program and the procedure. RANGE, ID, and BY variables are also available to the model program. Those variables not declared as any of the preceding classes are *program variables*.

Some classes of variables can be lagged; that is, their value at each observation is remembered, and previous values can be referred to by the lagging functions. Other classes have only a single value and are not affected by lagging functions. For example, parameters have only one value and are not affected by lagging functions; therefore, if P is a parameter, DIF*n*(P) is always 0, and LAG*n*(P) is always the same as P for all values of *n*.

The different variable classes and their roles in the model are described in the following.

Model Variables

Model variables are declared by VAR, ENDOGENOUS, or EXOGENOUS statements, or by FIT and SOLVE statements. The model variables are the variables that the model is intended to explain or predict.

PROC MODEL allows you to use expressions on the left-hand side of the equal sign to define model equations. For example, a log linear model for Y can now be written as

```
log( y ) = a + b * x;
```

Previously, only a variable name was allowed on the left-hand side of the equal sign.

The text on the left hand side of the equation serves as the equation name used to identify the equation in printed output, in the OUT= data sets, and in FIT or SOLVE statements. To refer to equations specified using left-hand side expressions (on the FIT statement, for example), place the left-hand side expression in quotes. For example, the following statements fit a log linear model to the dependent variable Y:

```
proc model data=in;
   log( y ) = a + b * x;
   fit "log(y)";
run;
```

The estimation and simulation is performed by transforming the models into general form equations. No actual or predicted value is available for general form equations so no R^2 or adjusted R^2 will be computed.

Equation Variables

An equation variable is one of several special variables used by PROC MODEL to control the evaluation of model equations. An equation variable name consists of one of the prefixes EQ, RESID, ERROR, PRED, or ACTUAL, followed by a period and the name of a model equation.

Equation variable names can appear on parts of the PROC MODEL printed output, and they can be used in the model program. For example, RESID-prefixed variables can be used in LAG functions to define equations with moving-average error terms. See the "Autoregressive Moving-Average Error Processes" section earlier in this chapter for details.

The meaning of these prefixes is detailed in the "Equation Translations" section.

Parameters

Parameters are variables that have the same value for each observation. Parameters can be given values or can be estimated by fitting the model to data. During the SOLVE stage, parameters are treated as constants. If no estimation is performed, the SOLVE stage uses the initial value provided in either the ESTDATA= data set, the MODEL= file, or on the PARAMETER statement, as the value of the parameter.

The PARAMETERS statement declares the parameters of the model. Parameters are not lagged, and they cannot be changed by the model program.

Control Variables

Control variables supply constant values to the model program that can be used to control the model in various ways. The CONTROL statement declares control variables and specifies their values. A control variable is like a parameter except that it has a fixed value and is not estimated from the data.

Control variables are not reinitialized before each pass through the data and can thus be used to retain values between passes. You can use control variables to vary the program logic. Control variables are not affected by lagging functions.

For example, if you have two versions of an equation for a variable Y, you could put both versions in the model and, using a CONTROL statement to select one of them, produce two different solutions to explore the effect the choice of equation has on the model:

```
      select (case);
          when (1) y =  ...first version of equation... ;
          when (2) y =  ...second version of equation... ;
      end;
      control case 1;
      solve / out=case1;
   run;
      control case 2;
      solve / out=case2;
   run;
```

RANGE, ID, and BY Variables

The RANGE statement controls the range of observations in the input data set that is processed by PROC MODEL. The ID statement lists variables in the input data set that are used to identify observations on the printout and in the output data set. The BY statement can be used to make PROC MODEL perform a separate analysis for each BY group. The variable in the RANGE statement, the ID variables, and the BY variables are available for the model program to examine, but their values should not be changed by the program. The BY variables are not affected by lagging functions.

BY Processing Improvements

Prior to version 6.11, the BY processing in the SOLVE statement was performed only for the DATA= data set. The last values in the ESTDATA= and SDATA= data sets were used regardless of the existence of BY variables in those two data sets. This constraint is now removed. If the BY variables are identical in the DATA= data set and the ESTDATA= data set, then the two data sets are syncronized and the simulations are performed using the data and parameters for each BY group. This holds for BY variables in the SDATA= data set as well. If, at some point, the BY variables don't match, BY processing is abandoned in either the ESTDATA= data set or the SDATA= data set, whichever has the missing BY value. If the DATA= data set does not contain BY variables and the ESTDATA= data set or the SDATA= data set does, then BY processing is performed for the ESTDATA= data set and the SDATA= data set by reusing the data in the DATA= data set for each BY group.

Internal Variables

You can use several internal variables in the model program to communicate with the procedure. For example, if you wanted PROC MODEL to list the values of all the variables when more than 10 iterations are performed and the procedure is past the 20th observation, you can write

```
if _obs_ > 20 then if _iter_ > 10 then _list_ = 1;
```

Internal variables are not affected by lagging functions, and they cannot be changed by the model program except as noted. The following internal variables are available. The variables are all numeric except where noted.

ERRORS	a flag that is set to 0 at the start of program execution and is set to a nonzero value whenever an error occurs. The program can also set the _ERRORS_ variable.
ITER	the iteration number. For FIT tasks, the value of _ITER_ is negative for preliminary grid-search passes. The iterative phase of the estimation starts with iteration 0. After the estimates have converged, a final pass is made to collect statistics with _ITER_ set to a missing value. Note that at least one pass, and perhaps several subiteration passes as well, is made for each iteration. For SOLVE tasks, _ITER_ counts the iterations used to compute the simultaneous solution of the system.
LAG	the number of dynamic lags that contribute to the solution at the current observation. _LAG_ is always 0 for FIT tasks and for

	STATIC solutions. _LAG_ is set to a missing value during the lag starting phase.
LIST	list flag that is set to 0 at the start of program execution. The program can set _LIST_ to a nonzero value to request a listing of the values of all the variables in the program after the program has finished executing.
METHOD	is the solution method in use for SOLVE tasks. _METHOD_ is set to a blank value for FIT tasks. _METHOD_ is a character-valued variable. Values are NEWTON, JACOBI, SIEDEL, or ONEPASS.
MODE	takes the value ESTIMATE for FIT tasks and the value SIMULATE or FORECAST for SOLVE tasks. _MODE_ is a character-valued variable.
NMISS	the number of missing or otherwise unusable observations during the model estimation. For FIT tasks, _NMISS_ is initially set to 0; at the start of each iteration, _NMISS_ is set to the number of unusable observations for the previous iteration. For SOLVE tasks, _NMISS_ is set to a missing value.
NUSED	the number of nonmissing observations used in the estimation. For FIT tasks, PROC MODEL initially sets _NUSED_ to the number of parameters; at the start of each iteration, _NUSED_ is reset to the number of observations used in the previous iteration. For SOLVE tasks, _NUSED_ is set to a missing value.
OBS	counts the observations being processed. _OBS_ is negative or 0 for observations in the lag starting phase.
REP	the replication number for Monte Carlo simulation when the RANDOM= option is specified in the SOLVE statement. _REP_ is 0 when the RANDOM= option is not used and for FIT tasks. When _REP_=0, the random-number generator functions always return 0.
WEIGHT	the weight of the observation. For FIT tasks, _WEIGHT_ provides a weight for the observation in the estimation. _WEIGHT_ is initialized to 1.0 at the start of execution for FIT tasks. For SOLVE tasks, _WEIGHT_ is ignored.

Program Variables

Variables not in any of the other classes are called program variables. Program variables are used to hold intermediate results of calculations. Program variables are reinitialized to missing values before each observation is processed. Program variables can be lagged. The RETAIN statement can be used to give program variables initial values and enable them to keep their values between observations.

Character Variables

PROC MODEL supports both numeric and character variables. Character variables are not involved in the model specification but can be used to label observations, to write debugging messages, or for documentation purposes. All variables are numeric unless they are the following.

- character variables in a DATA= SAS data set
- program variables assigned a character value
- declared to be character by a LENGTH or ATTRIB statement.

Equation Translations

Equations written in normalized form are always automatically converted to general form equations. For example, when a normalized-form equation such as

```
y = a + b*x;
```

is encountered, it is translated into the equations

```
PRED.y = a + b*x;
RESID.y = PRED.y - ACTUAL.y;
ERROR.y = PRED.y - y;
```

If the same system is expressed as the following general-form equation, then this equation is used unchanged.

```
EQ.y = y -  a + b*x;
```

This makes it easy to solve for arbitrary variables and to modify the error terms for autoregressive or moving average models.

Use the LIST option to see how this transformation is performed. For example, the following statements produce the listing shown in Figure 14.72.

```
proc model data=line list;
    y = a1 + b1*x1 + c1*x2;
    fit y;
run;
```

```
                              The MODEL Procedure

                        Listing of Compiled Program Code
          Stmt    Line:Col      Statement as Parsed

             1    15820:39      PRED.y = a1 + b1 * x1 + c1 * x2;
             1    15820:39      RESID.y = PRED.y - ACTUAL.y;
             1    15820:39      ERROR.y = PRED.y - y;
```

Figure 14.72. LIST Output

PRED.Y is the predicted value of Y, and ACTUAL.Y is the value of Y in the data set. The predicted value minus the actual value, RESID.Y, is then the error term, ϵ, for the original Y equation. ACTUAL.Y and Y have the same value for parameter estimation. For solve tasks, ACTUAL.Y is still the value of Y in the data set but Y becomes the solved value; the value that satisfies PRED.Y - Y = 0.

The following are the equation variable definitions.

EQ.	The value of an EQ-prefixed equation variable (normally used to define a general-form equation) represents the failure of the equation to hold. When the EQ.*name* variable is 0, the *name* equation is satisfied.
RESID.	The RESID.*name* variables represent the stochastic parts of the equations and are used to define the objective function for the estimation process. A RESID.-prefixed equation variable is like an EQ-prefixed variable but makes it possible to use or transform the stochastic part of the equation. The RESID. equation is used in place of the ERROR. equation for model solutions if it has been reassigned or used in the equation.
ERROR.	An ERROR.*name* variable is like an EQ-prefixed variable, except that it is used only for model solution and does not affect parameter estimation.
PRED.	For a normalized-form equation (specified by assignment to a model variable), the PRED.*name* equation variable holds the predicted value, where *name* is the name of both the model variable and the corresponding equation. (PRED-prefixed variables are not created for general-form equations.)
ACTUAL.	For a normalized-form equation (specified by assignment to a model variable), the ACTUAL.*name* equation variable holds the value of the *name* model variable read from the input data set.
DERT.	The DERT.*name* variable defines a differential equation. Once defined, it may be used on the right-hand side of another equation.
H.	The H.*name* variable specifies the functional form for the variance of the named equation.
GMM_H.	This is created for H.*vars* and is the moment equation for the variance for GMM. This variable is used only for GMM.

```
GMM_H.name = RESID.name**2 - H.name;
```

MSE.	The MSE.*y* variable contains the value of the mean square error for *y* at each iteration. An MSE. variable is created for each dependent/endogenous variable in the model. These variables can be used to specify the lagged values in the estimation and simulation of GARCH type models.

```
demret = intercept ;
if ( _OBS_ = 1 ) then
   h.demret = arch0 + arch1 * mse.demret +
              garch1 * mse.demret;
else
  h.demret = arch0 +
             arch1 * zlag( resid.demret ** 2) +
             garch1 * zlag(h.demret) ;
```

NRESID. This is created for H.*vars* and is the normalized residual of the variable <*name*>. The formula is

```
NRESID.name = RESID.name/ sqrt(H.name);
```

The three equation variable prefixes, RESID., ERROR., and EQ. allow for control over the objective function for the FIT, the SOLVE, or both the FIT and the SOLVE stages. For FIT tasks, PROC MODEL looks first for a RESID.*name* variable for each equation. If defined, the RESID-prefixed equation variable is used to define the objective function for the parameter estimation process. Otherwise, PROC MODEL looks for an EQ-prefixed variable for the equation and uses it instead.

For SOLVE tasks, PROC MODEL looks first for an ERROR.*name* variable for each equation. If defined, the ERROR-prefixed equation variable is used for the solution process. Otherwise, PROC MODEL looks for an EQ-prefixed variable for the equation and uses it instead. To solve the simultaneous equation system, PROC MODEL computes values of the solution variables (the model variables being solved for) that make all of the ERROR.name and EQ.*name* variables close to 0.

Derivatives

Nonlinear modeling techniques require the calculation of derivatives of certain variables with respect to other variables. The MODEL procedure includes an analytic differentiator that determines the model derivatives and generates program code to compute these derivatives. When parameters are estimated, the MODEL procedure takes the derivatives of the equation with respect to the parameters. When the model is solved, Newton's method requires the derivatives of the equations with respect to the variables solved for.

PROC MODEL uses exact mathematical formulas for derivatives of non-user-defined functions. For other functions, numerical derivatives are computed and used.

The differentiator differentiates the entire model program, including conditional logic and flow of control statements. Delayed definitions, as when the LAG of a program variable is referred to before the variable is assigned a value, are also differentiated correctly.

The differentiator includes optimization features that produce efficient code for the calculation of derivatives. However, when flow of control statements such as GOTO statements are used, the optimization process is impeded, and less efficient code for derivatives may be produced. Optimization is also reduced by conditional statements, iterative DO loops, and multiple assignments to the same variable.

The table of derivatives is printed with the LISTDER option. The code generated for the computation of the derivatives is printed with the LISTCODE option.

Derivative Variables

When the differentiator needs to generate code to evaluate the expression for the derivative of a variable, the result is stored in a special derivative variable. Derivative variables are not created when the derivative expression reduces to a previously computed result, a variable, or a constant. The names of derivative variables, which may sometimes appear in the printed output, have the form *@obj/@wrt*, where *obj* is the variable whose derivative is being taken and *wrt* is the variable that the differentiation is with respect to. For example, the derivative variable for the derivative of *Y* with respect to *X* is named *@Y/@X*.

The derivative variables cannot be accessed or used as part of the model program.

Mathematical Functions

The following is a brief summary of SAS functions useful for defining models. Additional functions and details are in *SAS Language: Reference*. Information on creating new functions can be found in *SAS/TOOLKIT Software: Usage and Reference*, chapter 15, "Writing a SAS Function or Call Routine."

