1. (10 points) Your friend is a something-not-mathematics major, and tells you that it is possible to count the real numbers in \([0, 1]\) by writing them in binary form (so each number \(x_j \in [0, 1]\) is of the form \(0.x_{j1}x_{j2}x_{j3}...\) where \(x_{jk}\) is either 0 or 1). Your friend gives the count as

\[
\begin{align*}
x_1 &= 0.x_{11}x_{12}x_{13}x_{14}... \\
x_2 &= 0.x_{21}x_{22}x_{23}x_{24}... \\
x_3 &= 0.x_{31}x_{32}x_{33}x_{34}... \\
x_4 &= 0.x_{41}x_{42}x_{43}x_{44}... \\
\end{align*}
\]

and so forth

Is your friend correct or wrong? Please explain why.

2. (10 points) By the greatest lower bound property of \(\mathbb{R}\) (the real numbers), a decreasing sequence of real numbers that is bounded below must converge to a limit \(L\) which is also a real number. Does a decreasing sequence of rational numbers always converge to a rational number? Please give a counterexample to show that \(\mathbb{Q}\) (the rational numbers) has no such greatest lower bound property. Hint: Think about the examples we have discussed in class and the homework exercises.