1. Consider the following Initial Value Problem for a damped spring-mass model.

\[ y'' + 5y' + 4y = 0, \quad y(0) = 1, y'(0) = -6 \]

(a) Find the roots of the characteristic equation and use them to classify this system as overdamped, underdamped, or critically damped.

(b) Find the solution to the IVP. If there is a value of \( t > 0 \) so that \( y(t) = 0 \), find that value of \( t \).

2. Consider the following forced spring-mass model. In your work on this problem, you may use the formulas presented in class or in the text.

\[ y'' + 2y' + 16y = \cos(\omega t) \]

(a) Explain what is meant by resonance in reference to the forced, damped, spring-mass model.

(b) Show that resonance occurs in this model and find the value of \( \omega \) that maximizes the amplitude of the forced response.

3. Answer the following questions.

(a) Suppose that \( \phi_1(x) \) and \( \phi_2(x) \) are two solutions to the linear second order differential equation \( y'' + p_1(x)y' + p_0(x)y = 0 \). If you know that the Wronskian \( W(\phi_1, \phi_2) \) is not equal to zero at \( x = 1 \) explain why this means that \( c_1\phi_1 + c_2\phi_2 \) is a general solution to the differential equation. You may assume that \( p_1 \) and \( p_2 \) are continuous for all \( x \) so that a unique solution to any IVP is guaranteed.

(b) Suppose that \( L_1 \) and \( L_2 \) are linear operators. If the composition \( L_1 \circ L_2 \) is defined by \( (L_1 \circ L_2)[f] = L_1[L_2[f]] \), show that \( L_1 \circ L_2 \) is a linear operator.

4. Consider the real, constant coefficient linear differential operator given below.

\[ L(D) = (D^2 + 4D + 20)^2 (D - 3)^2 (D + 1)^3 \]

(a) Find a general solution to the homogeneous differential equation \( L(D)y = 0 \).

(b) Write down the form for a particular solution to the non-homogeneous equation

\[ L(D) = (D^2 + 4D + 20)^2 (D - 3)^2 (D + 1)^3 \cdot 24x^3 e^{-x} \]

Do not try to solve for the coefficients! Make sure you explain your work clearly. Your particular solution must not contain any terms that are solutions of the homogeneous equation.

5. Find a particular solution of \( (D - 1)^2(D^2 + 1)y = 12xe^x \).