ABS(*x*)	the absolute value of *x*
ARCOS(*x*)	the arccosine in radians of *x*. *x* should be between -1 and 1.
ARSIN(*x*)	the arcsine in radians of *x*. *x* should be between -1 and 1.
ATAN(*x*)	the arctangent in radians of *x*
COS(*x*)	the cosine of *x*. *x* is in radians.
COSH(*x*)	the hyperbolic cosine of *x*
EXP(*x*)	e^x
LOG(*x*)	the natural logarithm of *x*
LOG10(*x*)	the log base ten of *x*
LOG2(*x*)	the log base two of *x*
SIN(*x*)	the sine of *x*. *x* is in radians.
SINH(*x*)	the hyperbolic sine of *x*
SQRT(*x*)	the square root of *x*
TAN(*x*)	the tangent of *x*. *x* is in radians and is not an odd multiple of $\pi/2$.
TANH(*x*)	the hyperbolic tangent of *x*

Random-Number Functions

The MODEL procedure provides several functions for generating random numbers for Monte Carlo simulation. These functions use the same generators as the corresponding SAS DATA step functions.

The following random-number functions are supported: RANBIN, RANCAU, RANEXP, RANGAM, RANNOR, RANPOI, RANTBL, RANTRI, and RANUNI. For more information, refer to *SAS Language: Reference*.

Each reference to a random-number function sets up a separate pseudo-random sequence. Note that this means that two calls to the same random function with the same seed produce identical results. This is different from the behavior of the random-number functions used in the SAS DATA step. For example, the statements

```
x=rannor(123);
y=rannor(123);
z=rannor(567);
```

produce identical values for X and Y, but Z is from an independent pseudo-random sequence.

For FIT tasks, all random-number functions always return 0. For SOLVE tasks, when Monte Carlo simulation is requested, a random-number function computes a new random number on the first iteration for an observation (if it is executed on that iteration) and returns that same value for all later iterations of that observation. When Monte Carlo simulation is not requested, random-number functions always return 0.

Functions Across Time

PROC MODEL provides four types of special built-in functions that refer to the values of variables and expressions in previous time periods. These functions have the form

LAG*n*(*i* , *x*)	returns the *i*th lag of *x*, where *n* is the maximum lag;
DIF*n*(*x*)	difference of *x* at lag *n*
ZLAG*n*(*i* , *x*)	returns the *i*th lag of *x*, where *n* is the maximum lag, with missing lags replaced with zero;
ZDIF*n*(*x*)	difference with lag length truncated and missing values converted to zero;
MOVAVGn(*x*)	the width of the moving average is *n*, and *x* is the variable or expression to compute the moving average of. Missing values of *x* are omitted in computing the average.

where *n* represents the number of periods, and *x* is any expression. The argument *i* is a variable or expression giving the lag length ($0 <= i <= n$), if the index value *i* is omitted, the maximum lag length *n* is used.

If you do not specify *n*, the number of periods is assumed to be one. For example, LAG(X) is the same as LAG1(X). No more than four digits can be used with a lagging function; that is, LAG9999 is the greatest LAG function, ZDIF9999 is the greatest ZDIF function, and so on.

The LAG functions get values from previous observations and make them available to the program. For example, LAG(X) returns the value of the variable X as it was computed in the execution of the program for the preceding observation. The expression LAG2(X+2*Y) returns the value of the expression X+2*Y, computed using the values of the variables X and Y that were computed by the execution of the program for the observation two periods ago.

The DIF functions return the difference between the current value of a variable or expression and the value of its LAG. For example, DIF2(X) is a short way of writing X-LAG2(X), and DIF15(SQRT(2*Z)) is a short way of writing SQRT(2*Z)-LAG15(SQRT(2*Z)).

The ZLAG and ZDIF functions are like the LAG and DIF functions, but they are not counted in the determination of the program lag length, and they replace missing values with 0s. The ZLAG function returns the lagged value if the lagged value is nonmissing, or 0 if the lagged value is missing. The ZDIF function returns the differenced value if the differenced value is nonmissing, or 0 if the value of the differenced value is missing. The ZLAG function is especially useful for models with ARMA error processes. See "Lag Logic", which follows for details.

Lag Logic

The LAG and DIF lagging functions in the MODEL procedure are different from the queuing functions with the same names in the DATA step. Lags are determined by the final values that are set for the program variables by the execution of the model program for the observation. This can have upsetting consequences for programs that take lags of program variables that are given different values at various places in the program, for example,

```
temp = x + w;
t    = lag( temp );
temp = q - r;
s    = lag( temp );
```

The expression LAG(TEMP) always refers to LAG(Q-R), never to LAG(X+W), since Q-R is the final value assigned to the variable TEMP by the model program. If LAG(X+W) is wanted for T, it should be computed as T=LAG(X+W) and not T=LAG(TEMP), as in the preceding example.

Care should also be exercised in using the DIF functions with program variables that may be reassigned later in the program. For example, the program

```
temp =  x ;
s    = dif( temp );
temp = 3 * y;
```

computes values for S equivalent to

```
s =  x  - lag( 3 * y );
```

Note that in the preceding examples, TEMP is a program variable, *not* a model variable. If it were a model variable, the assignments to it would be changed to assignments to a corresponding equation variable.

Note that whereas LAG1(LAG1(X)) is the same as LAG2(X), DIF1(DIF1(X)) is *not* the same as DIF2(X). The DIF2 function is the difference between the current period value at the point in the program where the function is executed and the final value at the end of execution two periods ago; DIF2 is not the second difference. In contrast, DIF1(DIF1(X)) is equal to DIF1(X)-LAG1(DIF1(X)), which equals X-2*LAG1(X)+LAG2(X), which is the second difference of X.

More information on the differences between PROC MODEL and the DATA step LAG and DIF functions is found in Chapter 2.

Lag Lengths

The lag length of the model program is the number of lags needed for any relevant equation. The program lag length controls the number of observations used to initialize the lags.

PROC MODEL keeps track of the use of lags in the model program and automatically determines the lag length of each equation and of the model as a whole. PROC MODEL sets the program lag length to the maximum number of lags needed to compute any equation to be estimated, solved, or needed to compute any instrument variable used.

In determining the lag length, the ZLAG and ZDIF functions are treated as always having a lag length of 0. For example, if Y is computed as

```
y = lag2( x + zdif3( temp ) );
```

then Y has a lag length of 2 (regardless of how TEMP is defined). If Y is computed as

```
y = zlag2( x + dif3( temp ) );
```

then Y has a lag length of 0.

This is so that ARMA errors can be specified without causing the loss of additional observations to the lag starting phase and so that recursive lag specifications, such as moving-average error terms, can be used. Recursive lags are not permitted unless the ZLAG or ZDIF functions are used to truncate the lag length. For example, the following statement produces an error message:

```
t = a + b * lag( t );
```

The program variable T depends recursively on its own lag, and the lag length of T is therefore undefined.

In the following equation RESID.Y depends on the predicted value for the Y equation but the predicted value for the Y equation depends on the LAG of RESID.Y, and, thus, the predicted value for the Y equation depends recursively on its own lag.

```
y = yhat + ma * lag( resid.y );
```

The lag length is infinite, and PROC MODEL prints an error message and stops. Since this kind of specification is allowed, the recursion must be truncated at some point. The ZLAG and ZDIF functions do this.

The following equation is legal and results in a lag length for the Y equation equal to the lag length of YHAT:

```
y = yhat + ma * zlag( resid.y );
```

Initially, the lags of RESID.Y are missing, and the ZLAG function replaces the missing residuals with 0s, their unconditional expected values.

The ZLAG0 function can be used to zero out the lag length of an expression. ZLAG0(*x*) returns the current period value of the expression *x*, if nonmissing, or else returns 0, and prevents the lag length of *x* from contributing to the lag length of the current statement.

Initializing Lags

At the start of each pass through the data set or BY group, the lag variables are set to missing values and an initialization is performed to fill the lags. During this phase, observations are read from the data set, and the model variables are given values from the data. If necessary, the model is executed to assign values to program variables that are used in lagging functions. The results for variables used in lag functions are saved. These observations are not included in the estimation or solution.

If, during the execution of the program for the lag starting phase, a lag function refers to lags that are missing, the lag function returns missing. Execution errors that occur while starting the lags are not reported unless requested. The modeling system automatically determines whether the program needs to be executed during the lag starting phase.

If L is the maximum lag length of any equation being fit or solved, then the first L observations are used to prime the lags. If a BY statement is used, the first L observations in the BY group are used to prime the lags. If a RANGE statement is used, the first L observations prior to the first observation requested in the RANGE statement are used to prime the lags. Therefore, there should be at least L observations in the data set.

Initial values for the lags of model variables can also be supplied in VAR, ENDOGENOUS, and EXOGENOUS statements. This feature provides initial lags of solution variables for dynamic solution when initial values for the solution variable are not available in the input data set. For example, the statement

```
   var x 2 3 y 4 5 z 1;
```

feeds the initial lags exactly like these values in an input data set:

Lag	X	Y	Z
2	3	5	.
1	2	4	1

If initial values for lags are available in the input data set and initial lag values are also given in a declaration statement, the values in the VAR, ENDOGENOUS, or EXOGENOUS statements take priority.

The RANGE statement is used to control the range of observations in the input data set that are processed by PROC MODEL. In the statement

```
   range date = ’01jan1924’d to ’01dec1943’d;
```

‘01jan1924’ specifies the starting period of the range, and ‘01dec1943’ specifies the ending period. The observations in the data set immediately prior to the start of the range are used to initialize the lags.

Language Differences

For the most part, PROC MODEL programming statements work the same as they do in the DATA step as documented in *SAS Language: Reference*. However, there are several differences that should be noted.

DO Statement Differences

The DO statement in PROC MODEL does not allow a character index variable. Thus, the following DO statement is not valid in PROC MODEL, although it is supported in the DATA step:

```
   do i = 'A', 'B', 'C';         /* invalid PROC MODEL code */
```

IF Statement Differences

The IF statement in PROC MODEL does not allow a character-valued condition. For example, the following IF statement is not supported by PROC MODEL:

```
   if 'this' then  statement;
```

Comparisons of character values are supported in IF statements, so the following IF statement is acceptable:

```
   if 'this' < 'that' then  statement};
```

PROC MODEL allows for embedded conditionals in expressions. For example the following two statements are equivalent:

```
   flag = if time = 1 or time = 2 then conc+30/5 + dose*time
                else if time > 5 then (0=1) else (patient * flag);

   if time = 1 or time = 2 then flag= conc+30/5 + dose*time;
          else if time > 5 then flag=(0=1); else flag=patient*flag;
```

Note that the ELSE operator only involves the first object or token after it so that the following assignments are not equivalent:

```
   total = if sum > 0 then sum else sum + reserve;
   total = if sum > 0 then sum else (sum + reserve);
```

The first assignment makes TOTAL always equal to SUM plus RESERVE.

PUT Statement Differences

The PUT statement, mostly used in PROC MODEL for program debugging, only supports some of the features of the DATA step PUT statement. It also has some new features that the DATA step PUT statement does not support.

The PROC MODEL PUT statement does not support line pointers, factored lists, iteration factors, overprinting, the _INFILE_ option, or the colon (:) format modifier.

The PROC MODEL PUT statement does support expressions but an expression must be enclosed in parentheses. For example, the following statement prints the square root of x:

```
put (sqrt(x));
```

Subscripted array names must be enclosed in parentheses. For example, the following statement prints the *i*th element of the array A:

```
put (a i);
```

However, the following statement is an error:

```
put a i;
```

The PROC MODEL PUT statement supports the print item _PDV_ to print a formatted listing of all the variables in the program. For example, the following statement prints a much more readable listing of the variables than does the _ALL_ print item:

```
put _pdv_;
```

To print all the elements of the array A, use the following statement:

```
put a;
```

To print all the elements of A with each value labeled by the name of the element variable, use the statement

```
put a=;
```

ABORT Statement Difference

In the MODEL procedure, the ABORT statement does not allow any arguments.

SELECT/WHEN/OTHERWISE Statement Differences

The WHEN and OTHERWISE statements allow more than one target statement. That is, DO groups are not necessary for multiple statement WHENs. For example in PROC MODEL, the following syntax is valid:

```
select;
   when(exp1)
      stmt1;
      stmt2;
   when(exp2)
      stmt3;
      stmt4;
end;
```

The ARRAY Statement

ARRAY *arrayname [{dimensions}] [$ [length]] [variables and constants];*

The ARRAY statement is used to associate a name with a list of variables and constants. The array name can then be used with subscripts in the model program to refer to the items in the list.

In PROC MODEL, the ARRAY statement does not support all the features of the DATA step ARRAY statement. Implicit indexing cannot be used; all array references must have explicit subscript expressions. Only exact array dimensions are allowed; lower-bound specifications are not supported. A maximum of six dimensions is allowed.

On the other hand, the ARRAY statement supported by PROC MODEL does allow both variables and constants to be used as array elements. You cannot make assignments to constant array elements. Both dimension specification and the list of elements are optional, but at least one must be supplied. When the list of elements is not given or fewer elements than the size of the array are listed, array variables are created by suffixing element numbers to the array name to complete the element list.

The following are valid PROC MODEL array statements:

```
array x[120];           /* array X of length 120           */
array q[2,2];           /* Two dimensional array Q         */
array b[4] va vb vc vd; /* B[2] = VB, B[4] = VD            */
array x x1-x30;         /* array X of length 30, X[7] = X7  */
array a[5] (1 2 3 4 5); /* array A initialized to 1,2,3,4,5 */
```

RETAIN Statement

RETAIN *variables initial-values ;*

The RETAIN statement causes a program variable to hold its value from a previous observation until the variable is reassigned. The RETAIN statement can be used to initialize program variables.

The RETAIN statement does not work for model variables, parameters, or control variables because the values of these variables are under the control of PROC MODEL and not programming statements. Use the PARMS and CONTROL statements to initialize parameters and control variables. Use the VAR, ENDOGENOUS, or EXOGENOUS statement to initialize model variables.

