Show all work needed to reach your answers.

1. (18 points) Several short-answer questions:
   
   (a) What do each of the following symbols mean? (Please write meaning after symbol.)
   \[\exists \quad \forall \quad \exists!\]

   (b) What is the meaning of \(\aleph_0\) (aleph naught)?

   (c) If \(x \in \mathcal{P}(A)\) what is the relationship between \(x\) and \(A\)? (\(\mathcal{P}(A)\) is the power set of \(A\).)

2. (17 points) Suppose that \(U\) is some universal set, and suppose that \(A, B \subset U\). Please prove that \(A^c \cap B \subset (A \cup B^c)^c\).
3. (20 points) Which of the following sequences (a) must converge, (b) which must diverge, and (c) which might converge or might diverge?

(a) A decreasing sequence of positive real numbers.

(b) An oscillating sequence of real numbers whose oscillation amplitude decreases to 1.

(c) An increasing sequence of real numbers.

(d) A decreasing sequence of rational numbers whose greatest lower bound is a negative real number.

(e) A sequence of real numbers which is bounded both above and below.

4. (15 points) Suppose that the sum of the digits of a number \( n \in \mathbb{Z} \) is divisible by 9. Please show that \( 9 | n \). Hint: If \( n \) has \( N + 1 \) digits, let

\[
n = a_N a_{N-1} \cdots a_2 a_1 a_0 = \sum_{k=0}^{N} a_k 10^k
\]
5. (15 points) Please give the contrapositive of the following statement:

If $x > 0$ but $y < 0$, then $z = 0$ or $q \in \mathbb{Q}$.

6. (15 points) A graph which is the union of disjoint trees is sometimes called a forest. Suppose that $T := \{T_1, T_2, \ldots, T_k, \ldots\}$ be a set of trees, and suppose that

$$F := \bigcup_{k=1}^{[T]} T_k$$

is such a forest (here $[T]$ be the number of trees in the forest). Let $|V_k|$ and $|E_k|$ be the numbers of vertices and edges in $T_k$, the $k$-th tree; let $|V|$ and $|E|$ be the total numbers of vertices and edges overall in the forest. Please find a formula relating $|T|$, $|V|$ and $|E|$. Also starting from the Euler formula, please prove that your formula is correct.