Chapter 35
The LATTICE Procedure

Chapter Table of Contents

Chapter 35
The LATTICE Procedure

Overview

The LATTICE procedure computes the analysis of variance and analysis of simple covariance for data from an experiment with a lattice design. PROC LATTICE analyzes balanced square lattices, partially balanced square lattices, and some rectangular lattices.

In balanced square lattices, the number of treatments is equal to the square of the number of units per block. Incomplete blocks are grouped to form mutually orthogonal replications. The number of replicates in the basic plan is always 1 plus the number of units per block.

The partially balanced square lattices are similar to balanced lattices, although the number of replicates may vary. Partially balanced designs are constructed of the replicates in the basic plan, but not all replicates are included the same number of times, and some may not be included at all.

In rectangular lattices, there are k units per block and $k(k+1)$ treatments. As in square lattices, blocks are grouped to form mutually orthogonal replicates in the basic plan. PROC LATTICE can analyze simple rectangular lattices (two orthogonal replications) and triple rectangular lattices (three orthogonal replications). The experiment can include several repetitions of the basic plan.

The LATTICE procedure determines from the data set which type of design has been used. It also checks to see if the design is valid and displays an appropriate message if it is not.

Getting Started

An example of a balanced square design is an experiment to investigate the effects of nine diets on the growth rate of pigs.

In some breeds of pigs, past experience has shown that a large part of the total variation in growth rates between animals can be attributed to the litter. Therefore, this experiment is planned so that litter differences do not contribute to the intrablock error.

First, the pigs are separated into sets of three litter-mates. Each block is assigned two sets of the three litter-mates. In a given block, one pig from each set receives a diet. Therefore, the experimental unit is a pair of pigs feeding in a particular pen on one of the nine diets. The response variable, growth rate, is the sum of the growth rates for the two pigs in a particular pen. To get the adjusted diet mean per pig, the adjusted treatment mean for the pen must be divided by 2.

The special numeric SAS variables named Group, Block, Treatmnt, and Rep must be used to define the design. In this example, the Treatmnt variable ranges from 1 to 9 and indicates the particular diet. The Block variable is 1, 2, or 3 and indicates the pen containing the two pigs. The Group variable ranges from 1 to 4 and specifies which replication within the basic plan includes the experimental unit. In this example, you would not use the Rep variable since the entire basic plan is not replicated.

You can use the following DATA step and PROC LATTICE statement to analyze this experiment. The response variable is Weight.

```
title 'Examining the Growth Rate of Pigs';
data Pigs;
   input Group Block Treatmnt Weight @@;
   datalines;
1 1 1 2.20  1 1 2 1.84  1 1 3 2.18
1 2 4 2.05  1 2 5 0.85  1 2 6 1.86
1 3 7 0.73  1 3 8 1.60  1 3 9 1.76
2 1 1 1.19  2 1 4 1.20  2 1 7 1.15
2 2 2 2.26  2 2 5 1.07  2 2 8 1.45
2 3 3 2.12  2 3 6 2.03  2 3 9 1.63
3 1 1 1.81  3 1 5 1.16  3 1 9 1.11
3 2 2 1.76  3 2 6 2.16  3 2 7 1.80
3 3 3 1.71  3 3 4 1.57  3 3 8 1.13
4 1 1 1.77  4 1 6 1.57  4 1 8 1.43
4 2 2 1.50  4 2 4 1.60  4 2 9 1.42
4 3 3 2.04  4 3 5 0.93  4 3 7 1.78
;

proc lattice data=Pigs;
   var Weight;
run;
```