Storing Programs in Model Files

Models can be saved and recalled from SAS catalog files. SAS catalogs are special files that can store many kinds of data structures as separate units in one SAS file. Each separate unit is called an entry, and each entry has an entry type that identifies its structure to the SAS system.

In general, to save a model, use the OUTMODEL=*name* option on the PROC MODEL statement, where *name* is specified as *libref.catalog.entry*, *libref.entry*, or *entry*. The *libref*, *catalog*, and *entry* names must be valid SAS names no more than eight characters long. The *catalog* name is restricted to seven characters on the CMS operating system. If not given, the *catalog* name defaults to MODELS, and the *libref* defaults to WORK. The entry type is always MODEL. Thus, OUTMODEL=X writes the model to the file WORK.MODELS.X.MODEL.

The MODEL= option is used to read in a model. A list of model files can be specified in the MODEL= option, and a range of names with numeric suffixes can be given, as in MODEL=(MODEL1-MODEL10). When more than one model file is given, the list must be placed in parentheses, as in MODEL=(A B C), except in the case of a single name. If more than one model file is specified, the files are combined in the order listed in the MODEL= option.

When the MODEL= option is specified in the PROC MODEL statement and model definition statements are also given later in the PROC MODEL step, the model files are read in first, in the order listed, and the model program specified in the PROC MODEL step is appended after the model program read from the MODEL= files. The class assigned to a variable, when multiple model files are used, is the last declaration of that variable. For example, if Y1 was declared endogenous in the model file M1 and exogenous in the model file M2, the following statement will cause Y1 to be declared exogenous.

```
   proc model model=(m1 m2);
```

The INCLUDE statement can be used to append model code to the current model code. In contrast, when the MODEL= option is used on the RESET statement, the current model is deleted before the new model is read.

No model file is output by default if the PROC MODEL step performs any FIT or SOLVE tasks, or if the MODEL= option or the NOSTORE option is used. However, to ensure compatibility with previous versions of SAS/ETS software, when the PROC MODEL step does nothing but compile the model program, no input model file is read, and the NOSTORE option is not used, a model file is written. This model file is the default input file for a later PROC SYSNLIN or PROC SIMNLIN step. The default output model filename in this case is WORK.MODELS._MODEL_.MODEL.

If FIT statements are used to estimate model parameters, the parameter estimates written to the output model file are the estimates from the last estimation performed for each parameter.

Diagnostics and Debugging

PROC MODEL provides several features to aid in finding errors in the model program. These debugging features are not usually needed; most models can be developed without them.

The example model program that follows will be used in the following sections to illustrate the diagnostic and debugging capabilities. This example is the estimation of a segmented model.

```
*---------Fitting a Segmented Model using MODEL----*
|       |                                          |
|   y   | quadratic          plateau               |
|       | y=a+b*x+c*x*x      y=p                   |
|       |                     .................... |
|       |              .     :                     |
|       |          .         :                     |
|       |       .            :                     |
|       |     .              :                     |
|       |    .               :                     |
|       +-----------------------------------------X |
|                           x0                     |
|                                                  |
| continuity restriction: p=a+b*x0+c*x0**2         |
| smoothness restriction: 0=b+2*c*x0 so x0=-b/(2*c)|
*--------------------------------------------------*;
title 'QUADRATIC MODEL WITH PLATEAU';
data a;
   input y x @@;
   datalines;
.46 1  .47  2 .57  3 .61  4 .62  5 .68  6 .69  7
.78 8  .70  9 .74 10 .77 11 .78 12 .74 13 .80 13
.80 15 .78 16
;
proc model data=a;
parms a 0.45 b 0.5 c -0.0025;

x0 = -.5*b / c;      /* join point */
if x < x0 then       /* Quadratic part of model */
   y = a + b*x + c*x*x;
else                 /* Plateau part of model */
   y = a + b*x0 + c*x0*x0;

fit y;
run;
```

Program Listing

The LIST option produces a listing of the model program. The statements are printed one per line with the original line number and column position of the statement.

The program listing from the example program is shown in Figure 14.73.

```
                          QUADRATIC MODEL WITH PLATEAU

                              The MODEL Procedure

                        Listing of Compiled Program Code
          Stmt    Line:Col      Statement as Parsed

             1    15888:74      x0 = (-0.5 * b) / c;
             2    15888:96      if x < x0 then
             3    15888:124     PRED.y = a + b * x + c * x * x;
             3    15888:124     RESID.y = PRED.y - ACTUAL.y;
             3    15888:124     ERROR.y = PRED.y - y;
             4    15888:148     else
             5    15888:176     PRED.y = a + b * x0 + c * x0 * x0;
             5    15888:176     RESID.y = PRED.y - ACTUAL.y;
             5    15888:176     ERROR.y = PRED.y - y;
```

Figure 14.73. LIST Output for Segmented Model

The LIST option also shows the model translations that PROC MODEL performs. LIST output is useful for understanding the code generated by the %AR and the %MA macros.

Cross-Reference

The XREF option produces a cross-reference listing of the variables in the model program. The XREF listing is usually used in conjunction with the LIST option. The XREF listing does not include derivative (@-prefixed) variables. The XREF listing does not include generated assignments to equation variables, PRED, RESID, and ERROR-prefixed variables, unless the DETAILS option is used.

The cross-reference from the example program is shown in Figure 14.74.

```
                              The MODEL Procedure

                      Cross Reference Listing For Program
Symbol-----------    Kind     Type     References (statement)/(line):(col)

a                    Var      Num      Used: 3/15913:130 5/15913:182
b                    Var      Num      Used: 1/15913:82 3/15913:133 5/15913:185
c                    Var      Num      Used: 1/15913:85 3/15913:139 5/15913:192
x0                   Var      Num      Assigned: 1/15913:85
                                       Used: 2/15913:103 5/15913:185
                                       5/15913:192 5/15913:195
x                    Var      Num      Used: 2/15913:103 3/15913:133
                                       3/15913:139 3/15913:141
PRED.y               Var      Num      Assigned: 3/15913:136 5/15913:189
```

Figure 14.74. XREF Output for Segmented Model

Compiler Listing

The LISTCODE option lists the model code and derivatives tables produced by the compiler. This listing is useful only for debugging and should not normally be needed.

LISTCODE prints the operator and operands of each operation generated by the compiler for each model program statement. Many of the operands are temporary variables generated by the compiler and given names such as #temp1. When derivatives are taken, the code listing includes the operations generated for the derivatives calculations. The derivatives tables are also listed.

A LISTCODE option prints the transformed equations from the example shown in Figure 14.75 and Figure 14.76.

```
                              The MODEL Procedure

                        Listing of Compiled Program Code
          Stmt    Line:Col      Statement as Parsed

             1    16459:83      x0 = (-0.5 * b) / c;
             1    16459:83      @x0/@b = -0.5 / c;
             1    16459:83      @x0/@c = (0 - x0) / c;
             2    16459:105     if x < x0 then
             3    16459:133     PRED.y = a + b * x + c * x * x;
             3    16459:133     @PRED.y/@a = 1;
             3    16459:133     @PRED.y/@b = x;
             3    16459:133     @PRED.y/@c = x * x;
             3    16459:133     RESID.y = PRED.y - ACTUAL.y;
             3    16459:133     @RESID.y/@a = @PRED.y/@a;
             3    16459:133     @RESID.y/@b = @PRED.y/@b;
             3    16459:133     @RESID.y/@c = @PRED.y/@c;
             3    16459:133     ERROR.y = PRED.y - y;
             4    16459:157     else
             5    16459:185     PRED.y = a + b * x0 + c * x0 * x0;
             5    16459:185     @PRED.y/@a = 1;
             5    16459:185     @PRED.y/@b = x0 + b * @x0/@b + (c
                                * @x0/@b * x0 + c * x0 * @x0/@b);
             5    16459:185     @PRED.y/@c = b * @x0/@c + ((x0 + c
                                * @x0/@c) * x0 + c * x0 * @x0/@c);
             5    16459:185     RESID.y = PRED.y - ACTUAL.y;
             5    16459:185     @RESID.y/@a = @PRED.y/@a;
             5    16459:185     @RESID.y/@b = @PRED.y/@b;
             5    16459:185     @RESID.y/@c = @PRED.y/@c;
             5    16459:185     ERROR.y = PRED.y - y;
```

Figure 14.75. LISTCODE Output for Segmented Model - Statements as Parsed

```
                              The MODEL Procedure


    1 Stmt ASSIGN     line 5619 column
                      83. (1) arg=x0
                      argsave=x0
                      Source Text:          x0 = -.5*b / c;
        Oper *        at 5619:91 (30,0,2).  * : #temp1 <- -0.5 b
        Oper /        at 5619:94 (31,0,2).  / : x0 <- #temp1 c
        Oper eeocf    at 5619:94 (18,0,1).  eeocf : _DER_ <- _DER_
        Oper /        at 5619:94 (31,0,2).  / : @x0/@b <- -0.5 c
        Oper -        at 5619:94 (33,0,2).  - : @1dt1_2 <- 0 x0
        Oper /        at 5619:94 (31,0,2).  / : @x0/@c <- @1dt1_2 c

    2 Stmt IF         line 5619 column       ref.st=ASSIGN stmt
                      105. (2) arg=#temp1   number 5 at 5619:185
                      argsave=#temp1
                      Source Text:          if x < x0 then
        Oper <        at 5619:112           < : #temp1 <- x x0
                      (36,0,2).

    3 Stmt ASSIGN     line 5619 column
                      133. (1) arg=PRED.y
                      argsave=y
                      Source Text:          y = a + b*x + c*x*x;
        Oper *        at 5619:142           * : #temp1 <- b x
                      (30,0,2).
        Oper +        at 5619:139           + : #temp2 <- a #temp1
                      (32,0,2).
        Oper *        at 5619:148           * : #temp3 <- c x
                      (30,0,2).
        Oper *        at 5619:150           * : #temp4 <- #temp3 x
                      (30,0,2).
        Oper +        at 5619:145           + : PRED.y <- #temp2 #temp4
                      (32,0,2).
        Oper eeocf    at 5619:150           eeocf : _DER_ <- _DER_
                      (18,0,1).
        Oper *        at 5619:150           * : @1dt1_1 <- x x
                      (30,0,2).
        Oper =        at 5619:145 (1,0,1).  = : @PRED.y/@a <- 1
        Oper =        at 5619:145 (1,0,1).  = : @PRED.y/@b <- x
        Oper =        at 5619:145 (1,0,1).  = : @PRED.y/@c <- @1dt1_1

    3 Stmt Assign     line 5619 column
                      133. (1) arg=RESID.y
                      argsave=y
        Oper -        at 5619:133           - : RESID.y <- PRED.y ACTUAL.y
                      (33,0,2).
        Oper eeocf    at 5619:133           eeocf : _DER_ <- _DER_
                      (18,0,1).
        Oper =        at 5619:133 (1,0,1).  = : @RESID.y/@a <- @PRED.y/@a
        Oper =        at 5619:133 (1,0,1).  = : @RESID.y/@b <- @PRED.y/@b
        Oper =        at 5619:133 (1,0,1).  = : @RESID.y/@c <- @PRED.y/@c

    3 Stmt Assign     line 5619 column
                      133. (1) arg=ERROR.y
                      argsave=y
        Oper -        at 5619:133           - : ERROR.y <- PRED.y y
                      (33,0,2).

    4 Stmt ELSE       line 5619 column
                      157. (9)
                      Source Text:          else
```

Figure 14.76. LISTCODE Output for Segmented Model - Compiled Code

Analyzing the Structure of Large Models

PROC MODEL provides several features to aid in analyzing the structure of the model program. These features summarize properties of the model in various forms.

The following Klein's model program is used to introduce the LISTDEP, BLOCK, and GRAPH options.

```
proc model  out=m data=klein listdep graph block;
   endogenous c p w i x wsum k y;
   exogenous  wp g t year;
   parms c0-c3 i0-i3 w0-w3;
   a: c = c0 + c1 * p + c2 * lag(p) + c3 * wsum;
   b: i = i0 + i1 * p + i2 * lag(p) + i3 * lag(k);
   c: w = w0 + w1 * x + w2 * lag(x) + w3 * year;
   x = c + i + g;
   y = c + i + g-t;
   p = x-w-t;
   k = lag(k) + i;
   wsum = w + wp;
   id year;
run;
```

Dependency List

The LISTDEP option produces a dependency list for each variable in the model program. For each variable, a list of variables that depend on it and a list of variables it depends on is given. The dependency list produced by the example program is shown in Figure 14.77.

```
                              The MODEL Procedure

                        Dependency Listing For Program
         Symbol-----------    Dependencies

         c                    Current values affect: ERROR.c PRED.x
                              RESID.x ERROR.x PRED.y RESID.y ERROR.y
         p                    Current values affect: PRED.c RESID.c
                              ERROR.c PRED.i RESID.i ERROR.i ERROR.p
                              Lagged values affect: PRED.c PRED.i
         w                    Current values affect: ERROR.w
                              PRED.p RESID.p ERROR.p PRED.wsum
                              RESID.wsum ERROR.wsum
         i                    Current values affect: ERROR.i PRED.x
                              RESID.x ERROR.x PRED.y RESID.y
                              ERROR.y PRED.k RESID.k ERROR.k
         x                    Current values affect: PRED.w RESID.w
                              ERROR.w ERROR.x PRED.p RESID.p ERROR.p
                              Lagged values affect: PRED.w
         wsum                 Current values affect: PRED.c
                              RESID.c ERROR.c ERROR.wsum
         k                    Current values affect: ERROR.k
                              Lagged values affect: PRED.i RESID.i
                              ERROR.i PRED.k RESID.k
```

Figure 14.77. A Portion of the LISTDEP Output for Klein's Model

BLOCK Listing

The BLOCK option prints an analysis of the program variables based on the assignments in the model program. The output produced by the example is shown in Figure 14.78.

```
                              The MODEL Procedure
                            Model Structure Analysis
              (Based on Assignments to Endogenous Model Variables)

                 Exogenous Variables     wp g t year
                 Endogenous Variables    c p w i x wsum k y
NOTE: The System Consists of 2 Recursive Equations and 1 Simultaneous Blocks.


                          Block Structure of the System

                           Block 1    c p w i x wsum


                        Dependency Structure of the System

                 Block 1    Depends On All_Exogenous
                 k          Depends On Block 1 All_Exogenous
                 y          Depends On Block 1 All_Exogenous
```

Figure 14.78. The BLOCK Output for Klein's Model

One use for the block output is to put a model in recursive form. Simulations of the model can be done with the SEIDEL method, which is efficient if the model is recursive and if the equations are in recursive order. By examining the block output, you can determine how to reorder the model equations for the most efficient simulation.

