Show all work needed to reach your answers.

1. (10 points) Please compute the Wronskian for \( \{ \sin ax, \cos ax \} \) where \( a \) is a constant. Are these functions linearly dependent or linearly independent?

\[
W(\sin ax, \cos ax) = \begin{vmatrix}
\sin ax & \cos ax \\
a \cos ax & -a \sin ax
\end{vmatrix}
\]

\[
= -a \sin^2 ax - a \cos^2 ax = -a
\]

\[
W[\sin ax, \cos ax] = -a
\]

Linearly Dependent or Linearly Independent? 

2. (15 points) Which of the following set of the functions are linearly independent (LI) and which are linearly dependent (LD)? Please write LI in front of the sets that are linearly independent, and LD in front of the others. You do not need to compute the Wronskians or justify your answers.

(a) \( \text{L.D.} \) \( \{1, \cos^2 x, \sin^2 x\} \) on the interval \( (-\infty, \infty) \)

\[
1(1) - 1(\cos^2 x) - 1(\sin^2 x) = 0
\]

(b) \( \text{L.D.} \) \( \{x, 0, 2x + 1\} \) on the interval \( (-\infty, \infty) \)

(c) \( \text{L.I.} \) \( \{x^k, x^m, x^n\} \) on the interval \( (-\infty, \infty) \) when \( k, m, n \) are distinct integers.

(d) \( \text{L.D.} \) \( \{x, |x|\} \) on the interval \( (0, 1) \)

\[
|\sin 0, 1| = 1
\]

(e) \( \text{L.I.} \) \( \{x, |x|\} \) on the interval \( (-1, 1) \)