The SAS code produces the output shown in Figure 35.1.

```
                  Examining the Growth Rate of Pigs

                       The Lattice Procedure

                  Analysis of Variance for Weight

                                        Sum of        Mean
     Source                        DF    Squares      Square

     Replications                   3    0.07739      0.02580
     Blocks within Replications (Adj.)  8    1.4206       0.1776
        Component B                 8    1.4206       0.1776
     Treatments (Unadj.)            8    3.2261       0.4033
     Intra Block Error             16    1.2368       0.07730
     Randomized Complete Block Error  24    2.6574       0.1107
     Total                         35    5.9609       0.1703


                  Additional Statistics for Weight

             Description                       Value

             Variance of Means in Same Block    0.04593
             LSD at .01 Level                   0.6259
             LSD at .05 Level                   0.4543
             Efficiency Relative to RCBD        120.55
```

```
                  Examining the Growth Rate of Pigs

                       The Lattice Procedure

                       Adjusted Treatment
                        Means for Weight

                  Treatment        Mean

                       1         1.8035
                       2         1.7544
                       3         1.9643
                       4         1.7267
                       5         0.9393
                       6         1.8448
                       7         1.3870
                       8         1.4347
                       9         1.5004
```

Figure 35.1. Output from Example LATTICE Procedure

Diet 3 yields the highest mean growth rate at 1.9643 pounds for the two pigs (0.9822 per pig), while diet 5 has the lowest rate at 0.9393 (0.4696 per pig). The efficiency of the experiment relative to a randomized complete block design is 120.55 percent, so using the lattice design increased precision, producing more accurate estimates of the treatment effects. The different elements of the LATTICE procedure's output are discussed in the "Displayed Output" section on page 1752.

Syntax

The following statements are available in PROC LATTICE.

> **PROC LATTICE** < *options* >;
>   **BY** *variables* ;
>   **VAR** *variables* ;

Three specific numeric SAS variables, Group, Block, and Treatmnt, *must* be present in the data set to which PROC LATTICE is applied. A fourth numeric variable named Rep must be present when the design involves repetition of the entire basic plan. (See the "Input Data Set" section on page 1751 for more information.)

Every numeric variable other than Group, Block, Treatmnt, or Rep in the input SAS data set may be considered a response variable. A VAR statement tells PROC LATTICE that only the variables listed in the VAR statement are to be considered response variables. If the VAR statement is omitted, then all numeric variables, excluding Group, Block, Treatmnt, and Rep, are considered response variables. PROC LATTICE performs an analysis for each response variable.

PROC LATTICE Statement

> **PROC LATTICE** < *options* >;

You can specify the following options in the PROC LATTICE statement.

DATA=*SAS-data-set*
> names the SAS data set to be used by PROC LATTICE. If you omit the DATA= option, the most recently created SAS data set is used.

COVARIANCE
COV
> calculates sums of products for every possible pair of response variables. A sum of products is given for each source of variation in the analysis of variance table. For each pair of response variables, the one appearing later in the data set (or in the VAR statement) is the covariable.

VAR Statement

> **VAR** *variables* ;

The VAR statement specifies the response variables. If you do not include a VAR statement, all numeric variables in the data set are considered response variables (except Group, Block, Treatmnt, and Rep).

BY Statement

> **BY** *variables* **;**

You can specify a BY statement with PROC LATTICE to obtain separate analyses on observations in groups defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. The *variables* are one or more variables in the input data set.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data using the SORT procedure with a similar BY statement.

- Specify the BY statement option NOTSORTED or DESCENDING in the BY statement for the ANOVA procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.

- Create an index on the BY variables using the DATASETS procedure (in base SAS software).

For more information on the BY statement, refer to the discussion in *SAS Language Reference: Concepts*. For more information on the DATASETS procedure, refer to the discussion in the *SAS Procedures Guide*.

Details

Input Data Set

Four numeric SAS variables, Group, Block, Treatmnt, and Rep, are used in the input data set to define the lattice design. The Group, Block, and Treatmnt variables are required in the data set to which PROC LATTICE is applied. The Rep variable must be present when the design involves repetition of the entire basic plan.

Group
: specifies which orthogonal replication in the basic plan includes the experimental unit. Values of Group must be 1, 2, . . . , *n*, where *n* is the number of replicates in the basic plan.

Block
: specifies the block in which the experimental unit is present. Values of Block must be 1, 2, . . . , *m*, where *m* is the number of blocks in a replication.