Adjacency Graph

The GRAPH option displays the same information as the BLOCK option with the addition of an adjacency graph. An X in a column in an adjacency graph indicates that the variable associated with the row depends on the variable associated with the column. The output produced by the example is shown in Figure 14.79.

```
                   The MODEL Procedure

          Adjacency Matrix for Graph of System
                                    w           y
                                    s           e
                                    u     w     a
        Variable          c p w i x m k y p g t r

                                          * * * *
        c                 X X . . . X . . . . . .
        p                 . X X . X . . . . . X .
        w                 . . X . X . . . . . . X
        i                 . X . X . . . . . . . .
        x                 X . . X X . . . . X . .
        wsum              . . X . . X . . X . . .
        k                 . . . X . . X . . . . .
        y                 X . . X . . . X . X X .
        wp              * . . . . . . . . X . . .
        g               * . . . . . . . . . X . .
        t               * . . . . . . . . . . X .
        year            * . . . . . . . . . . . X

             (Note: * = Exogenous Variable.)


        Transitive Closure Matrix of Sorted System
                                             w
                                             s
                                             u
        Block    Variable          c p w i x m k y

            1    c                 X X X X X X . .
            1    p                 X X X X X X . .
            1    w                 X X X X X X . .
            1    i                 X X X X X X . .
            1    x                 X X X X X X . .
            1    wsum              X X X X X X . .
                 k                 X X X X X X X .
                 y                 X X X X X X . X


           Adjacency Matrix for Graph of System
                 Including Lagged Impacts
                                         w           y
                                         s           e
                                         u     w     a
    Block    Variable          c p w i x m k y p g t r

                                               * * * *
        1    c                 X L . . . X . . . . . .
        1    p                 . X X . X . . . . . X .
        1    w                 . . X . L . . . . . . X
        1    i                 . L . X . . L . . . . .
        1    x                 X . . X X . . . . X . .
        1    wsum              . . X . . X . . X . . .
             k                 . . . X . . L . . . . .
             y                 X . . X . . . X . X X .
             wp              * . . . . . . . . X . . .
             g               * . . . . . . . . . X . .
             t               * . . . . . . . . . . X .
             year            * . . . . . . . . . . . X

             (Note: * = Exogenous Variable.)
```

Figure 14.79. The GRAPH Output for Klein's Model

The first and last graphs are straightforward. The middle graph represents the dependencies of the nonexogenous variables after transitive closure has been performed (that is, A depends on B, and B depends on C, so A depends on C). The preceding transitive closure matrix indicates that K and Y do not directly or indirectly depend on each other.

Examples

Example 14.1. OLS Single Nonlinear Equation

This example illustrates the use of the MODEL procedure for nonlinear ordinary least-squares (OLS) regression. The model is a logistic growth curve for the population of the United States. The data is the population in millions recorded at ten year intervals starting in 1790 and ending in 1990. For an explanation of the starting values given by the START= option, see "Troubleshooting Convergence Problems" earlier in this chapter. Portions of the output from the following code are shown in Output 14.1.1 and Output 14.1.2.

```
title 'Logistic Growth Curve Model of U.S. Population';
data uspop;
   input pop :6.3 @@;
   retain year 1780;
   year=year+10;
   label pop='U.S. Population in Millions';
   datalines;
3929  5308  7239   9638  12866  17069  23191  31443  39818 50155
62947 75994 91972 105710 122775 131669 151325 179323 203211
226542 248710
;

proc model data=uspop;
   label a = 'Maximum Population'
         b = 'Location Parameter'
         c = 'Initial Growth Rate';
   pop = a / ( 1 + exp( b - c * (year-1790) ) );
   fit pop start=(a 1000  b 5.5  c .02)/ out=resid outresid;
run;
```

Output 14.1.1. Logistic Growth Curve Model Summary

```
                 Logistic Growth Curve Model of U.S. Population

                              The MODEL Procedure

                                 Model Summary

                           Model Variables         1
                           Parameters              3
                           Equations               1
                           Number of Statements    1


                    Model Variables  pop
                         Parameters  a(1000) b(5.5) c(0.02)
                          Equations  pop
```

```
                 Logistic Growth Curve Model of U.S. Population

                              The MODEL Procedure

                          The Equation to Estimate is

                               pop =  F(a, b, c)
```

Output 14.1.2. Logistic Growth Curve Estimation Summary

```
                 Logistic Growth Curve Model of U.S. Population

                              The MODEL Procedure

                    Nonlinear OLS Summary of Residual Errors

                      DF      DF                                           Adj
   Equation        Model   Error         SSE         MSE   R-Square       R-Sq

   pop                 3      18       345.6     19.2020     0.9972     0.9969


                        Nonlinear OLS Parameter Estimates

                                 Approx                  Approx
Parameter      Estimate     Std Err    t Value     Pr > |t|   Label

a              387.9307     30.0404      12.91       <.0001   Maximum Population
b              3.990385      0.0695      57.44       <.0001   Location Parameter
c              0.022703     0.00107      21.22       <.0001   Initial Growth Rate
```

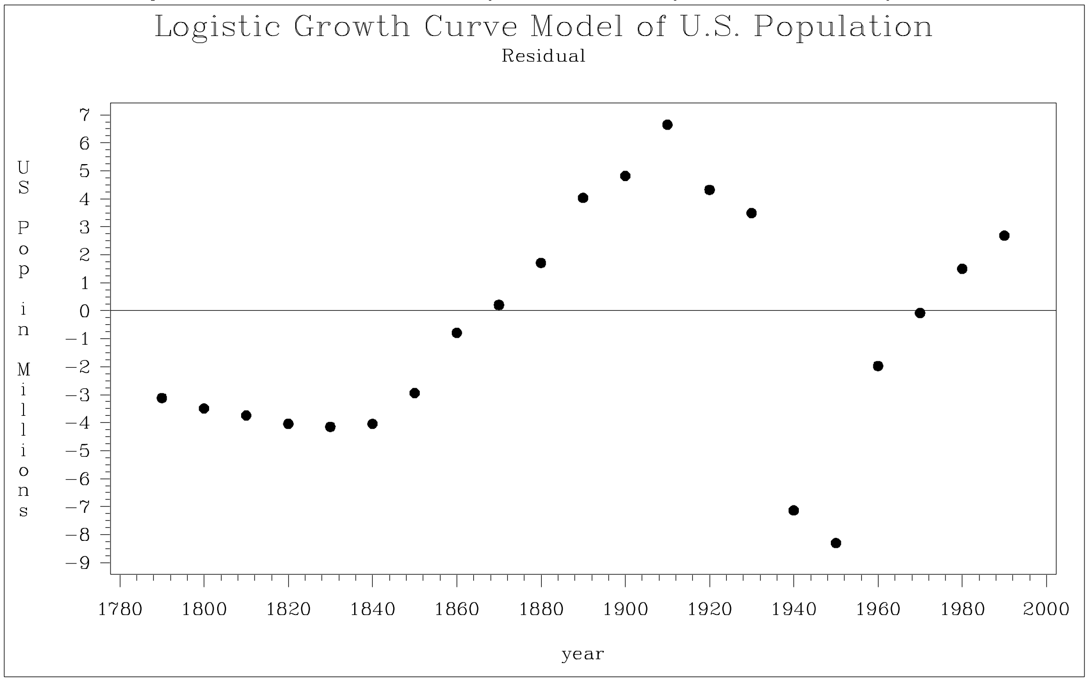
The adjusted R^2 value indicates the model fits the data well. There are only 21 observations and the model is nonlinear, so significance tests on the parameters are only approximate. The significance tests and associated approximate probabilities indicate that all the parameters are significantly different from 0.

The FIT statement included the options OUT=RESID and OUTRESID so that the residuals from the estimation are saved to the data set RESID. The residuals are plotted to check for heteroscedasticity using PROC GPLOT as follows.

```
proc gplot data=resid;
    plot pop*year / vref=0;
    title "Residual";
    symbol1 v=plus;
   run;
```

The plot is shown in Output 14.1.3.

Output 14.1.3. Residual for Population Model (Actual - Predicted)



The residuals do not appear to be independent, and the model could be modified to explain the remaining nonrandom errors.

Example 14.2. A Consumer Demand Model

This example shows the estimation of a system of nonlinear consumer demand equations based on the translog functional form using seemingly unrelated regression (SUR). Expenditure shares and corresponding normalized prices are given for three goods.

Since the shares add up to one, the system is singular; therefore, one equation is omitted from the estimation process. The choice of which equation to omit is arbitrary. The parameter estimates of the omitted equation (share3) can be recovered from the other estimated parameters. The nonlinear system is first estimated in unrestricted form.

```
title1 'Consumer Demand--Translog Functional Form';
title2 'Nonsymmetric Model';
proc model data=tlog1;
```

```
         var share1 share2 p1 p2 p3;
         parms a1 a2 b11 b12 b13 b21 b22 b23 b31 b32 b33;
         bm1 = b11 + b21 + b31;
         bm2 = b12 + b22 + b32;
         bm3 = b13 + b23 + b33;
         lp1 = log(p1);
         lp2 = log(p2);
         lp3 = log(p3);
         share1 = ( a1 + b11 * lp1 + b12 * lp2 + b13 * lp3 ) /
                  ( -1 + bm1 * lp1 + bm2 * lp2 + bm3 * lp3 );
         share2 = ( a2 + b21 * lp1 + b22 * lp2 + b23 * lp3 ) /
                  ( -1 + bm1 * lp1 + bm2 * lp2 + bm3 * lp3 );
         fit share1 share2
             start=( a1 -.14 a2 -.45 b11 .03 b12 .47 b22 .98 b31 .20
                     b32 1.11 b33 .71 ) / outsused = smatrix sur;
      run;
```

A portion of the printed output produced in the preceding example is shown in Output 14.2.1 .

Output 14.2.1. Estimation Results from the Unrestricted Model

```
                   Consumer Demand--Translog Functional Form
                               Nonsymmetric Model

                              The MODEL Procedure

                                 Model Summary

                           Model Variables          5
                           Parameters              11
                           Equations                2
                           Number of Statements     8


Model Variables  share1 share2 p1 p2 p3
     Parameters  a1(-0.14) a2(-0.45) b11(0.03) b12(0.47) b13 b21
                 b22(0.98) b23 b31(0.2) b32(1.11) b33(0.71)
      Equations  share1 share2
```

```
                   Consumer Demand--Translog Functional Form
                               Nonsymmetric Model

                              The MODEL Procedure

                           The 2 Equations to Estimate

          share1 =  F(a1, b11, b12, b13, b21, b22, b23, b31, b32, b33)
          share2 =  F(a2, b11, b12, b13, b21, b22, b23, b31, b32, b33)


        NOTE: At SUR Iteration 2 CONVERGE=0.001 Criteria Met.
```

```
                   Consumer Demand--Translog Functional Form
                               Nonsymmetric Model

                              The MODEL Procedure

                   Nonlinear SUR Summary of Residual Errors

                DF     DF                                                Adj
Equation      Model  Error        SSE        MSE  Root MSE  R-Square     R-Sq

share1          5.5   38.5    0.00166   0.000043   0.00656    0.8067   0.7841
share2          5.5   38.5    0.00135   0.000035   0.00592    0.9445   0.9380


                       Nonlinear SUR Parameter Estimates

                                     Approx                    Approx
        Parameter       Estimate     Std Err     t Value     Pr > |t|

        a1              -0.14881     0.00225      -66.08       <.0001
        a2              -0.45776     0.00297     -154.29       <.0001
        b11             0.048382      0.0498        0.97       0.3379
        b12              0.43655      0.0502        8.70       <.0001
        b13             0.248588      0.0516        4.82       <.0001
        b21             0.586326      0.2089        2.81       0.0079
        b22             0.759776      0.2565        2.96       0.0052
        b23             1.303821      0.2328        5.60       <.0001
        b31             0.297808      0.1504        1.98       0.0550
        b32             0.961551      0.1633        5.89       <.0001
        b33               0.8291      0.1556        5.33       <.0001


             Number of Observations     Statistics for System

             Used               44    Objective         1.7493
             Missing             0    Objective*N      76.9697
```

The model is then estimated under the restriction of symmetry (b_{ij}=b_{ji}).

Hypothesis testing requires that the **S** matrix from the unrestricted model be imposed on the restricted model, as explained in "Tests on Parameters" in this chapter. The **S** matrix saved in the data set SMATRIX is requested by the SDATA= option.

A portion of the printed output produced in the following example is shown in Output 14.2.2.

```
title2 'Symmetric Model';
proc model data=tlog1;
   var share1 share2 p1 p2 p3;
   parms a1 a2 b11 b12 b22 b31 b32 b33;
   bm1 = b11 + b12 + b31;
   bm2 = b12 + b22 + b32;
   bm3 = b31 + b32 + b33;
   lp1 = log(p1);
   lp2 = log(p2);
   lp3 = log(p3);
   share1 = ( a1 + b11 * lp1 + b12 * lp2 + b31 * lp3 ) /
            ( -1 + bm1 * lp1 + bm2 * lp2 + bm3 * lp3 );
   share2 = ( a2 + b12 * lp1 + b22 * lp2 + b32 * lp3 ) /
```

```
                 ( -1 + bm1 * lp1 + bm2 * lp2 + bm3 * lp3 );
      fit share1 share2
          start=( a1 -.14 a2 -.45 b11 .03 b12 .47 b22 .98 b31 .20
                  b32 1.11 b33 .71 ) / sdata=smatrix sur;
   run;
```

A chi-square test is used to see if the hypothesis of symmetry is accepted or rejected. (*Oc*-*Ou*) has a chi-square distribution asymptotically, where *Oc* is the constrained OBJECTIVE*N and *Ou* is the unconstrained OBJECTIVE*N. The degrees of freedom is equal to the difference in the number of free parameters in the two models.

