1. Find the general solutions of the following differential equations. Where initial conditions are given also solve the particular initial value problem.

(a) \( y'' + 5y' + 4y = 0 \)

(b) \( y'' + 4y' + 20y = 0 \) with \( y(0) = -2, \ y'(0) = 4 \)

2. Answer the following questions about linear, first order differential equations.

(a) Write down the general form for a first-order, linear differential equation.

(b) Write down the property that characterizes an integrating factor, \( \mu(x) \), for a first order, linear differential equation and explain how this property allows the first order linear differential equation to be solved.

3. Consider the following differential equation.

\[
y' = \frac{x^2}{y^2 + 1}
\]

First, use our basic existence result to show that there is a unique solution of the differential equation through any point \((a, b)\). Then solve the differential equation using separation of variables.

4. At time \( t = 0 \), a thermometer reading 68°F is placed in a water bath whose temperature is maintained at 100°F. After 2 minutes in the bath, the thermometer reads 84°F. Use Newton’s law of cooling to set up and solve a differential equation that models the temperature reading of the thermometer as a function of time. Also, determine the value of the constant \( k \).

5. Consider the following differential equation.

\[
y' + \frac{1}{x + 2}y = \frac{2}{(x + 2)^2}
\]

(a) Find the general solution to this differential equation. You may assume \( x > -2 \).

(b) Use your general solution to solve the IVP consisting of the differential equation above and the initial condition \( y(0) = 5 \).

6. Consider the following DE.

\[
y'' - 6y' + 9y = q(x)
\]

(a) Find a general solution of the homogeneous equation.

(b) If \( q(x) = 11x^2e^{3x} \), write down the form for the particular solution. Do not solve for the coefficients.

(c) If \( q(x) = e^{2x} \), find the particular solution.