Treatmnt
: specifies which treatment was applied to the experimental unit. Values of Treatmnt must be 1, 2, . . . , *i*, where *i* is the number of treatments in a replication.

Rep   specifies which repetition of the basic plan includes the experimental unit. Values of Rep must be 1, 2, ... , p, where p is the number of replications of the entire basic plan. Thus, the experiment has a total of *np* replicates.

Missing Values

If a value of Group, Block, Treatmnt, or Rep is missing, the analysis is not performed and an appropriate error message is displayed.

If a value of a response variable is missing, this entire variable is dropped from the analysis. If other response variables exist that do not have missing values, they are analyzed.

Displayed Output

For each response variable, PROC LATTICE displays

- an "Analysis of Variance" table and related statistics, including the following as separate sources of variations:

  - Replications
  - Blocks within Replications (adjusted for treatments)
  - Treatments (unadjusted)
  - Intra Block Error
  - Randomized Complete Block Error

  The Blocks within Replications sum of squares is further broken down into "Component A" and "Component B." If there is no repetition of the basic plan, the Component B sum of squares is the same as the Blocks within Replications sum of squares. If there is repetition of the basic plan, the Component A sum of squares reflects the variation among blocks that contain the same treatments.

  The source of variation called Randomized Complete Block Error is the sum of the Blocks within Replications sum of squares and the Intra Block Error sum of squares. It is the appropriate error term if the experimental design is a randomized complete block design, with the replications filling the roles of complete blocks.

- two values for the Variance of Means. For some lattice designs, these are only approximations. The first value is applicable when the two treatments appear in the same block; the other (when it appears) applies when the two treatments never appear in the same block (a possibility in partially balanced and rectangular designs).

- an Average of Variance. Except with small designs, it is sufficient to use this average variance of means for tests between treatments (whether the two treatments appear in the same block or not); refer to Cochran and Cox (1957).

- the Least Significant Differences (LSDs) at the .01 and .05 levels of significance, based on the Average of Variance

- Efficiency Relative to RCBD, the efficiency of the lattice design relative to a randomized complete block design. The efficiency is the ratio of the randomized complete block mean squared error to the effective error variance; refer to Cochran and Cox (1957).

- the Adjusted Treatment Means. These are adjusted for blocks if the relative precision is greater than 105 percent.

When you specify the COVARIANCE option, PROC LATTICE produces Sums of Products and the Mean Product for each source of variation in the analysis of variance table.

ODS Table Names

PROC LATTICE assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in the following table. For more information on ODS, see Chapter 15, "Using the Output Delivery System."

Table 35.1. ODS Tables Produced in PROC LATTICE

ODS Table Name	Description	PROC LATTICE Option
ANOVA	Analysis of variance	default
AdjTreatmentMeans	Adjusted treatment means	default
Statistics	Additional statistics	default

Example

Example 35.1. Analysis of Variance through PROC LATTICE

In the following example, from Cochran and Cox (1957, p. 406), the data are yields
(Yield) in bushels per acre of 25 varieties (Treatmnt) of soybeans. The data are
collected in two replications (Group) of 25 varieties in five blocks (Block) containing
five varieties each. This is an example of a partially balanced square lattice design.

```
data Soy;
  do Group = 1 to 2;
    do Block = 1 to 5;
      do Plot = 1 to 5;
          input Treatmnt Yield @@;
          output;
        end;
      end;
    end;
  drop Plot;
  datalines;
1    6    2    7    3    5    4    8    5    6
6    16   7    12   8    12   9    13   10   8
11   17   12   7    13   7    14   9    15   14
16 18     17   16   18   13   19   13   20   14
21   14   22   15   23   11   24   14   25   14
1    24   6    13   11   24   16   11   21   8
2    21   7    11   12   14   17   11   22   23
3    16   8    4    13   12   18   12   23   12
4    17   9    10   14   30   19   9    24   23
5    15   10   15   15   22   20   16   25   19
;

proc print data=Soy;
   id Treatmnt;
run;

proc lattice data=Soy;
run;
```

The results from these statements are shown in Output 35.1.1 and Output 35.1.2.