In this example, Ou is 76.9697 and Oc is 78.4097, resulting in a difference of 1.44 with 3 degrees of freedom. You can obtain the probability value by using the following statements:

```
   data _null_;
                   /* reduced-full, nrestrictions */
      p = 1-probchi( 1.44, 3 );
      put p=;
   run;
```

The output from this DATA step run is ‘P=0.6961858724’. With this probability you cannot reject the hypothesis of symmetry. This test is asymptotically valid.

Output 14.2.2. Estimation Results from the Restricted Model

```
                   Consumer Demand--Translog Functional Form
                                Symmetric Model

                              The MODEL Procedure

                          The 2 Equations to Estimate

                 share1 =  F(a1, b11, b12, b22, b31, b32, b33)
                 share2 =  F(a2, b11, b12, b22, b31, b32, b33)
```

```
                   Consumer Demand--Translog Functional Form
                                Symmetric Model

                              The MODEL Procedure

                   Nonlinear SUR Summary of Residual Errors

                  DF     DF                                            Adj
Equation       Model  Error        SSE        MSE  Root MSE  R-Square     R-Sq

share1             4     40    0.00166  0.000041   0.00644    0.8066   0.7920
share2             4     40    0.00139  0.000035   0.00590    0.9428   0.9385


                       Nonlinear SUR Parameter Estimates

                                     Approx                     Approx
        Parameter       Estimate     Std Err     t Value     Pr > |t|

        a1              -0.14684     0.00135     -108.99       <.0001
        a2               -0.4597     0.00167     -275.34       <.0001
        b11              0.02886     0.00741        3.89       0.0004
        b12             0.467827      0.0115       40.57       <.0001
        b22             0.970079      0.0177       54.87       <.0001
        b31             0.208143     0.00614       33.88       <.0001
        b32             1.102415      0.0127       86.51       <.0001
        b33             0.694245      0.0168       41.38       <.0001


             Number of Observations     Statistics for System

             Used               44    Objective         1.7820
             Missing             0    Objective*N      78.4097
```

Example 14.3. Vector AR(1) Estimation

This example shows the estimation of a two-variable vector AR(1) error process for the Grunfeld model (Grunfeld 1960) using the %AR macro. First, the full model is estimated. Second, the model is estimated with the restriction that the errors are univariate AR(1) instead of a vector process. The following produces Output 14.3.1 and Output 14.3.2.

```
data grunfeld;
   input year gei gef gec whi whf whc;
   label gei = ’Gross Investment GE’
         gec = ’Capital Stock Lagged GE’
         gef = ’Value of Outstanding Shares GE Lagged’
         whi = ’Gross Investment WH’
         whc = ’Capital Stock Lagged WH’
         whf = ’Value of Outstanding Shares Lagged WH’;
   datalines;
1935     33.1      1170.6    97.8      12.93     191.5     1.8
1936     45.0      2015.8    104.4     25.90     516.0     .8
1937     77.2      2803.3    118.0     35.05     729.0     7.4
1938     44.6      2039.7    156.2     22.89     560.4     18.1
1939     48.1      2256.2    172.6     18.84     519.9     23.5
1940     74.4      2132.2    186.6     28.57     628.5     26.5
1941     113.0     1834.1    220.9     48.51     537.1     36.2
1942     91.9      1588.0    287.8     43.34     561.2     60.8
1943     61.3      1749.4    319.9     37.02     617.2     84.4
```

```
1944     56.8      1687.2    321.3     37.81     626.7     91.2
1945     93.6      2007.7    319.6     39.27     737.2     92.4
1946     159.9     2208.3    346.0     53.46     760.5     86.0
1947     147.2     1656.7    456.4     55.56     581.4     111.1
1948     146.3     1604.4    543.4     49.56     662.3     130.6
1949     98.3      1431.8    618.3     32.04     583.8     141.8
1950     93.5      1610.5    647.4     32.24     635.2     136.7
1951     135.2     1819.4    671.3     54.38     723.8     129.7
1952     157.3     2079.7    726.1     71.78     864.1     145.5
1953     179.5     2371.6    800.3     90.08     1193.5    174.8
1954     189.6     2759.9    888.9     68.60     1188.9    213.5
;

title1 'Example of Vector AR(1) Error Process
            Using Grunfeld''s Model';
/* Note: GE stands for General Electric
            and WH for Westinghouse     */

proc model outmodel=grunmod;
   var gei whi gef gec whf whc;
   parms ge_int ge_f ge_c wh_int wh_f wh_c;
   label ge_int = 'GE Intercept'
         ge_f   = 'GE Lagged Share Value Coef'
         ge_c   = 'GE Lagged Capital Stock Coef'
         wh_int = 'WH Intercept'
         wh_f   = 'WH Lagged Share Value Coef'
         wh_c   = 'WH Lagged Capital Stock Coef';
   gei = ge_int + ge_f * gef + ge_c * gec;
   whi = wh_int + wh_f * whf + wh_c * whc;
run;
```

The preceding PROC MODEL step defines the structural model and stores it in the model file named GRUNMOD.

The following PROC MODEL step reads in the model, adds the vector autoregressive terms using %AR, and requests SUR estimation using the FIT statement.

```
title2 'With Unrestricted Vector AR(1) Error Process';
proc model data=grunfeld model=grunmod;
   %ar( ar, 1, gei whi )
   fit gei whi / sur;
run;
```

The final PROC MODEL step estimates the restricted model.

```
title2 'With restricted AR(1) Error Process';
proc model data=grunfeld model=grunmod;
   %ar( gei, 1 )
   %ar( whi, 1)
   fit gei whi / sur;
run;
```

Output 14.3.1. Results for the Unrestricted Model (Partial Output)

```
          Example of Vector AR(1) Error Process Using Grunfeld's Model
                  With Unrestricted Vector AR(1) Error Process

                              The MODEL Procedure

                                 Model Summary

                           Model Variables          6
                           Parameters              10
                           Equations                2
                           Number of Statements     6


Model Variables  gei whi gef gec whf whc
     Parameters  ge_int ge_f ge_c wh_int wh_f wh_c ar_l1_1_1(0)
                 ar_l1_1_2(0) ar_l1_2_1(0) ar_l1_2_2(0)
      Equations  gei whi
```

```
          Example of Vector AR(1) Error Process Using Grunfeld's Model
                  With Unrestricted Vector AR(1) Error Process

                              The MODEL Procedure

                           The 2 Equations to Estimate

    gei =  F(ge_int, ge_f, ge_c, wh_int, wh_f, wh_c, ar_l1_1_1, ar_l1_1_2)
    whi =  F(ge_int, ge_f, ge_c, wh_int, wh_f, wh_c, ar_l1_2_1, ar_l1_2_2)


       NOTE: At SUR Iteration 9 CONVERGE=0.001 Criteria Met.
```

```
          Example of Vector AR(1) Error Process Using Grunfeld's Model
                  With Unrestricted Vector AR(1) Error Process

                              The MODEL Procedure

                   Nonlinear SUR Summary of Residual Errors

                     DF      DF                                          Adj
   Equation        Model   Error        SSE        MSE   R-Square      R-Sq

   gei                 5      15     9374.5      625.0     0.7910    0.7352
   whi                 5      15     1429.2    95.2807     0.7940    0.7391


                       Nonlinear SUR Parameter Estimates

                               Approx                  Approx
Parameter      Estimate    Std Err   t Value    Pr > |t|   Label

ge_int         -42.2858    30.5284     -1.39      0.1863   GE Intercept
ge_f           0.049894     0.0153      3.27      0.0051   GE Lagged Share
                                                           Value Coef
ge_c           0.123946     0.0458      2.70      0.0163   GE Lagged Capital
                                                           Stock Coef
wh_int         -4.68931     8.9678     -0.52      0.6087   WH Intercept
wh_f           0.068979     0.0182      3.80      0.0018   WH Lagged Share
                                                           Value Coef
wh_c           0.019308     0.0754      0.26      0.8015   WH Lagged Capital
                                                           Stock Coef
ar_l1_1_1      0.990902     0.3923      2.53      0.0233   AR(ar) gei: LAG1
                                                           parameter for gei
ar_l1_1_2      -1.56252     1.0882     -1.44      0.1716   AR(ar) gei: LAG1
                                                           parameter for whi
ar_l1_2_1      0.244161     0.1783      1.37      0.1910   AR(ar) whi: LAG1
                                                           parameter for gei
ar_l1_2_2      -0.23864     0.4957     -0.48      0.6372   AR(ar) whi: LAG1
                                                           parameter for whi
```

Output 14.3.2. Results for the Restricted Model (Partial Output)

```
          Example of Vector AR(1) Error Process Using Grunfeld's Model
                      With Restricted AR(1) Error Process

                              The MODEL Procedure

                                 Model Summary

                          Model Variables         6
                          Parameters              8
                          Equations               2
                          Number of Statements    6


     Model Variables  gei whi gef gec whf whc
          Parameters  ge_int ge_f ge_c wh_int wh_f wh_c gei_l1(0) whi_l1(0)
           Equations  gei whi
          Example of Vector AR(1) Error Process Using Grunfeld's Model
                        With Restricted AR(1) Error Process

                               The MODEL Procedure

                     Nonlinear SUR Summary of Residual Errors

                      DF      DF                                              Adj
   Equation        Model   Error         SSE         MSE   R-Square      R-Sq

   gei                 4      16     10558.8       659.9     0.7646    0.7204
   whi                 4      16      1669.8       104.4     0.7594    0.7142


                         Nonlinear SUR Parameter Estimates

                                  Approx                  Approx
Parameter      Estimate    Std Err   t Value    Pr > |t|   Label

ge_int         -30.1239    29.7227     -1.01      0.3259   GE Intercept
ge_f           0.043527     0.0149      2.93      0.0099   GE Lagged Share
                                                           Value Coef
ge_c           0.119206     0.0423      2.82      0.0124   GE Lagged Capital
                                                           Stock Coef
wh_int         3.112671     9.2765      0.34      0.7416   WH Intercept
wh_f           0.053932     0.0154      3.50      0.0029   WH Lagged Share
                                                           Value Coef
wh_c           0.038246     0.0805      0.48      0.6410   WH Lagged Capital
                                                           Stock Coef
gei_l1         0.482397     0.2149      2.24      0.0393   AR(gei) gei lag1
                                                           parameter
whi_l1         0.455711     0.2424      1.88      0.0784   AR(whi) whi lag1
                                                           parameter
```

Example 14.4. MA(1) Estimation

This example estimates parameters for an MA(1) error process for the Grunfeld model, using both the unconditional least-squares and the maximum-likelihood methods. The ARIMA procedure estimates for Westinghouse equation are shown for comparison. The output of the following code is summarized in Output 14.4.1:

```
title1 'Example of MA(1) Error Process Using Grunfeld''s Model';
title2 'MA(1) Error Process Using Unconditional Least Squares';
proc model data=grunfeld model=grunmod;
   %ma(gei,1, m=uls);
   %ma(whi,1, m=uls);
   fit whi gei start=( gei_m1 0.8 -0.8) / startiter=2;
run;
```

Output 14.4.1. PROC MODEL Results Using ULS Estimation

```
             Example of MA(1) Error Process Using Grunfeld's Model
             MA(1) Error Process Using Unconditional Least Squares

                              The MODEL Procedure

                   Nonlinear OLS Summary of Residual Errors

                     DF      DF                                         Adj
   Equation       Model   Error         SSE         MSE   R-Square      R-Sq

   whi                4      16      1874.0       117.1     0.7299    0.6793
   resid.whi                 16      1295.6     80.9754
   gei                4      16     13835.0       864.7     0.6915    0.6337
   resid.gei                 16      7646.2       477.9


                       Nonlinear OLS Parameter Estimates

                              Approx                  Approx
Parameter      Estimate     Std Err    t Value     Pr > |t|   Label

ge_int          -26.839     32.0908      -0.84       0.4153   GE Intercept
ge_f           0.038226      0.0150       2.54       0.0217   GE Lagged Share
                                                              Value Coef
ge_c           0.137099      0.0352       3.90       0.0013   GE Lagged Capital
                                                              Stock Coef
wh_int         3.680835      9.5448       0.39       0.7048   WH Intercept
wh_f           0.049156      0.0172       2.85       0.0115   WH Lagged Share
                                                              Value Coef
wh_c           0.067271      0.0708       0.95       0.3559   WH Lagged Capital
                                                              Stock Coef
gei_m1         -0.87615      0.1614      -5.43       <.0001   MA(gei) gei lag1
                                                              parameter
whi_m1         -0.75001      0.2368      -3.17       0.0060   MA(whi) whi lag1
                                                              parameter
```

The estimation summary from the following PROC ARIMA statements is shown in Output 14.4.2.

```
title2 'PROC ARIMA Using Unconditional Least Squares';

proc arima data=grunfeld;
   identify var=whi cross=(whf whc ) noprint;
   estimate q=1 input=(whf whc) method=uls maxiter=40;
run;
```

Output 14.4.2. PROC ARIMA Results Using ULS Estimation

```
            Example of MA(1) Error Process Using Grunfeld's Model
                 PROC ARIMA Using Unconditional Least Squares

                             The ARIMA Procedure

                    Unconditional Least Squares Estimation

                       Approx Std
Parameter    Estimate       Error     t Value  Pr > |t|   Lag  Variable  Shift

MU            3.68608     9.54425        0.39    0.7044     0  whi           0
MA1,1        -0.75005     0.23704       -3.16    0.0060     1  whi           0
NUM1          0.04914     0.01723        2.85    0.0115     0  whf           0
NUM2          0.06731     0.07077        0.95    0.3557     0  whc           0


                       Constant Estimate      3.686077
                       Variance Estimate      80.97535
                       Std Error Estimate     8.998631
                       AIC                    149.0044
                       SBC                    152.9873
                       Number of Residuals          20
```

The model stored in Example 14.3 is read in using the MODEL= option and the moving average terms are added using the %MA macro.

