1. (20 points) Consider the implication $A \Rightarrow B$ where $A$ and $B$ are themselves statements. For this implication, please state the following:

(a) contrapositive:

(b) negation:

(c) direct implication:

(d) converse:

2. (20 points) **Theorem**: If a sequence $\{a_n\} \subseteq \mathbb{R}$ converges, then the limit is unique.

**Proof**: Suppose that $a_n \to a$ for some $a \in \mathbb{R}$ and that $a_n \to \alpha$ for some $\alpha \in \mathbb{R}$. To prove that the limit is unique, one must show that 

$\alpha = a$. So, given any $\epsilon > 0$, there exists $N$ such that $|a_n - \alpha| < \epsilon/2$ and also $|a_n - a| < \epsilon/2$ for all $n > N$. By the triangle inequality,

$|a - \alpha| \leq \epsilon$

Since $\epsilon$ is arbitrarily small, $\ldots$ and thus the limit is unique.
3. (15 points) Let $f : A \to B$ be a function, and let $S \subseteq B$ and $T \subseteq B$. Please prove that $f^{-1}(S \cap T) \subseteq f^{-1}(S) \cap f^{-1}(T)$.

4. (15 points) Please show that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

**Hint:** Induction.
5. (15 points) Please negate the following statement about a sequence \( \{a_n\} \) and its limit \( a \):

\[
\forall N \in \mathbb{Z}^+, \exists \epsilon > 0 \text{ s.t. } |a_n - a| < \epsilon \forall n > N
\]

6. (15 points) Consider the sum of any six consecutive positive integers. What can be said about the divisors (factors) of this sum? Please prove your answer.