1. **10 points** A failure-time distribution has hazard function

\[ h(t) = \beta/(t + \gamma) + \delta t, \text{ where } \beta > 0, \gamma > 0, \text{ and } \delta > 0. \]

(a) What kind(s) of shape(s) is this hazard function capable of (e.g., increasing, decreasing, bathtub, other)? Prove your assertion(s).

(b) Tell which parameter values \( \beta, \gamma, \text{ and } \delta \) result in each shape identified in (a). Prove your assertion(s).

(c) Obtain the survival function, cumulative hazard function, cumulative distribution function, and probability density function for this failure-time distribution.

2. **30 points** A factorial experiment was conducted to evaluate the life of a type of glass capacitor as a function of voltage and operating temperature (degrees C). For each of two levels of temperature and four of voltage, four capacitors were run until failure. The test was stopped at 1000 hours. The data are in the data set t2q2 at the course web site.

(a) Fit a linear regression model to the logged lifetimes, with predictors (1) centered temperature, (2) centered voltage, and (3) their product. You may have to do this for more than one failure time distribution.

(b) Analyze the residuals of the fits to determine an appropriate distribution to use for maximum likelihood estimation. Obtain a final maximum likelihood fit for the chosen distribution.

(c) Assess the quality of fit of the chosen model.

(d) Conduct a likelihood ratio test to determine the significance of temperature and the voltage-temperature interaction.

(e) If these predictors are not found significant, fit an appropriate reduced model, and assess the fit.

(f) Interpret the final model.

3. **40 points** A survival study enrolled 73 individuals with acute lymphoma. Of these, survival times were obtained for 60 and censored times for 13. The data are in the data set t2q3, found at the course web site.

(a) Determine if an exponential distribution is appropriate for these data. Whether or not you conclude the exponential distribution is appropriate, base your analysis in parts (b) and (c) on the exponential distribution.

(b) Assume the observations were obtained by terminating the study after the sixty-sixth death.
   
   i. Obtain an appropriate small-sample 95% confidence interval for \( \theta \), the mean of the exponential distribution. Interpret this interval in terms a physician can understand.
   
   ii. Translate this interval into a confidence interval for the probability of surviving at least one (non-leap) year. Interpret this interval in terms a physician can understand.

(c) Assume the observations were obtained by terminating the study after day 1855.
   
   i. Obtain an appropriate small-sample 95% confidence interval for \( \theta \), the mean of the exponential distribution. Interpret this interval in terms a physician can understand.
   
   ii. Compare this result with the interval in (b).

4. **20 points** Another survival study enrolled 30 individuals with cervical cancer. Sixteen were assigned at random to the standard therapy and the rest to a new therapy. The data, which consist of days of survival or a censored time, are found in the data set t2q4 on the course web site.

(a) Use an appropriate nonparametric procedure to estimate the survivor functions for the two groups. Graph and compare these functions.

(b) Use one or more nonparametric tests to test whether the survival functions are the same. What do you conclude?