The MA(1) model using maximum likelihood is estimated using the following:

```
title2 'MA(1) Error Process Using Maximum Likelihood ';
proc model data=grunfeld model=grunmod;
   %ma(gei,1, m=ml);
   %ma(whi,1, m=ml);
   fit whi gei;
run;
```

For comparison, the model is estimated using PROC ARIMA as follows:

```
title2 'PROC ARIMA Using Maximum Likelihood ';
proc arima data=grunfeld;
   identify var=whi cross=(whf whc) noprint;
   estimate q=1 input=(whf whc) method=ml;
run;
```

PROC ARIMA does not estimate systems so only one equation is evaluated.

The estimation results are shown in Output 14.4.3 and Output 14.4.4. The small differences in the parameter values between PROC MODEL and PROC ARIMA can be eliminated by tightening the convergence criteria for both procedures.

Output 14.4.3. PROC MODEL Results Using ML Estimation

```
             Example of MA(1) Error Process Using Grunfeld's Model
                 MA(1) Error Process Using Maximum Likelihood

                              The MODEL Procedure

                    Nonlinear OLS Summary of Residual Errors

                      DF      DF                                        Adj
   Equation        Model   Error        SSE        MSE   R-Square      R-Sq

   whi                 4      16     1857.5      116.1     0.7323    0.6821
   resid.whi                  16     1344.0    84.0012
   gei                 4      16    13742.5      858.9     0.6936    0.6361
   resid.gei                  16     8095.3      506.0


                        Nonlinear OLS Parameter Estimates

                            Approx                  Approx
Parameter      Estimate    Std Err   t Value    Pr > |t|   Label

ge_int          -25.002    34.2933     -0.73      0.4765   GE Intercept
ge_f            0.03712     0.0161      2.30      0.0351   GE Lagged Share
                                                           Value Coef
ge_c           0.137788     0.0380      3.63      0.0023   GE Lagged Capital
                                                           Stock Coef
wh_int         2.946761     9.5638      0.31      0.7620   WH Intercept
wh_f           0.050395     0.0174      2.89      0.0106   WH Lagged Share
                                                           Value Coef
wh_c           0.066531     0.0729      0.91      0.3749   WH Lagged Capital
                                                           Stock Coef
gei_m1         -0.78516     0.1942     -4.04      0.0009   MA(gei) gei lag1
                                                           parameter
whi_m1         -0.69389     0.2540     -2.73      0.0148   MA(whi) whi lag1
                                                           parameter
```

Output 14.4.4. PROC ARIMA Results Using ML Estimation

```
             Example of MA(1) Error Process Using Grunfeld's Model
                      PROC ARIMA Using Maximum Likelihood

                              The ARIMA Procedure

                         Maximum Likelihood Estimation

                        Approx Std
 Parameter    Estimate       Error     t Value  Pr > |t|   Lag  Variable  Shift

 MU            2.95645     9.20752        0.32    0.7481     0  whi           0
 MA1,1        -0.69305     0.25307       -2.74    0.0062     1  whi           0
 NUM1          0.05036     0.01686        2.99    0.0028     0  whf           0
 NUM2          0.06672     0.06939        0.96    0.3363     0  whc           0


                        Constant Estimate      2.956449
                        Variance Estimate      81.29645
                        Std Error Estimate     9.016455
                        AIC                    148.9113
                        SBC                    152.8942
                        Number of Residuals          20
```

Example 14.5. Polynomial Distributed Lags Using %PDL

This example shows the use of the %PDL macro for polynomial distributed lag models. Simulated data is generated so that Y is a linear function of six lags of X, with the lag coefficients following a quadratic polynomial. The model is estimated using a fourth-degree polynomial, both with and without endpoint constraints. The example uses simulated data generated from the following model:

$$y_t = 10 + \sum_{z=0}^{6} f(z) x_{t-z} + \epsilon$$

$$f(z) = -5z^2 + 1.5z$$

The LIST option prints the model statements added by the %PDL macro.

```
/*---------------------------------------------------------------*/
/*  Generate Simulated Data for a Linear Model with a PDL on X  */
/*        y = 10 + x(6,2) + e                                    */
/*        pdl(x) = -5.*(lg)**2 + 1.5*(lg) + 0.                   */
/*---------------------------------------------------------------*/
data pdl;
   pdl2=-5.; pdl1=1.5; pdl0=0;
   array zz(i) z0-z6;
   do i=1 to 7;
      z=i-1;
      zz=pdl2*z**2 + pdl1*z + pdl0;
      end;
   do n=-11 to 30;
      x  =10*ranuni(1234567)-5;
      pdl=z0*x + z1*xl1 + z2*xl2 + z3*xl3 + z4*xl4 + z5*xl5 + z6*xl6;
      e  =10*rannor(123);
      y  =10+pdl+e;
      if n>=1 then output;
      xl6=xl5; xl5=xl4; xl4=xl3; xl3=xl2; xl2=xl1; xl1=x;
      end;
run;

title1 'Polynomial Distributed Lag Example';

title3 'Estimation of PDL(6,4) Model-- No Endpoint Restrictions';
proc model data=pdl;
   parms int;                  /* declare the intercept parameter */
   %pdl( xpdl, 6, 4 )          /* declare the lag distribution */
   y = int + %pdl( xpdl, x );  /* define the model equation */
   fit y / list;               /* estimate the parameters */
run;
```

Output 14.5.1. PROC MODEL Listing of Generated Program

```
                    Polynomial Distributed Lag Example

         Estimation of PDL(6,4) Model-- No Endpoint Restrictions

                           The MODEL Procedure

                     Listing of Compiled Program Code
          Stmt    Line:Col      Statement as Parsed

             1    25242:14      XPDL_L0 = XPDL_0;
             2    25254:14      XPDL_L1 = XPDL_0 + XPDL_1 +
                                XPDL_2 + XPDL_3 + XPDL_4;
             3    25283:14      XPDL_L2 = XPDL_0 + XPDL_1 *
                                2 + XPDL_2 * 2 ** 2 + XPDL_3
                                * 2 ** 3 + XPDL_4 * 2 ** 4;
             4    25331:14      XPDL_L3 = XPDL_0 + XPDL_1 *
                                3 + XPDL_2 * 3 ** 2 + XPDL_3
                                * 3 ** 3 + XPDL_4 * 3 ** 4;
             5    25379:14      XPDL_L4 = XPDL_0 + XPDL_1 *
                                4 + XPDL_2 * 4 ** 2 + XPDL_3
                                * 4 ** 3 + XPDL_4 * 4 ** 4;
             6    25427:14      XPDL_L5 = XPDL_0 + XPDL_1 *
                                5 + XPDL_2 * 5 ** 2 + XPDL_3
                                * 5 ** 3 + XPDL_4 * 5 ** 4;
             7    25475:14      XPDL_L6 = XPDL_0 + XPDL_1 *
                                6 + XPDL_2 * 6 ** 2 + XPDL_3
                                * 6 ** 3 + XPDL_4 * 6 ** 4;
             8    25121:204     PRED.y = int + XPDL_L0 * x + XPDL_L1 *
                                LAG1( x ) + XPDL_L2 * LAG2( x ) +
                                XPDL_L3 * LAG3( x ) + XPDL_L4
                                * LAG4( x ) + XPDL_L5 * LAG5(
                                x ) + XPDL_L6 * LAG6( x );
             8    25121:204     RESID.y = PRED.y - ACTUAL.y;
             8    25121:204     ERROR.y = PRED.y - y;
             9    25218:15      ESTIMATE XPDL_L0, XPDL_L1, XPDL_L2,
                                XPDL_L3, XPDL_L4, XPDL_L5, XPDL_L6;
            10    25218:15      _est0 = XPDL_L0;
            11    25221:15      _est1 = XPDL_L1;
            12    25224:15      _est2 = XPDL_L2;
            13    25227:15      _est3 = XPDL_L3;
            14    25230:15      _est4 = XPDL_L4;
            15    25233:15      _est5 = XPDL_L5;
            16    25238:14      _est6 = XPDL_L6;
```

Output 14.5.2. PROC MODEL Results Specifying No Endpoint Restrictions

```
                              Polynomial Distributed Lag Example

                     Estimation of PDL(6,4) Model-- No Endpoint Restrictions

                                     The MODEL Procedure

                          Nonlinear OLS Summary of Residual Errors

                          DF     DF                                                Adj
  Equation             Model  Error        SSE        MSE   Root MSE  R-Square      R-Sq

  y                        6     18     2070.8      115.0    10.7259    0.9998    0.9998


                              Nonlinear OLS Parameter Estimates

                                  Approx                   Approx
Parameter      Estimate       Std Err    t Value    Pr > |t|   Label

int            9.621969        2.3238       4.14      0.0006
XPDL_0         0.084374        0.7587       0.11      0.9127   PDL(XPDL,6,4)
                                                               parameter for (L)**0
XPDL_1         0.749956        2.0936       0.36      0.7244   PDL(XPDL,6,4)
                                                               parameter for (L)**1
XPDL_2           -4.196        1.6215      -2.59      0.0186   PDL(XPDL,6,4)
                                                               parameter for (L)**2
XPDL_3         -0.21489        0.4253      -0.51      0.6195   PDL(XPDL,6,4)
                                                               parameter for (L)**3
XPDL_4         0.016133        0.0353       0.46      0.6528   PDL(XPDL,6,4)
                                                               parameter for (L)**4
```

The LIST output for the model without endpoint restrictions is shown in Output 14.5.1 and Output 14.5.2. The first seven statements in the generated program are the polynomial expressions for lag parameters XPDL_L0 through XPDL_L6. The estimated parameters are INT, XPDL_0, XPDL_1, XPDL_2, XPDL_3, and XPDL_4.

Portions of the output produced by the following PDL model with endpoints of the model restricted to 0 are presented in Output 14.5.3 and Output 14.5.4.

```
   title3 'Estimation of PDL(6,4) Model-- Both Endpoint Restrictions';
   proc model data=pdl ;
      parms int;                 /* declare the intercept parameter */
      %pdl( xpdl, 6, 4, r=both )  /* declare the lag distribution */
      y = int + %pdl( xpdl, x );  /* define the model equation */
      fit y /list;                /* estimate the parameters */
   run;
```

Output 14.5.3. PROC MODEL Results Specifying Both Endpoint Restrictions

```
                       Polynomial Distributed Lag Example

           Estimation of PDL(6,4) Model-- Both Endpoint Restrictions

                              The MODEL Procedure

                   Nonlinear OLS Summary of Residual Errors

                      DF     DF                                            Adj
  Equation         Model  Error        SSE        MSE  Root MSE  R-Square     R-Sq

  y                    4     20     449868    22493.4     150.0    0.9596   0.9535


                       Nonlinear OLS Parameter Estimates

                                 Approx                  Approx
Parameter      Estimate    Std Err    t Value    Pr > |t|   Label

int            17.08581    32.4032       0.53      0.6038
XPDL_2         13.88433     5.4361       2.55      0.0189   PDL(XPDL,6,4)
                                                            parameter for (L)**2
XPDL_3          -9.3535     1.7602      -5.31      <.0001   PDL(XPDL,6,4)
                                                            parameter for (L)**3
XPDL_4         1.032421     0.1471       7.02      <.0001   PDL(XPDL,6,4)
                                                            parameter for (L)**4
```

Note that XPDL_0 and XPDL_1 are not shown in the estimate summary. They were used to satisfy the endpoint restrictions analytically by the generated %PDL macro code. Their values can be determined by back substitution.

To estimate the PDL model with one or more of the polynomial terms dropped, specify the largest degree of the polynomial desired with the %PDL macro and use the DROP= option on the FIT statement to remove the unwanted terms. The dropped parameters should be set to 0. The following PROC MODEL code demonstrates estimation with a PDL of degree 2 without the 0th order term.

```
title3 'Estimation of PDL(6,2) Model-- With XPDL_0 Dropped';
proc model data=pdl list;
   parms int;                  /* declare the intercept parameter */
   %pdl( xpdl, 6, 2 )          /* declare the lag distribution */
   y = int + %pdl( xpdl, x );  /* define the model equation */
   xpdl_0 =0;
   fit y drop=xpdl_0;          /* estimate the parameters */
run;
```

The results from this estimation are shown in Output 14.5.4.

Output 14.5.4. PROC MODEL Results Specifying %PDL(XPDL, 6, 2)

```
                       Polynomial Distributed Lag Example

               Estimation of PDL(6,2) Model-- With XPDL_0 Dropped

                              The MODEL Procedure

                   Nonlinear OLS Summary of Residual Errors

                   DF     DF                                                 Adj
  Equation      Model  Error        SSE        MSE  Root MSE  R-Square      R-Sq

  y                 3     21     2114.1      100.7   10.0335    0.9998    0.9998


                       Nonlinear OLS Parameter Estimates

                            Approx                Approx
Parameter      Estimate    Std Err   t Value    Pr > |t|   Label

int            9.536382     2.1685      4.40      0.0003
XPDL_1         1.883315     0.3159      5.96      <.0001   PDL(XPDL,6,2)
                                                           parameter for (L)**1
XPDL_2         -5.08827     0.0656    -77.56      <.0001   PDL(XPDL,6,2)
                                                           parameter for (L)**2
```

Example 14.6. General-Form Equations

Data for this example are generated. General-form equations are estimated and forecast using PROC MODEL. The system is a basic supply-demand model. Portions of the output from the following code is shown in Output 14.6.1 through Output 14.6.4.

```
title1 "General Form Equations for Supply-Demand Model";

proc model;
   var price quantity income unitcost;
   parms d0-d2 s0-s2;
   eq.demand=d0+d1*price+d2*income-quantity;
   eq.supply=s0+s1*price+s2*unitcost-quantity;

/* estimate the model parameters */
   fit supply demand / data=history outest=est n2sls;
   instruments income unitcost year;
run;

/* produce forecasts for income and unitcost assumptions */
   solve price quantity / data=assume out=pq;
run;

/* produce goal-seeking solutions for
     income and quantity assumptions*/
   solve price unitcost / data=goal out=pc;
run;

title2 "Parameter Estimates for the System";
proc print data=est;
run;
```

```
title2 "Price Quantity Solution";
proc print data=pq;
run;

title2 "Price Unitcost Solution";
proc print data=pc;
run;
```

Three data sets were used in this example. The first data set, HISTORY, was used to estimate the parameters of the model. The ASSUME data set was used to produce a forecast of PRICE and QUANTITY. Notice that the ASSUME data set does not have to contain the variables PRICE and QUANTITY.

```
data history;
   input year income unitcost price quantity;
   datalines;
1976    2221.87    3.31220     0.17903    266.714
1977    2254.77    3.61647     0.06757    276.049
1978    2285.16    2.21601     0.82916    285.858
1979    2319.37    3.28257     0.33202    295.034
1980    2369.38    2.84494     0.63564    310.773
1981    2395.26    2.94154     0.62011    319.185
1982    2419.52    2.65301     0.80753    325.970
1983    2475.09    2.41686     1.01017    342.470
1984    2495.09    3.44096     0.52025    348.321
1985    2536.72    2.30601     1.15053    360.750
;

data assume;
   input year income unitcost;
   datalines;
1986    2571.87    2.31220
1987    2609.12    2.45633
1988    2639.77    2.51647
1989    2667.77    1.65617
1990    2705.16    1.01601
;
```

The output produced by the first SOLVE statement is shown in Output 14.6.3.

