Exercise Set II

1. Let $A$ and $B$ be subsets of a universe $U$. Please prove the second De Morgan’s law:

$$(A \cap B)^c = A^c \cup B^c$$

2. Prove that if $A$, $B$ and $C$ are sets, and if $A \subset B$ and $B \subset C$, then $A \subset C$.

3. If $U := [0, 10]$, $A := [3, 7)$ and $B := \{3, 6, 9\}$, then what are $A_U^c$, $A_R^c$ and $B_U^c$?

4. Let $A$ and $B$ be sets. Please prove or disprove:

$\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

Hint: Counterexample

5. Prove that for each $n \in \mathbb{Z}^+$,

1. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

2. $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

6. Please find two distinct proofs that for any $n \in \mathbb{Z}^+$, then 6 divides $n^3 - n$, that is, $6|(n^3 - n)$.

7. Suppose $A$ and $B$ are sets with $A \subset B$. Given the standard definition of $A_B^c$, use the axioms to show that this complement exists.

8. In terms of axiomatic set theory, please explain why a “set” containing all sets is not a set.

9. Is $\emptyset$ the same as $\{\emptyset\}$? Explain why or why not. Hint: Cardinality.

10. Please construct on the basis of the axioms a set containing exactly three elements.