1. Find the general solutions of the following differential equations. Where initial conditions are
given also solve the particular initial value problem.

(a) \( y'' + 2y' + 10y = 0 \)
(b) \( y'' + 8y' + 16y = 0 \) with \( y(0) = 3, \ y'(0) = -5 \)

2. Answer the following questions about linear, first order differential equations.

(a) Write down the general form for a first-order, linear differential equation.
(b) Solve the homogeneous first-order, linear differential equation by separation of variables.

3. Describe our basic existence and uniqueness theorem for a first order differential equation
\( y' = F(x, y) \).

4. A tank initially contains 300 gallons of brine containing 60 pounds of dissolved salt. Brine with
a concentration of 0.1 pounds per gallon flows into the tank at a rate of 3 gallons per minute
and brine flows out at the same rate. State and solve an initial value problem for the amount of
salt in the tank as a function of time. At what time will the amount of salt in the tank reach 40
pounds?

5. Consider the following differential equation.

\[ y' + \frac{4}{x}y = \frac{8e^{-2x}}{x^4} \]

(a) Find the general solution to this differential equation. You may assume \( x > 0 \).
(b) Use your general solution to solve the IVP consisting of the differential equation above and
the initial condition \( y(1) = 6 \). What is the interval of existence for your solution?

6. Consider the following DE.

\[ y'' + 5y' + 4y = q(x) \]

(a) Find a general solution of the homogeneous equation.
(b) If \( q(x) = 12xe^{-4x} \), write down the form for the particular solution. Do not solve for the
coefficients.
(c) If \( q(x) = x^2 \), find the particular solution.