The third data set, GOAL, is used in a forecast of PRICE and UNITCOST as a function of INCOME and QUANTITY.

```
data goal;
   input year income quantity;
   datalines;
1986    2571.87     371.4
1987    2721.08     416.5
1988    3327.05     597.3
1989    3885.85     764.1
1990    3650.98     694.3
;
```

The output from the final SOLVE statement is shown in Output 14.6.4.

Output 14.6.1. Printed Output from the FIT Statement

```
                 General Form Equations for Supply-Demand Model

                              The MODEL Procedure

                           The 2 Equations to Estimate

                    supply =  F(s0(1), s1(price), s2(unitcost))
                    demand =  F(d0(1), d1(price), d2(income))
                 Instruments  1 income unitcost year
```

```
                 General Form Equations for Supply-Demand Model

                              The MODEL Procedure

                  Nonlinear 2SLS Summary of Residual Errors

                    DF     DF                                               Adj
Equation         Model  Error        SSE       MSE  Root MSE  R-Square     R-Sq

supply               3      7     3.3240    0.4749    0.6891
demand               3      7     1.0829    0.1547    0.3933


                       Nonlinear 2SLS Parameter Estimates

                                             Approx                  Approx
        Parameter       Estimate      Std Err     t Value     Pr > |t|

        d0              -395.887       4.1841      -94.62       <.0001
        d1              0.717328       0.5673        1.26       0.2466
        d2              0.298061      0.00187      159.65       <.0001
        s0               -107.62       4.1780      -25.76       <.0001
        s1              201.5711       1.5977      126.16       <.0001
        s2              102.2116       1.1217       91.12       <.0001
```

Output 14.6.2. Listing of OUTEST= Data Set Created in the FIT Statement

```
                 General Form Equations for Supply-Demand Model
                       Parameter Estimates for the System


                 _
                 S       _
   _  _          T       N
   N  T          A       U
   A  Y          T       S
 O M  P          U       E
 b E  E          S       D     d       d       d        s       s       s
 s _  _          _       _     0       1       2        0       1       2

 1   2SLS 0 Converged 10 -395.887 0.71733 0.29806 -107.620 201.571 102.212
```

Output 14.6.3. Listing of OUT= Data Set Created in the First SOLVE Statement

```
                 General Form Equations for Supply-Demand Model
                            Price Quantity Solution

 Obs   _TYPE_    _MODE_   _ERRORS_   price   quantity   income  unitcost  year

  1   PREDICT  SIMULATE      0     1.20473   371.552  2571.87   2.31220  1986
  2   PREDICT  SIMULATE      0     1.18666   382.642  2609.12   2.45633  1987
  3   PREDICT  SIMULATE      0     1.20154   391.788  2639.77   2.51647  1988
  4   PREDICT  SIMULATE      0     1.68089   400.478  2667.77   1.65617  1989
  5   PREDICT  SIMULATE      0     2.06214   411.896  2705.16   1.01601  1990
```

Output 14.6.4. Listing of OUT= Data Set Created in the Second SOLVE Statement

```
                 General Form Equations for Supply-Demand Model
                            Price Unitcost Solution

 Obs   _TYPE_    _MODE_   _ERRORS_   price   quantity   income  unitcost  year

  1   PREDICT  SIMULATE      0     0.99284    371.4   2571.87   2.72857  1986
  2   PREDICT  SIMULATE      0     1.86594    416.5   2721.08   1.44798  1987
  3   PREDICT  SIMULATE      0     2.12230    597.3   3327.05   2.71130  1988
  4   PREDICT  SIMULATE      0     2.46166    764.1   3885.85   3.67395  1989
  5   PREDICT  SIMULATE      0     2.74831    694.3   3650.98   2.42576  1990
```

Example 14.7. Spring and Damper Continuous System

This model simulates the mechanical behavior of a spring and damper system shown in Figure 14.80.

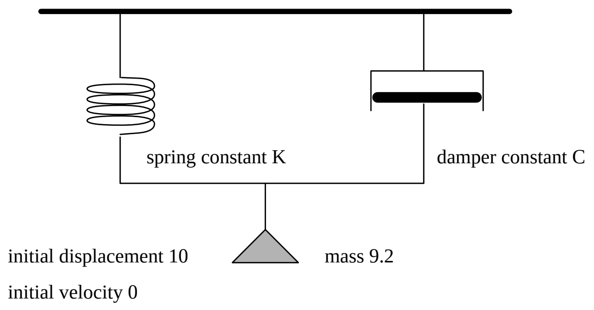


Figure 14.80. Spring and Damper System Model

A mass is hung from a spring with spring constant K. The motion is slowed by a damper with damper constant C. The damping force is proportional to the velocity, while the spring force is proportional to the displacement.

This is actually a continuous system; however, the behavior can be approximated by a discrete time model. We approximate the differential equation

$$\frac{\partial\, disp}{\partial\, time} = velocity$$

with the difference equation

$$\frac{\Delta\, disp}{\Delta\, time} = velocity$$

This is rewritten

$$\frac{disp - \mathrm{LAG(disp)}}{\mathrm{dt}} = velocity$$

where *dt* is the time step used. In PROC MODEL, this is expressed with the program statement

```
disp = lag(disp) + vel * dt;
```

This statement is simply a computing formula for Euler's approximation for the integral

$$disp = \int velocity\ dt$$

If the time step is small enough with respect to the changes in the system, the approximation is good. Although PROC MODEL does not have the variable step-size and error-monitoring features of simulators designed for continuous systems, the procedure is a good tool to use for less challenging continuous models.

This model is unusual because there are no exogenous variables, and endogenous data are not needed. Although you still need a SAS data set to count the simulation periods, no actual data are brought in.

Since the variables DISP and VEL are lagged, initial values specified in the VAR statement determine the starting state of the system. The mass, time step, spring constant, and damper constant are declared and initialized by a CONTROL statement.

```
title1 'Simulation of Spring-Mass-Damper System';

/*- Generate some obs. to drive the simulation time periods ---*/
data one;
   do n=1 to 100;
      output;
   end;
run;

proc model data=one;
   var      force -200  disp  10  vel  0  accel -20  time 0;
   control  mass   9.2  c    1.5  dt  .1  k      20;
   force = -k * disp -c * vel;
   disp  = lag(disp) + vel * dt;
   vel   = lag(vel) + accel * dt;
   accel = force / mass;
   time  = lag(time) + dt;
```

The displacement scale is zeroed at the point where the force of gravity is offset, so the acceleration of the gravity constant is omitted from the force equation. The control variable C and K represent the damper and the spring constants respectively.

The model is simulated three times, and the simulation results are written to output data sets. The first run uses the original initial conditions specified in the VAR statement. In the second run, the time step is reduced by half. Notice that the path of the displacement is close to the old path, indicating that the original time step is short enough to yield an accurate solution. In the third run, the initial displacement is doubled; the results show that the period of the motion is unaffected by the amplitude. These simulations are performed by the following statements:

```
/*- Simulate the model for the base case -------------------*/
   control run ’1’;
   solve / out=a;
run;

/*- Simulate the model with half the time step -------------*/
   control run ’2’ dt .05;
   solve / out=b;
run;

/*- Simulate the model with twice the initial displacement -*/
   control run ’3’;
   var disp 20;
   solve / out=c;
run;
```

The output SAS data sets containing the solution results are merged and the displacement time paths for the three simulations are plotted. The three runs are identified on the plot as 1, 2, and 3. The following code produces Output 14.7.1 through Output 14.7.2.

```
/*- Plot the results ---------------------------------------*/
data p;
   set a b c;
run;

title2 ’Overlay Plot of All Three Simulations’;
proc gplot data=p;
   plot disp*time=run;
run;
```

Output 14.7.1. Printed Output Produced by PROC MODEL SOLVE Statements

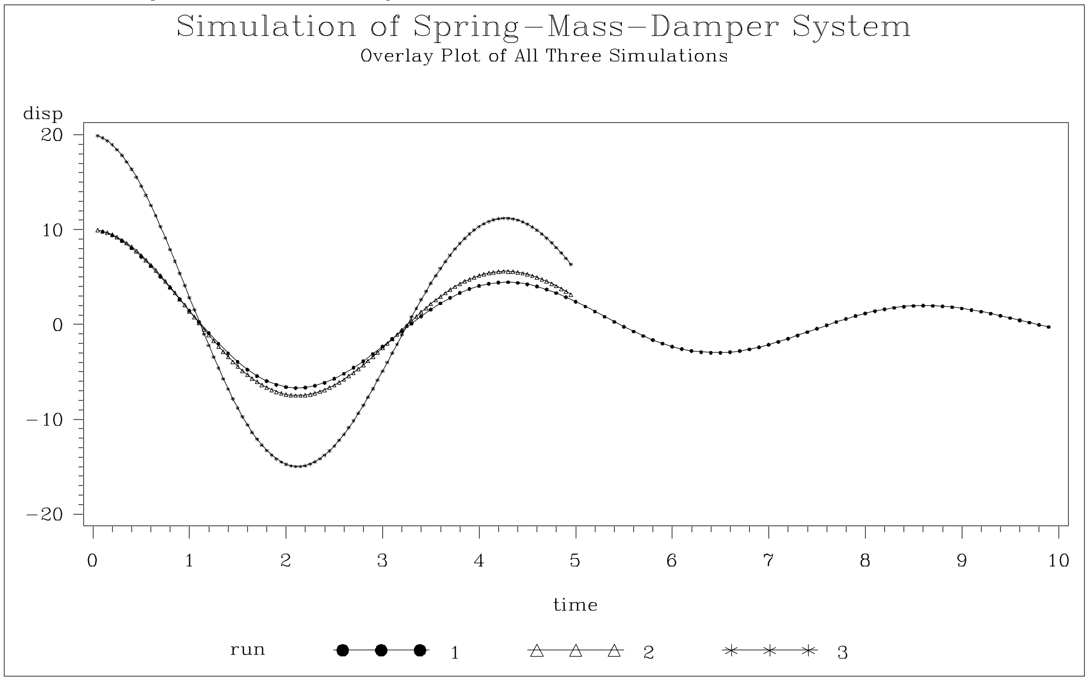
```
                    Simulation of Spring-Mass-Damper System

                              The MODEL Procedure

                                 Model Summary

                           Model Variables         5
                           Control Variables       5
                           Equations               5
                           Number of Statements    5
                           Program Lag Length      1


         Model Variables  force(-200) disp(10) vel(0) accel(-20) time(0)
       Control Variables  mass(9.2) c(1.5) dt(0.1) k(20) run(1)
               Equations  force disp vel accel time
```

```
                    Simulation of Spring-Mass-Damper System

                              The MODEL Procedure
                        Dynamic Simultaneous Simulation

                               Data Set Options

                                 DATA=    ONE
                                 OUT=     A


                               Solution Summary

                       Variables Solved               5
                       Simulation Lag Length          1
                       Solution Method           NEWTON
                       CONVERGE=                   1E-8
                       Maximum CC              8.68E-15
                       Maximum Iterations             1
                       Total Iterations              99
                       Average Iterations             1


                            Observations Processed

                                 Read      100
                                 Lagged      1
                                 Solved     99
                                 First       2
                                 Last      100


               Variables Solved For    force disp vel accel time
```

```
                    Simulation of Spring-Mass-Damper System

                              The MODEL Procedure
                        Dynamic Simultaneous Simulation

                                Data Set Options

                                  DATA=    ONE
                                  OUT=     B


                               Solution Summary

                        Variables Solved                5
                        Simulation Lag Length           1
                        Solution Method            NEWTON
                        CONVERGE=                    1E-8
                        Maximum CC               1.32E-15
                        Maximum Iterations              1
                        Total Iterations               99
                        Average Iterations              1


                             Observations Processed

                                 Read      100
                                 Lagged      1
                                 Solved     99
                                 First       2
                                 Last      100


                Variables Solved For    force disp vel accel time
```

```
                  Simulation of Spring-Mass-Damper System

                           The MODEL Procedure
                      Dynamic Simultaneous Simulation

                             Data Set Options

                              DATA=    ONE
                              OUT=     C


                             Solution Summary

                      Variables Solved               5
                      Simulation Lag Length          1
                      Solution Method           NEWTON
                      CONVERGE=                   1E-8
                      Maximum CC              3.93E-15
                      Maximum Iterations             1
                      Total Iterations              99
                      Average Iterations             1


                           Observations Processed

                             Read      100
                             Lagged      1
                             Solved     99
                             First       2
                             Last      100


              Variables Solved For    force disp vel accel time
```