Example 35.1.   Analysis of Variance through PROC LATTICE ⬩ 1755

Output 35.1.1. Displayed Output from PROC PRINT

Treatmnt	Group	Block	Yield
1	1	1	6
2	1	1	7
3	1	1	5
4	1	1	8
5	1	1	6
6	1	2	16
7	1	2	12
8	1	2	12
9	1	2	13
10	1	2	8
11	1	3	17
12	1	3	7
13	1	3	7
14	1	3	9
15	1	3	14
16	1	4	18
17	1	4	16
18	1	4	13
19	1	4	13
20	1	4	14
21	1	5	14
22	1	5	15
23	1	5	11
24	1	5	14
25	1	5	14
1	2	1	24
6	2	1	13
11	2	1	24
16	2	1	11
21	2	1	8
2	2	2	21
7	2	2	11
12	2	2	14
17	2	2	11
22	2	2	23
3	2	3	16
8	2	3	4
13	2	3	12
18	2	3	12
23	2	3	12
4	2	4	17
9	2	4	10
14	2	4	30
19	2	4	9
24	2	4	23
5	2	5	15
10	2	5	15
15	2	5	22
20	2	5	16
25	2	5	19

Output 35.1.2. Displayed Output from PROC LATTICE

```
                    The Lattice Procedure

               Analysis of Variance for Yield

                                        Sum of        Mean
Source                           DF     Squares      Square

Replications                      1     212.18       212.18
Blocks within Replications (Adj.) 8     501.84       62.7300
   Component B                    8     501.84       62.7300
Treatments (Unadj.)              24     559.28       23.3033
Intra Block Error                16     218.48       13.6550
Randomized Complete Block Error  24     720.32       30.0133
Total                            49    1491.78       30.4445


               Additional Statistics for Yield

        Description                        Value

        Variance of Means in Same Block    15.7915
        Variance of Means in Different Bloc 17.9280
        Average of Variance                17.2159
        LSD at .01 Level                   12.1189
        LSD at .05 Level                    8.7959
        Efficiency Relative to RCBD        174.34
```

```
                    The Lattice Procedure

                   Adjusted Treatment
                    Means for Yield

                 Treatment      Mean

                     1       19.0681
                     2       16.9728
                     3       14.6463
                     4       14.7687
                     5       12.8470
                     6       13.1701
                     7        9.0748
                     8        6.7483
                     9        8.3707
                    10        8.4489
                    11       23.5511
                    12       12.4558
                    13       12.6293
                    14       20.7517
                    15       19.3299
                    16       12.6224
                    17       10.5272
                    18       10.7007
                    19        7.3231
                    20       11.4013
                    21       11.6259
                    22       18.5306
                    23       12.2041
                    24       17.3265
                    25       15.4048
```

The efficiency of the experiment relative to a randomized complete block design is 174.34 percent. Precision is gained using the lattice design via the recovery of intrablock error information, enabling more accurate estimates of the treatment effects. Variety 8 of soybean had the lowest adjusted treatment mean (6.7483 bushels per acre), while variety 11 of soybean had the highest adjusted treatment mean (23.5511 bushels per acre).

References

Cochran, W.G. and Cox, G.M. (1957), *Experimental Designs*, Second Edition, New York: John Wiley & Sons, Inc.

Comstock, R.E., Peterson, W.J., and Stewart, H.A. (1948), "An Application of the Balanced Lattice Design in a Feeding Trial with Swine," *Journal of Animal Science*, 7, 320–331.

Cornelius, P.L. (1983), "Lattice Designs," *Encyclopedia of Statistical Sciences*, 4, 510–518.

Robinson, H.F. and Watson, G.S. (1949), "Analysis of Simple and Triple Rectangular Designs," *North Carolina Agricultural Experiment Station Technical Bulletin*, 88.