Output 14.7.2. Overlay Plot of all Three Simulations



Example 14.8. Nonlinear FIML Estimation

The data and model for this example were obtained from Bard (1974, p.133-138). The example is a two-equation econometric model used by Bodkin and Klein to fit U.S production data for the years 1909-1949. The model is the following:

$$g_1 = c_1 10^{c_2 z_4}(c_5 z_1^{-c_4} + (1-c_5)z_2^{-c_4})^{-c_3/c_4} - z_3 = 0$$

$$g_2 = [c_5/(1-c_5)](z_1/z_2)^{(-1-c_4)} - z_5 = 0$$

where z_1 is capital input, z_2 is labor input, z_3 is real output, z_4 is time in years with 1929 as year zero, and z_5 is the ratio of price of capital services to wage scale. The c_i's are the unknown parameters. z_1 and z_2 are considered endogenous variables. A FIML estimation is performed.

```
data bodkin;
   input z1 z2 z3 z4 z5;
datalines;
1.33135 0.64629 0.4026 -20 0.24447
1.39235 0.66302 0.4084 -19 0.23454
1.41640 0.65272 0.4223 -18 0.23206
1.48773 0.67318 0.4389 -17 0.22291
1.51015 0.67720 0.4605 -16 0.22487
1.43385 0.65175 0.4445 -15 0.21879
1.48188 0.65570 0.4387 -14 0.23203
1.67115 0.71417 0.4999 -13 0.23828
1.71327 0.77524 0.5264 -12 0.26571
1.76412 0.79465 0.5793 -11 0.23410
1.76869 0.71607 0.5492 -10 0.22181
1.80776 0.70068 0.5052  -9 0.18157
1.54947 0.60764 0.4679  -8 0.22931
1.66933 0.67041 0.5283  -7 0.20595
1.93377 0.74091 0.5994  -6 0.19472
1.95460 0.71336 0.5964  -5 0.17981
2.11198 0.75159 0.6554  -4 0.18010
2.26266 0.78838 0.6851  -3 0.16933
2.33228 0.79600 0.6933  -2 0.16279
2.43980 0.80788 0.7061  -1 0.16906
2.58714 0.84547 0.7567   0 0.16239
2.54865 0.77232 0.6796   1 0.16103
2.26042 0.67880 0.6136   2 0.14456
1.91974 0.58529 0.5145   3 0.20079
1.80000 0.58065 0.5046   4 0.18307
1.86020 0.62007 0.5711   5 0.18352
1.88201 0.65575 0.6184   6 0.18847
1.97018 0.72433 0.7113   7 0.20415
2.08232 0.76838 0.7461   8 0.18847
1.94062 0.69806 0.6981   9 0.17800
1.98646 0.74679 0.7722  10 0.19979
2.07987 0.79083 0.8557  11 0.21115
2.28232 0.88462 0.9925  12 0.23453
```

```
2.52779 0.95750 1.0877  13 0.20937
2.62747 1.00285 1.1834  14 0.19843
2.61235 0.99329 1.2565  15 0.18898
2.52320 0.94857 1.2293  16 0.17203
2.44632 0.97853 1.1889  17 0.18140
2.56478 1.02591 1.2249  18 0.19431
2.64588 1.03760 1.2669  19 0.19492
2.69105 0.99669 1.2708  20 0.17912
;

proc model data=bodkin;
   parms c1-c5;
   endogenous z1 z2;
   exogenous z3 z4 z5;

   eq.g1 = c1 * 10 **(c2 * z4) * (c5*z1**(-c4)+
           (1-c5)*z2**(-c4))**(-c3/c4) - z3;
   eq.g2 = (c5/(1-c5))*(z1/z2)**(-1-c4) -z5;

   fit g1 g2 / fiml ;
run;
```

When FIML estimation is selected, the log likelihood of the system is output as the objective value. The results of the estimation are show in Output 14.8.1.

Output 14.8.1. FIML Estimation Results for U.S. Production Data

```
                              The MODEL Procedure

                    Nonlinear FIML Summary of Residual Errors

                  DF     DF                                               Adj
Equation       Model  Error        SSE       MSE  Root MSE  R-Square     R-Sq

g1                 4     37     0.0529   0.00143    0.0378
g2                 1     40     0.0173  0.000431    0.0208


                         Nonlinear FIML Parameter Estimates

                                         Approx                   Approx
        Parameter       Estimate        Std Err     t Value     Pr > |t|

        c1               0.58395         0.0218       26.76       <.0001
        c2              0.005877       0.000673        8.74       <.0001
        c3                1.3636         0.1148       11.87       <.0001
        c4              0.473688         0.2699        1.75       0.0873
        c5              0.446748         0.0596        7.49       <.0001


           Number of Observations       Statistics for System

           Used                 41    Log Likelihood     110.7773
           Missing               0
```

Example 14.9. Circuit Estimation

Consider the nonlinear circuit shown in Figure 14.81.

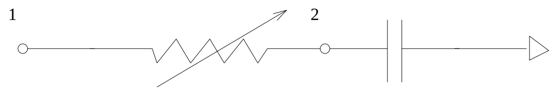


Figure 14.81. Nonlinear Resistor Capacitor Circuit

The theory of electric circuits is governed by Kirchhoff's laws: the sum of the currents flowing to a node is zero, and the net voltage drop around a closed loop is zero. In addition to Kirchhoff's laws, there are relationships between the current I through each element and the voltage drop V across the elements. For the circuit in Figure 14.81, the relationships are

$$C\frac{dV}{dt} = I$$

for the capacitor and

$$V = (R_1 + R_2(1 - \exp(-V)))I$$

for the nonlinear resistor. The following differential equation describes the current at node 2 as a function of time and voltage for this circuit:

label dvdt

$$C\frac{dV_2}{dt} - \frac{V_1 - V_2}{R_1 + R_2(1 - \exp(-V))} = 0$$

This equation can be written in the form

$$\frac{dV_2}{dt} = \frac{V_1 - V_2}{(R_1 + R_2(1 - \exp(-V)))C}$$

Consider the following data.

```
data circ;
   input v2 v1 time@@;
   datalines;
-0.00007 0.0 0.0000000001 0.00912 0.5 0.0000000002
 0.03091 1.0 0.0000000003 0.06419 1.5 0.0000000004
 0.11019 2.0 0.0000000005 0.16398 2.5 0.0000000006
 0.23048 3.0 0.0000000007 0.30529 3.5 0.0000000008
 0.39394 4.0 0.0000000009 0.49121 4.5 0.0000000010
 0.59476 5.0 0.0000000011 0.70285 5.0 0.0000000012
 0.81315 5.0 0.0000000013 0.90929 5.0 0.0000000014
 1.01412 5.0 0.0000000015 1.11386 5.0 0.0000000016
 1.21106 5.0 0.0000000017 1.30237 5.0 0.0000000018
 1.40461 5.0 0.0000000019 1.48624 5.0 0.0000000020
 1.57894 5.0 0.0000000021 1.66471 5.0 0.0000000022
;
```

You can estimate the parameters in the previous equation by using the following SAS statements:

```
proc model data=circ mintimestep=1.0e-23;
   parm R2 2000  R1 4000 C 5.0e-13;
   dert.v2 = (v1-v2)/((r1 + r2*(1-exp( -(v1-v2)))) * C);
   fit v2;
run;
```

The results of the estimation are shown in Output 14.9.1.

Output 14.9.1. Circuit Estimation

```
                              The MODEL Procedure

                       Nonlinear OLS Parameter Estimates

                                      Approx                     Approx
          Parameter       Estimate     Std Err     t Value     Pr > |t|

          R2              3002.465      1556.5        1.93       0.0688
          R1              4984.848      1504.9        3.31       0.0037
          C                  5E-13    1.01E-22     4.941E9       <.0001
```

Example 14.10. Systems of Differential Equations

The following is a simplified reaction scheme for the competitive inhibitors with recombinant human renin (Morelock et al. 1995).

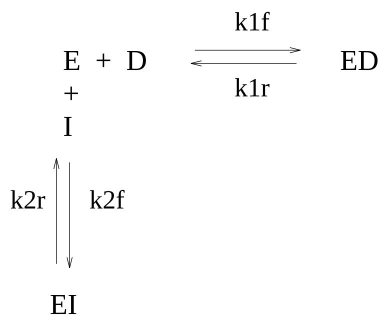


Figure 14.82. Competitive Inhibition of Recombinant Human Renin

In Figure 14.82, *E*= enzyme, *D*= probe, and *I*= inhibitor.

The differential equations describing this reaction scheme are

$$\frac{dD}{dt} = k1r{*}ED - k1f{*}E{*}D$$

$$\frac{dED}{dt} = k1f{*}E{*}D - k1r{*}ED$$

$$\frac{dE}{dt} = k1r{*}ED - k1f{*}E{*}D + k2r{*}EI - k2f{*}E{*}I$$

$$\frac{dEI}{dt} = k2f{*}E{*}I - k2r{*}EI$$

$$\frac{dI}{dt} = k2r{*}EI - k2f{*}E{*}I$$

For this system, the initial values for the concentrations are derived from equilibrium considerations (as a function of parameters) or are provided as known values.

The experiment used to collect the data was carried out in two ways; pre-incubation (type='disassoc') and no pre-incubation (type='assoc'). The data also contain repeated measurements. The data contain values for fluorescence F, which is a function of concentration. Since there are no direct data for the concentrations, all the differential equations are simulated dynamically.

The SAS statements used to fit this model are

```
proc model data=fit;

   parameters qf  = 2.1e8
              qb  = 4.0e9
              k2f = 1.8e5
              k2r = 2.1e-3
              l   = 0;

              k1f = 6.85e6;
              k1r = 3.43e-4;

      /* Initial values for concentrations */
   control dt 5.0e-7
           et 5.0e-8
           it 8.05e-6;

      /* Association initial values --------------*/
   if type = 'assoc' and time=0 then
      do;
         ed = 0;
            /* solve quadratic equation ----------*/
         a = 1;
         b = -(&it+&et+(k2r/k2f));
         c = &it*&et;
         ei = (-b-(((b**2)-(4*a*c))**.5))/(2*a);
         d = &dt-ed;
         i = &it-ei;
         e = &et-ed-ei;
      end;

      /* Disassociation initial values ----------*/
   if type = ’disassoc’ and time=0 then
      do;
         ei = 0;
         a = 1;
         b = -(&dt+&et+(&k1r/&k1f));
         c = &dt*&et;
         ed = (-b-(((b**2)-(4*a*c))**.5))/(2*a);
         d = &dt-ed;
         i = &it-ei;
         e = &et-ed-ei;
      end;

   if time ne 0 then
      do;
         dert.d = k1r* ed  - k1f *e *d;

         dert.ed = k1f* e *d - k1r*ed;

         dert.e = k1r* ed - k1f* e * d  + k2r * ei - k2f * e *i;

         dert.ei = k2f* e *i - k2r * ei;

         dert.i = k2r * ei - k2f* e *i;

      end;

      /* L - offset between curves  */
   if type = ’disassoc’ then
           F = (qf*(d-ed)) + (qb*ed) -L;
   else
           F = (qf*(d-ed)) + (qb*ed);

   Fit F / method=marquardt;
run;
```

This estimation requires the repeated simulation of a system of 42 differential equations (5 base differential equations and 36 differential equations to compute the partials with respect to the parameters).

The results of the estimation are shown in Output 14.10.1.

Output 14.10.1. Kinetics Estimation

```
                              The MODEL Procedure

                   Nonlinear OLS Summary of Residual Errors

                 DF      DF                                              Adj
Equation      Model   Error        SSE        MSE   Root MSE  R-Square     R-Sq

f                 5     797     2525.0     3.1681     1.7799    0.9980   0.9980


                       Nonlinear OLS Parameter Estimates

                                     Approx                      Approx
        Parameter       Estimate     Std Err    t Value     Pr > |t|

        qf              2.0413E8      681443     299.55       <.0001
        qb              4.2263E9     9133179     462.74       <.0001
        k2f              6451229      867011       7.44       <.0001
        k2r             0.007808     0.00103       7.55       <.0001
        l               -5.76981      0.4138     -13.94       <.0001
```

Example 14.11. Monte Carlo Simulation

This example illustrates how the form of the error in a ODE model affects the results from a static and dynamic estimation. The differential equation studied is

$$\frac{dy}{dt} = a - ay$$

The analytical solution to this differential equation is

$$y = 1 - \exp(-\mathrm{at})$$

The first data set contains errors that are strictly additive and independent. The data for this estimation are generated by the following DATA step:

```
data drive1;
   a = 0.5;
   do iter=1 to 100;
      do time = 0 to 50;
         y = 1 - exp(-a*time) + 0.1 *rannor(123);
         output;
      end;
   end;
```

The second data set contains errors that are cumulative in form.

```
data drive2;
   a = 0.5;
   yp = 1.0 + 0.01 *rannor(123);
   do iter=1 to 100;
```

```
            do time = 0 to 50;
               y = 1 - exp(-a)*(1 - yp);
               yp = y + 0.01 *rannor(123);
               output;
            end;
         end;
```

The following statements perform the 100 static estimations for each data set:

```
      proc model data=drive1 noprint;
         parm a 0.5;
         dert.y = a - a * y;
         fit y / outest=est;
         by iter;
      run;
```

Similar code is used to produce 100 dynamic estimations with a fixed and an unknown initial value. The first value in the data set is used to simulate an error in the initial value. The following PROC UNIVARIATE code processes the estimations:

```
      proc univariate data=est noprint;
         var a;
         output out=monte mean=mean p5=p5 p95=p95;
      run;

      proc print data=monte; run;
```

The results of these estimations are summarized in Table 14.4.

Table 14.4. Monte Carlo Summary, A=0.5

Estimation Type	Additive Error			Cumulative Error		
	mean	p95	p5	mean	p95	p5
static	0.77885	1.03524	0.54733	0.57863	1.16112	0.31334
dynamic fixed	0.48785	0.63273	0.37644	3.8546E24	8.88E10	-51.9249
dynamic unknown	0.48518	0.62452	0.36754	641704.51	1940.42	-25.6054

For this example model, it is evident that the static estimation is the least sensitive to misspecification.

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